

ACTM Regional Math Contest  
Pre-Calculus/Trigonometry 2010

Select the best answer for each of the following questions and mark it on the answer sheet provided. Be sure to read all of the answer choices before making your selection. When you are finished with the multiple-choice, attempt the tiebreaker questions.

1. A child pulls a sled along level ground by exerting a force of 30 pounds on a rope that makes an angle of  $35^\circ$  with the ground. Approximately how much work is done pulling the sled 200 feet?

- a. 3915 ft-pounds    b. 4915 ft-pounds    c. 5915 ft-pounds    d. 4595 ft-pounds

2. Simplify the exponential expression  $(2x^2y^3)(3x^5y)$

- a.  $5x^{10}y^4$                       b.  $6x^{10}y^3$                       c.  $6x^7y^4$                       d.  $5x^7y^4$

3. Rationalize the denominator:  $\frac{3}{5+\sqrt{3}}$

- a.  $\frac{15-3\sqrt{3}}{22}$                       b.  $\frac{15+3\sqrt{3}}{22}$                       c.  $\frac{15+3\sqrt{3}}{28}$                       d.  $\frac{15-3\sqrt{3}}{28}$

4. Find b for  $\frac{\sin 52^\circ}{b} = \frac{\sin 30^\circ}{12}$

- a. 0.05                      b. 4.73                      c. 20.8                      d. 18.91

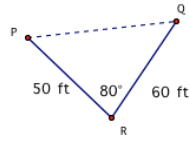
5. Find the exact value in quadrant III of  $\tan\left[\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$

- a.  $\sqrt{3}$                       b.  $-\sqrt{3}$                       c.  $\frac{1}{-\sqrt{3}}$                       d.  $\frac{1}{\sqrt{3}}$

6. A tower that is 125 feet tall casts a shadow of 172 feet. Find the angle of elevation of the sun to the nearest degree.

- a.  $40^\circ$                       b.  $37^\circ$                       c.  $36^\circ$                       d.  $41^\circ$

7. Two homes are located on opposite sides of a small hill at points P and Q. To measure the distance between them, a surveyor walks the distance of 50 feet from house P to point R, uses a transit to measure  $\angle PRQ$ , which is found to be  $80^\circ$ , and then walks to house Q, a distance of 60 feet. How far apart are the houses?



- a. 71.12 ft      b. 72.11 ft      c. 81.12 ft      d. 82.11 ft

8. Perform the indicated operations and simplify:  $\frac{4x + 8}{9x^3} \div \frac{x^2 - 4}{3x}$

- a.  $\frac{4(x + 2)^2(x - 2)}{27x^4}$       b.  $\frac{4}{3x^2(x - 2)}$       c.  $\frac{4x}{x - 2}$       d.  $\frac{4}{x^2(x - 2)}$

9. Write in terms of simplest logarithmic form:  $\log_b \left( \frac{x^4 y^2}{z^5} \right)$

- a.  $\frac{(4 \log_b x)(2 \log_b y)}{(5 \log_b z)}$       b.  $4 \log_b x + 2 \log_b y - 5 \log_b z$   
 c.  $\log_b x^4 + \log_b y^2 - \log_b z^5$       d.  $P_1 = (3, \sqrt{2})$

10. Find  $\begin{vmatrix} 0 & 2 & 1 \\ 3 & -1 & 2 \\ 4 & -4 & 1 \end{vmatrix}$

- a. 2      b. 9      c. 6      d. -3

11. Find the work done by a force of 5 pounds acting in the direction  $60^\circ$  to the horizontal in moving an object 20 feet from (0,0) to (20,0)

- a. 70 ft-lb      b. 60 ft-lb      c. 40 ft-lb      d. 50 ft-lb

12. Write the following expression in standard form  $a + bi$ :  $2(\cos 60^\circ + i \sin 60^\circ)$

- a.  $1 + i\sqrt{3}$       b.  $1 - i\sqrt{3}$       c.  $\sqrt{3} - i$       d.  $\sqrt{3} + i$

13. Which of the following is a solution to  $e^{2x} - 4e^x + 3 = 0$  for  $x$  in exact terms?

- a.  $\ln 3$       b.  $\ln 4$       c.  $\ln 2$       d.  $\ln 5$

14. Change  $r = \frac{2}{3 + 2\cos\theta}$  to a rectangular equation.

- a.  $9x^2 + 5y^2 + 8x - 4 = 0$       b.  $9x^2 - 5y^2 - 8x - 4 = 0$   
c.  $9x^2 - 5y^2 - 8x + 4 = 0$       d.  $5x^2 + 9y^2 + 8x - 4 = 0$

15. A gasoline station has weekly costs and revenue (the money taken in by the station) that are functions of the number of gallons of gasoline purchased and sold. If  $x$  gallons are purchased and sold, weekly costs are given by  $C(x) = 1.2x + 1080$  and weekly revenue by  $R(x) = 1.6x$ . How many gallons of gasoline must be sold weekly for the station to break even?

- a. 2500 gallons      b. 3700 gallons.      c. 2700 gallons      d. 3500 gallons

16. Find the product  $AB$  if  $A = \begin{bmatrix} 1 & -1 & 4 \\ 4 & -1 & 3 \\ 2 & 0 & -1 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 4 \\ 1 & -1 & 3 \end{bmatrix}$

- a.  $\begin{bmatrix} 2 & -2 & 4 \\ 5 & -1 & 5 \\ 0 & 4 & -6 \end{bmatrix}$       b.  $\begin{bmatrix} 4 & -5 & 8 \\ 5 & -1 & 5 \\ 4 & -2 & 0 \end{bmatrix}$       c.  $\begin{bmatrix} 4 & -5 & 8 \\ 4 & -4 & 2 \\ 1 & -1 & 0 \end{bmatrix}$       d.  $\begin{bmatrix} 4 & -5 & 8 \\ 6 & -1 & 5 \\ 1 & 3 & -3 \end{bmatrix}$

17. Find the exact value of  $\frac{\sin 50^\circ}{\cos 40^\circ}$

- a.  $\sqrt{2}$       b.  $\sqrt{3}$       c.  $-\sqrt{2}$       d. 1

18. Given  $f(x) = \frac{2}{1 + 2x^2}$  and  $g(x) = 3x$ . Find  $(f \circ g)(2)$

- a.  $\frac{73}{2}$       b.  $\frac{2}{3}$       c.  $\frac{2}{73}$       d.  $\frac{3}{2}$

19. A special window in the shape of a rectangle with two semi-circles at each end is to be constructed so that the outside dimensions are 100 feet in length. Find the dimensions of the rectangle that maximizes its area.

- a.  $50 \text{ ft} \times \frac{25}{\pi}$       b.  $50 \text{ ft} \times \frac{\pi}{25}$       c.  $25 \text{ ft} \times \frac{\pi}{50}$       d.  $25 \text{ ft} \times \frac{50}{\pi}$

20. Solve the inequality and write your final answer in interval notation:  $\frac{x+5}{x-3} \leq 0$

- a.  $(-\infty, -5] \cup [3, \infty)$       b.  $(-\infty, -5] \cup (3, \infty)$       c.  $[-5, 3)$       d.  $[-5, 3]$

21. Evaluate  $\frac{12!}{4!8!}$

- a. 1      b. 490      c. 3960      d. 495

22. A radioactive substance has a half-life of 40 days. There are initially 900 grams of the substance. Use the decay model  $A = A_0 e^{kt}$  to determine how much of the substance will remain after 10 days.

- a. 759.3 grams      b. 795.3 grams      c. 856.7 grams      d. 826.3 grams

23. Find the magnitude and direction of  $u = -2i + 3j$

- a. 3.6,  $56.3^\circ$       b. 2.5,  $127.3^\circ$       c. 4.5,  $123.7^\circ$       d. 3.6,  $123.7^\circ$

24. Solve for x:  $\sqrt{2x+4} - \sqrt{x+3} - 1 = 0$

- a.  $x = \{-2, 6\}$       b.  $x = 6$       c.  $x = -2$       d. no solution

25. Find all vertical asymptotes for the graph of  $f$  if  $f(x) = \frac{2x+5}{x^2-3x-18}$

- a.  $x = -3, x = 6$       b.  $y = 3, y = -6$       c.  $x = 3, x = -6$       d.  $y = -3, y = 6$

Name: \_\_\_\_\_ School: \_\_\_\_\_

ACTM Regional Exam PreCalculus/Trigonometry

Tiebreaker #1

A semielliptical archway over a one-way road has a height of 10 feet and a width of 40 feet. Your truck has a width of 10 feet and a height of 9 feet. Determine the height of the archway and show that your truck will clear the archway.

Name: \_\_\_\_\_ School: \_\_\_\_\_

Tiebreaker #2

A cake removed from the oven has a temperature of  $210^{\circ}\text{F}$ . It is left to cool in a room that has a temperature of  $70^{\circ}\text{F}$ . After 30 minutes, the temperature of the cake is  $140^{\circ}\text{F}$ . What is the temperature of the cake after 40 minutes?

Name: \_\_\_\_\_ School: \_\_\_\_\_

Tiebreaker #3

Two commuters drove to work a distance of 40 miles and then returned again on the same route. The average velocity on the return trip was 30 miles per hour faster than the average velocity on the outgoing trip. Express the total time required to complete the round trip,  $T$ , as a function of the average velocity on the outgoing trip,  $x$ .

## Solutions for ACTM Regional PreCalculus/Trigonometry

1. b
2. c
3. a
4. d
5. a
6. c
7. a
8. b
9. b
10. a
11. d
12. a
13. a
14. d
15. c
16. d
17. d
18. c
19. d
20. c
21. d
22. a
23. d
24. b
25. a

### Tiebreakers

1. Height is approximately 9.68 feet.

2. 126°F

3.  $T(x) = \frac{40}{x} + \frac{40}{x+30}$