Arkansas Council of Teachers of Mathematics
2012 State Competition
Trigonometry/Precalculus Exam

For questions 1 through 25, mark your answer choice on the answer sheet provided. After completing items 1 through 25, answer each of the tiebreaker items in sequential order (do #1 first, followed by #2, and then #3 last). Be sure that your name is printed on each of the tiebreaker pages. Congratulations for being selected to participate in the ACTM State Contest!

1. Determine the number of vertical and horizontal asymptotes of $f(x)=\dfrac{x^2-2x}{x^3-4x}$.

a.) 4 b.) 2 c.) 3 d.) 1 e.) 5

2. A theatre can seat 3000 people. At the current ticket price of $4.25, the theatre fills 1025 seats. The owner knows that he will lose 3 seats for each additional 30 cent increase in ticket price. Determine the largest possible revenue the theatre can generate from ticket sales. You should round your answer to the nearest hundred dollars.

a.) $\$28,500$
b.) $\$4,400$
c.) $\$12,800$
d.) $\$32,300$
e.) $\$23,700$

3. Find the inverse of $f(x)=e^{-2x}$.

a.) $f^{-1}(x)=\ln(2x)$ b.) $f^{-1}(x)=\ln(\sqrt{x})$

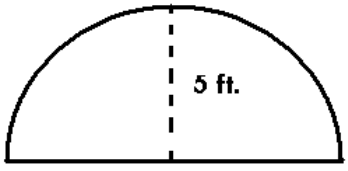
c.) $f^{-1}(x)=\ln\left(\dfrac{x}{2}\right)$ d.) $f^{-1}(x)=\ln\left(\dfrac{1}{\sqrt{x}}\right)$ e.) $f^{-1}(x)=\ln\left(x^2\right)$

4. Determine the foci of the ellipse given by $9(x-2)^2+25y^2=225$.

a.) $(-2,0)$ and $(6,0)$
b.) $(-4,0)$ and $(4,0)$
c.) $(-5,0)$ and $(5,0)$
d.) $(2,4)$ and $(-2,4)$
e.) $(-3,0)$ and $(3,0)$

5. A semicircular window with a height of five feet was constructed. What is the width of the window two feet down from its top?

a.) $\sqrt{21}\,ft.$
b.) $10\,ft.$
c.) $4\,ft.$
d.) $2\sqrt{21}\,ft.$
e.) $8\,ft.$



6. The solution to inequality $\left|x^2-4\right|<1$ in interval notation is given by $(a,b)\cup(c,d)$ with $a<b<c<d$. Determine $\dfrac{a+b}{c+d}$.

a.) 0 b.) -2 c.) 1 d.) 2 e.) -1

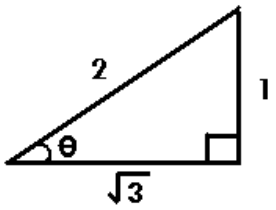
7. Determine the $\sin\left(\tan^{-1}\left(\dfrac{x}{3}\right)\right)$.

a.) $\dfrac{1}{3}$ b.) $\dfrac{x}{\sqrt{x^2+9}}$ c.) $\dfrac{3}{\sqrt{x^2+9}}$ d.) $\dfrac{x}{x+3}$ e.) $\dfrac{\sqrt{x^2+9}}{3}$

8. Consider the right triangle below. Find θ in radians and determine $1+\theta+\theta^2+\theta^3+\ldots$.

a.) $\dfrac{3}{3-\pi}$ b) $1-\dfrac{\pi}{6}$ c) $\dfrac{6}{6-\pi}$

d) 3π e) $\dfrac{1}{1-\pi}$



9. Let x be an angle and y its coterminal angle. Determine $\tan(y-x)$.

a.) 0 b.) 1 c.) -1 d.) $\dfrac{\sqrt{3}}{3}$ e.) $\sqrt{3}$

10. Determine the function that is odd.

a.) $f(x)=(\cos^2 x)(\sin^2 x)$
b.) $f(x)=\sin x-\cos x$
c.) $f(x)=\cos x\sin x$
d.) $f(x)=\csc^2 x\sec x$
e.) $f(x)=\sec x$

11. If $\tan x = -\frac{1}{3}$ and $\cos x > 0$, what is the value of $\csc x$?

a.) $\frac{-\sqrt{10}}{10}$ b.) $\frac{3\sqrt{10}}{10}$ c.) $\frac{-3\sqrt{10}}{10}$ d.) $\frac{1}{3}$ e.) $-\sqrt{10}$

12. Determine the rectangular coordinates that correspond to the polar coordinates $\left(4, -\frac{5\pi}{6}\right)$.

a.) $\left(2\sqrt{3}, -2\right)$

b.) $\left(-2\sqrt{3}, 2\right)$

c.) $\left(2\sqrt{3}, 2\right)$

d.) $\left(-2\sqrt{3}, -2\right)$

e.) $\left(-\sqrt{3}, -2\right)$

13. Determine the rectangular form of the polar equation $r = \tan\theta\sec\theta$

a.) $x^2 + y^2 = 1$

b.) $y = x$

c.) $y = -x$

d.) $y = \frac{1}{x}$

e.) $y = x^2$

14. Suppose the longest side of a triangle is twice as long as the shortest side. If the angle opposite the longest side is 120 degrees, then determine the tangent of the angle opposite to the shortest side.

a.) $\frac{1}{2}$ b.) $\frac{\sqrt{3}}{2}$ c.) $\frac{\sqrt{39}}{13}$ d.) $\frac{\sqrt{13}}{3}$ e.) $\frac{\sqrt{53}}{3}$

15. Determine the number of times the line $y = -x$ intersects $r = \theta$ for $0 \leq \theta \leq 2\pi$.

a.) 0 b.) 1 c.) 2 d.) 3 e) 4

16. A horse is pulling a sled with a force of 100 pounds. How much work is done in moving the sled 300 feet if the rope makes an angle of 15 degrees with the ground? You should round your answer to nearest foot-pound.

a.) $30,000$ ft-lb
b.) 78 ft-lb
c.) $25,456$ ft-lb
d.) 80 ft-lb
e.) $28,978$ ft-lb

17. Let $f(x) = x^2 - 8x + 12$ and let $g(x) = x^2 - x$. The equation $(f \circ g)(x) = 0$ has four solutions r_1, r_2, r_3, and r_4. Determine $r_1 + r_2 + r_3 + r_4$.

a.) 8
b.) 2
c.) 4
d.) 6
e.) 9

18. The domain of the function $f(x) = \dfrac{\ln(4 - x^2)}{x - 1}$ is given by $(a, b) \cup (c, d)$ with $a \le b \le c \le d$. Determine $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$.

a.) 0 b.) 1 c.) –5 d.) –3 e) –4

19. Solve the equation $\log_b(x^2 b^3) = 5$ for x in terms of b. It turns out that there are two possible solutions. The product of these two solutions is given by

a.) b b.) 0 c.) b^2 d.) 1 e) $-b^2$

20. The population on island A is 12,500 people, island B is 5,800 people, and island C is 700 people. The population is decreasing by 7.5% each year on A, increasing by 5% each year on island B, and increasing by 2.7% each year on island C. Determine how long it will take for the population on island A to be the same as the combined populations of islands B and C. To the nearest hundred people, the population of island A at this time is given by

a.) 7400 b.) 8300 c.) 6700 d.) 9100 e.) 8400

21. Determine the radius of the circle $x^2 + y^2 - 2ax + 2ab = b^2$

a.) $|a-b|$ b.) $|b|$ c.) $|a+b|$ d.) b^2 e.) a^2

22. You deposit 100 dollars into an account earning 0.1% per week. At the end of the first week you deposit enough to double the current amount in the account, at the end of the second week you deposit again enough to double the current amount in the account, and so on. Let A_n represent the amount in the account at the beginning of the nth week. Determine A_8. You should round your answer to the nearest dollar.

a.) $\$12,890$
b.) $\$25,780$
c.) $\$6,438$
d.) $\$12,903$
e.) $\$25,806$

23. Determine $\sum_{n=0}^{\infty} \cos^{2n}\left(\frac{\pi}{8}\right)$.

a.) $\sqrt{2}$ b) $2(2+\sqrt{2})$ c) $1+\frac{\sqrt{2}}{2}$ d) $2+\sqrt{2}$ e) 2

24. The function $f(x) = sinx - cosx$ can be written as $y = Asin(x - \frac{\pi}{a})$. Determine the possible product aA^2.

a.) 2
b.) 16
c.) 32
d.) 8
e.) 4

25. Consider all the solutions $x_1, x_2, \ldots, x_n$ of the equation $8\sin^{2013} x - \tan^{2013} x = x$ between -2π and 2π. Determine $cos(\sum_{i=1}^{n} x_i)$.

a.) 1 b.) $\frac{1}{2}$ c.) $\frac{\sqrt{3}}{2}$ d.) 0 e.) $-\frac{1}{2}$

Name_____ **School**_____

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Tiebreaker Questions

Your solutions should be written clearly. All work leading to your final answer must be included. The questions will be used in sequential order to resolve ties for first, second, and/or third place.

Tiebreaker #1.

The minute hand on a clock is twice as long as the hour hand. If it is 12:30 PM, then how far apart are the tips of hands on the clock if the hour hand is 1cm long? You should round your answer to two decimal places.

Tiebreaker #2.

A piece of wire 10 meters long is cut into two pieces. The left piece is bent into an equilateral triangle and the other is bent into a circle. How far from the left end of the wire should it be cut so that the total area enclosed is a minimum? You should round your answer to two decimal places.

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Tiebreaker #3.

X is deposited in an account which has an annual interest rate of 10%. At the end of each year $1000 is withdrawn. X/2 is deposited in an account which has an annual interest rate of 7%. At the end of each year $200 is deposited. At the start of the sixth year the account balances are the same. Determine X to the nearest cent.

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ANSWERS

1. B
2. A
3. D
4. A
5. E
6. E
7. B
8. C
9. A
10. C
11. E
12. D
13. E
14. C
15. D
16. E
17. B
18. C
19. E
20. B
21. A
22. A
23. B
24. D
25. A

Tiebreaker 1
2.98 cm

Tiebreaker 2
6.23 meters

Tiebreaker 3
$7,979.52