

ACTM Regional Math Contest
Pre-Calculus/Trigonometry 2009

Select the best answer for each of the following questions and mark it on the answer sheet provided. Be sure to read all of the answer choices before making your selection. When you are finished with the multiple-choice, attempt the tiebreaker questions.

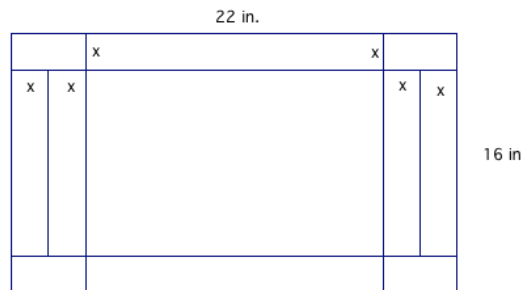
1. Determine the domain for $f(x) = \sqrt{25 - x^2}$

- a. $(-\infty, -5] \cup [5, \infty)$ b. $[-5, 5]$ c. $(-5, 5)$ d. $(-\infty, -5) \cup (5, \infty)$

2. An air freight company has determined that its cost of delivering x parcels per flight is $C(x) = 875 + 0.75x$. The price it charges to send a parcel is \$12.00 per parcel. Determine the profit function.

- a. $P(x) = 11.25x + 875$ b. $P(x) = 12x - 875$
c. $P(x) = 11.25x - 875$ d. $P(x) = 12x$

3. An open box is to be constructed from a rectangular sheet of cardboard that measures 16 inches by 22 inches. It is made by cutting along the long side of the rectangles that measure x inches by $2x$ inches at each corner of the rectangle. What value x (to the nearest 0.001 inch) will produce a box with maximum volume?



- a. 2.137 b. 2.367 c. 2.037 d. 2.223

4. Solve for x . Give exact answer without using a calculator: $\log(\log x) = 3$

- a. 10^3 b. 10^{100} c. 10^{10} d. 10^{1000}

5. The scrap value S of a product with an expected life span of n years is given by $S(n) = P(1 - r)^n$ where P is the original price of the product and r is the annual rate of depreciation. A taxicab is purchased for \$12,400 and is expected to last 3 years. What is its scrap value if it depreciates at a rate of 29% per year?

- a. \$5684.32 b. \$4438.10 c. \$4826.34 d. \$5224.53

6. Find the exact value of $\sec 150^\circ$

- a. $\frac{2}{\sqrt{3}}$ b. 2 c. -2 d. $-\frac{2}{\sqrt{3}}$

7. A car with a 16-inch radius wheel is moving with a speed of 50 mph. Find the angular speed (to the nearest radian per second) of the wheel in radians per second.

- a. 45 radians/sec b. 55 radians/sec c. 65 radians/sec d. 35 radians/sec

8. Given $\sin \phi = -\frac{\sqrt{3}}{2}$, $180^\circ < \phi < 270^\circ$ find the exact value of $\tan \phi$

- a. $\sqrt{3}$ b. $-\sqrt{3}$ c. $\frac{1}{\sqrt{3}}$ d. $-\frac{1}{\sqrt{3}}$

9. Find the exact value of $\tan\left(67\frac{1}{2}\right)^\circ$

- a. $\sqrt{3} + 1$ b. $\sqrt{2} + 1$ c. $\sqrt{2} - 1$ d. $\sqrt{3} - 1$

10. Write $2\sin 3x \cos 3x$ as a single trigonometric function.

- a. $2\sin 3x$ b. $\sin 3x$ c. $\sin 6x$ d. $2\sin 6x$

11. Solve $4\sin^2 x + 2\sqrt{3}\sin x - 2\sin x - \sqrt{3} = 0$ for x , where $0^\circ \leq x < 360^\circ$

- a. $60^\circ, 120^\circ, 210^\circ, 330^\circ$ b. $60^\circ, 150^\circ, 210^\circ, 300^\circ$
c. $30^\circ, 150^\circ, 240^\circ, 300^\circ$ d. $30^\circ, 120^\circ, 210^\circ, 330^\circ$

12. Find the magnitude and direction of $u = -2i + 3j$

- a. 3.6, 123.7° b. 2.5, 127.3° c. 4.5, 123.7° d. 2.6, 123.7°

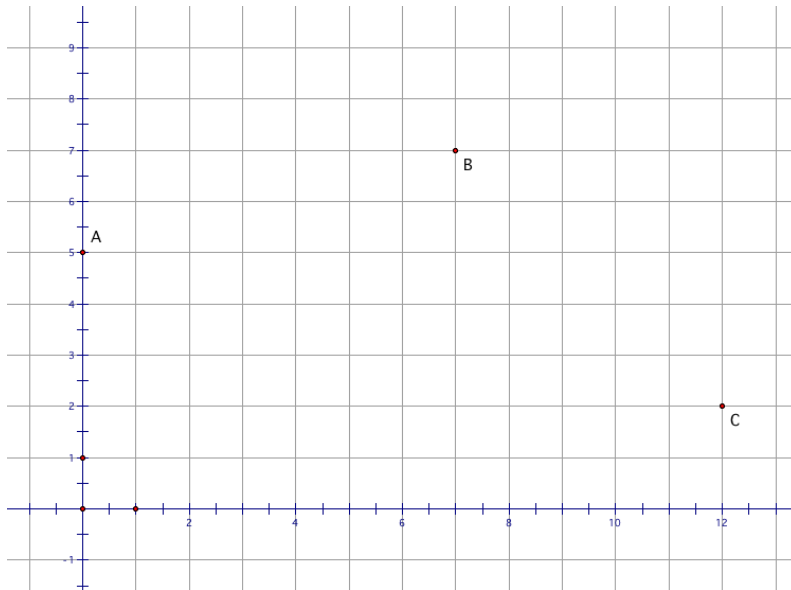
13. A plane is flying at an airspeed of 400 mph at a heading of 204° . A wind of 45 mph is blowing from the east. Find the ground speed of the plane.

- a. 410 mph b. 440 mph c. 420 mph d. 430 mph

14. Change $r = \frac{4}{1 - \cos \theta}$ to a rectangular equation.

- a. $x^2 = 8y + 16$ b. $y^2 = 8x + 16$ c. $y^2 = x + 2$ d. $x^2 = y + 2$

15. A ball is bounced on the floor and passes through the points A(0,5), B(7,7) and C(12,2) as seen in the figure below. Find the function that best models the bouncing ball.

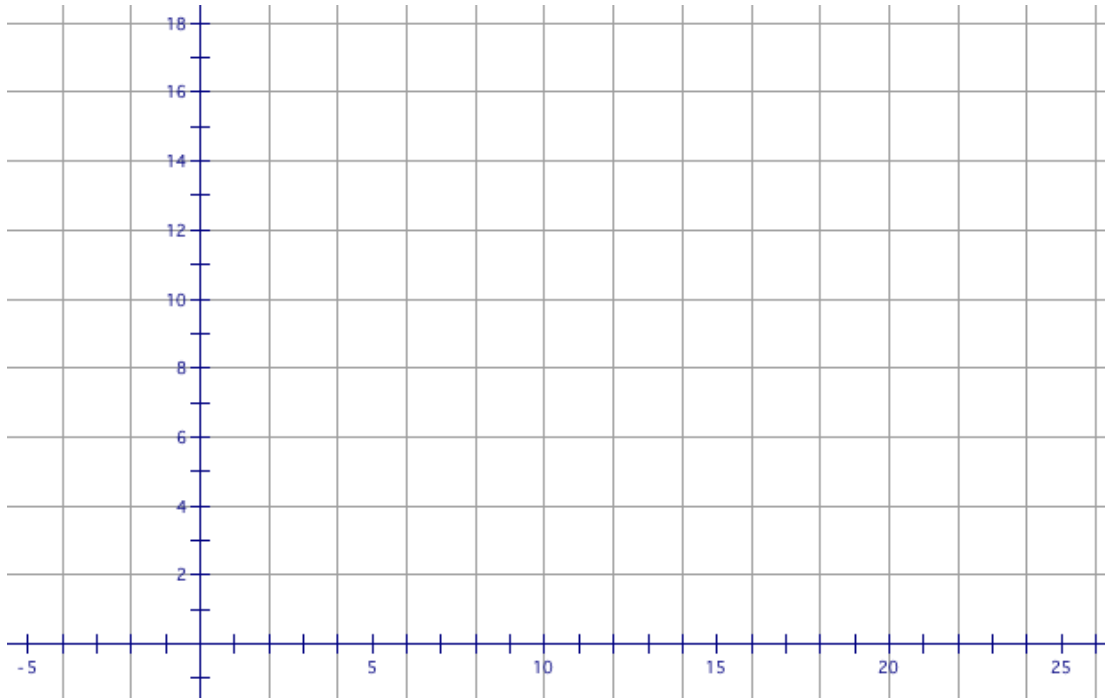


- a. $f(x) = \frac{3}{28}x^2 + \frac{29}{28}x - 5$ b. $f(x) = -\frac{3}{28}x^2 + \frac{29}{28}x + 5$
c. $f(x) = -\frac{3}{28}x^2 - \frac{29}{28}x + 5$ d. $f(x) = -\frac{3}{28}x^2 - \frac{29}{28}x + 5$

16. Given the objective function $P(x) = 6x + 3y$, and the constraints

$$\begin{cases} 5x + 2y \geq 20 \\ x + y \geq 7 \\ x + 2y \geq 10 \\ x \leq 15, y \leq 15 \end{cases}$$

Minimize the objective function.



a. $(4,3)$, $P = 33$

b. $(2.5,3.5)$, $P = 25.5$

c. $(0,0)$, $P = 0$

d. $(2,5)$, $P = 27$

17. Find the sum of $\sum_{i=1}^{10} \left(\frac{2}{3}\right)^i$

a. 1.9653

b. 0.01734

c. 0.6667

d. 1.3695

18. Use the binomial theorem to expand $(4a - b)^5$

a. $1024a^5 + 1280a^4b + 640a^3b^2 + 160a^2b^3 + 20ab^4 - b^5$

b. $1024a^5 - 1280a^4b - 640a^3b^2 - 160a^2b^3 - 20ab^4 + b^5$

c. $1024a^5 - 1280a^4b + 640a^3b^2 - 160a^2b^3 + 20ab^4 - b^5$

d. $1024a^5 + 1280a^4b - 640a^3b^2 + 160a^2b^3 - 20ab^4 + b^5$

19. A tree casts a shadow of 8.55 feet when the angle of elevation of the sun is 55.3° . Find the height of the tree.

- a. 25.6 feet b. 7.1 feet c. 4.9 feet d. 12.3 feet

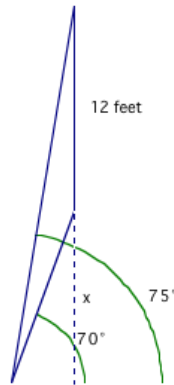
20. If $f(x) = x^2 + 4x$ and $g(x) = x - 8$, find $(f \circ g)(x)$

- a. $x^2 + 4x - 8$ b. $x^2 - 12x + 32$ c. $x^2 + 3x - 8$ d. $x^2 + 12x - 32$

21. Solve the quadratic equation $(2x - 5)^2 - (4x + 11)^2 = 0$

- a. $x = \{-8, -1\}$ b. $x = \{8, 1\}$ c. $x = \{8, -1\}$ d. $x = \{-8, 1\}$

22. A surveying team determines the height of a hill by placing a 12-foot pole at the top of the hill and measuring the angles of elevation to the bottom and the top of the pole. They find the angles of elevation as shown. Find the height of the hill.



- a. 33 feet b. 15 feet c. 23 feet d. 35 feet

23. Find the area of acres in a pasture whose shape is a triangle measuring 800 feet by 1020 feet by 680 feet. (An acre is 43,570 square feet)

- a. 7.65 acres b. 5.57 acres c. 4.76 acres d. 6.23 acres

24. The distance from Saturn to the sun at Saturn's aphelion is 934.34 million miles, and the distance from Saturn to the sun at its perihelion is 835.14 million miles. Find an equation of the orbit of Saturn.

a. $\frac{x^2}{883.35^2} + \frac{y^2}{884.74^2} = 1$

b. $\frac{x^2}{884.74^2} + \frac{y^2}{883.35^2} = 1$

c. $\frac{x^2}{883.35^2} - \frac{y^2}{884.74^2} = 1$

d. $\frac{x^2}{884.74^2} - \frac{y^2}{883.35^2} = 1$

25. Find the inverse of $f(x) = \frac{-3x}{x+4}$

a. $f^{-1}(x) = \frac{4x}{x-3}$

b. $f^{-1}(x) = \frac{4x}{x+3}$

c. $f^{-1}(x) = \frac{-4x}{x+3}$

d. $f^{-1}(x) = \frac{-4x}{x-3}$

ACTM Regional PreCalculus/Trigonometry

Name _____

Tiebreaker #1

Find the exact value of the equation $2^{2-x} = 3^{2x+1}$. Write your answer in terms of natural logarithms.

Name _____

Tiebreaker #2

Find a polynomial of degree 4 that has real coefficients and zeros $2i$ and $3 - 7i$ of the form $ax^4 + bx^3 + cx^2 + dx + e$

Name _____

Tiebreaker #3

Graph at least one period for $y = -2 \cos\left(\pi x + \frac{\pi}{2}\right) + 1$ finding the amplitude, period, phase shift and vertical shift.

Solutions for ACTM Regional PreCalculus/Trigonometry

1. b
2. c
3. a
4. d
5. b
6. d
7. b
8. a
9. b
10. c
11. c
12. a
13. c
14. b
15. b
16. d
17. a
18. c
19. d
20. b
21. a
22. a
23. d
24. b
25. c

Tiebreakers

1. $x = \frac{2\ln 2 - \ln 3}{2\ln 3 + \ln 2}$

2. $x^4 - 6x^3 + 62x^2 - 24x + 232$

3. Amplitude = 2 Period = 2 Phase Shift = $-\pi/2$ Vertical Shift = 1 unit up

