

ACTM Regional Pre-Calculus/Trig Exam
March 3, 2012

Place the letter of the correct answer in the space provided as well as on the answer sheet. Make sure you attempt the tie-breaker questions at the end of the test if you have time.

_____ 1. The equation $(\log_3 x)^2 - 4\log_3 x + 3 = 0$ has two roots r_1 and r_2 . Determine sum of these two roots $r_1 + r_2$.

- a.) -4 b.) 3 c.) 12 d.) 30

_____ 2. Determine $\sum_{n=0}^{100} \left(\frac{1}{2}\right)^n$.

- a.) 2 b.) $2 - \frac{1}{2^{100}}$ c.) $2 - \frac{1}{2^{101}}$ d.) $2 + \frac{1}{2^{100}}$

_____ 3. The minute hand on a clock is twice as long as the hour hand. If it is 3:00 PM, then how far apart are the tips of hands on the clock if the hour hand is 1cm long?

- a.) $\sqrt{3}$ cm b.) $\sqrt{5}$ cm c.) 2 cm d.) $2\sqrt{2}$ cm

_____ 4. The domain of the function $f(x) = \frac{\ln(x)}{x^2 - 4}$ is

- a.) $(0, \infty)$ b.) $(-\infty, \infty)$ c.) $(0, 2) \cup (2, \infty)$ d.) $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

_____ 5. Determine all vertical asymptotes of $f(x) = \frac{3x+6}{x^2+5x+6}$.

- a.) $x = -3$ and $x = -2$ b.) $x = 3$ c.) $x = 1$ and $x = 3$ d.) $x = -3$

_____ 6. Find the inverse of $f(x) = \ln(x^5)$.

- a.) $f^{-1}(x) = e^{-5x}$ b.) $f^{-1}(x) = e^{5x}$ c.) $f^{-1}(x) = e^{x^5}$ d.) $f^{-1}(x) = e^{\frac{x}{5}}$

- _____ 7. The following equation $\log_2(x^2) + \log_2(4) = 2$ has two solutions. The sum of these two solutions is given by
- a.) 1 b.) 0 c.) -2 d.) 2
- _____ 8. Determine the $\cos\left(\tan^{-1}\left(\frac{x}{2}\right)\right)$.
- a.) $\frac{1}{2}$ b.) $\frac{x}{\sqrt{x^2+4}}$ c.) $\frac{2}{\sqrt{x^2+4}}$ d.) $\frac{x}{x+2}$
- _____ 9. Determine the rectangular form of the polar equation $r = 5 \sec \theta$
- a.) $x^2 + y^2 = 25$ b.) $x = 5$ c.) $x^2 + y^2 = 5$ d.) $x + y = 5$
- _____ 10. Determine the radius of the circle $x^2 + y^2 - 2x + 6y + 1 = 0$
- a.) 3 b.) 8 c.) $2\sqrt{2}$ d.) 1
- _____ 11. Determine the rectangular coordinates that correspond to the polar coordinates $\left(-2, \frac{5\pi}{6}\right)$.
- a.) $(\sqrt{3}, -1)$ b.) $(-\sqrt{3}, 1)$ c.) $(\sqrt{3}, 1)$ d.) $(-\sqrt{3}, -1)$
- _____ 12. Two planes take off at the same time separated by 30 degrees. Plane A is travelling at 200 mph while plane B is traveling at 300 mph. Approximately how far apart are the planes after two hours?
- a.) 361 miles b.) 721 miles c.) 500 miles d.) 323 miles
- _____ 13. The line $y = x - 1$ intersects $3x^2 + y^2 = 1$ at two points. Determine the area of the triangle made by these two points and the point $\left(\frac{1}{2}, -1\right)$.
- a.) 1 b.) $\frac{1}{8}$ c.) $\frac{1}{2}$ d.) $\frac{1}{4}$

_____14. The sum of all the coefficients in $(3x-2)^6$ is

- a.) -64 b.) 15625 c.) 64 d.) 1

_____15. Determine $\sum_{n=0}^{\infty} \cos^n\left(\frac{\pi}{4}\right)$.

- a.) $\sqrt{2}$ b.) $2(2+\sqrt{2})$ c.) $1+\frac{\sqrt{2}}{2}$ d.) $2+\sqrt{2}$

_____16. Determine the function that is even.

- a.) $f(x) = \cos x - \sin^3 x$ b.) $f(x) = \sin x - \cos x$
 c.) $f(x) = \tan x \sin x$ d.) $f(x) = \csc x \sec x$

_____17. Let x be an angle and y its coterminal angle. Determine $\cos(y-x)$.

- a.) 0 b.) 1 c.) -1 d.) $\frac{1}{2}$

_____18. Which of the following is a horizontal asymptote of $f(x) = \tan^{-1}(x)$

- a.) $y = 0$ b.) $y = 1$
 c.) $y = \frac{\pi}{4}$ d.) $y = \frac{\pi}{2}$

_____19. The equation $8\sin^3 x - 1 = 0$ has one solution between 0 and $\frac{\pi}{2}$. Determine the cotangent of this solution.

- a.) 8 b.) 1 c.) $\sqrt{3}$ d.) $\frac{\sqrt{3}}{3}$

_____20. Solve the inequality and write the final answer in interval notation: $|x^2 - 6| < x$.

- a.) $(-\infty, -\sqrt{6}) \cup (\sqrt{6}, \infty)$ b.) (2, 3) c.) (2, ∞) d.) $(\sqrt{6}, \infty)$

_____ 21. If $\sin x = \frac{2}{3}$ and $\tan x < 0$, what is the value of $\sec x$?

- a.) $\frac{-3\sqrt{5}}{5}$ b.) $\frac{3\sqrt{5}}{5}$ c.) $\frac{-\sqrt{5}}{3}$ d.) $\frac{\sqrt{5}}{3}$

_____ 22. If the minute hand of a clock is 5 inches long, how far does the tip travel in 20 minutes?

- a.) 25π in b.) $\frac{25\pi}{2}$ in c.) $\frac{10\pi}{3}$ in d.) $\frac{5\pi}{3}$ in

_____ 23. Determine the period of $y = \tan\left(2x + \frac{\pi}{3}\right)$

- a.) $\frac{\pi}{3}$ b.) $\frac{\pi}{2}$ c.) π d.) 2π

_____ 24. Determine the sum of the infinite series

$$4 - \frac{4}{3} + \frac{4}{9} - \frac{4}{27} + \dots$$

- a.) 6 b.) 4 c.) $\frac{7}{2}$ d.) 3

_____ 25. Determine the vertex of the parabola passing through (0, -12), (1, -15) and (6, 0).

- a.) (-3, 10) b.) (2, -16) c.) (3, -15) d.) (-2, 0)

TIEBREAKERS

Name _____

Show all your work to receive maximum credit.

1. A bridge is to be built over a river. A surveyor placed pole A on one side of the river and pole B on the other side of the river. Pole C is placed 100 feet from pole A. At pole C, the surveyor measured the angle between pole B and pole A to be 68 degrees. At Pole A, the surveyor determined the angle between pole C and pole B to be 110 degrees. What is the distance between pole A and pole B to the nearest foot?

Name _____

2. X is deposited in an account which has an annual interest rate of 10%. At the end of each year \$1000 is withdrawn. After five years the account has a balance of zero. Determine X to the nearest cent.

Name _____

3. An equilateral triangular tube open at both ends is made by folding a rectangular piece of cardboard two times and taping the ends together. If the perimeter of the cardboard is 12 feet, then determine the length and width of the cardboard that will lead to the largest volume.

ANSWERS

1. D
2. B
3. B
4. C
5. D
6. D
7. B
8. C
9. B
10. A
11. A
12. D
13. B
14. D
15. D
16. C
17. B
18. D
19. C
20. B
21. A
22. C
23. B
24. D
25. B

Tiebreaker 1

Distance between pole A and pole B is 2,657 feet.

Tiebreaker 2

$X = \$3,790.79$

Tiebreaker 3

4 feet and 2 feet