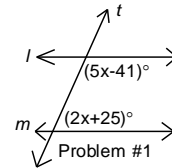


**Arkansas Council of Teachers of Mathematics
2012 State Competition
Geometry Exam**

For questions 1 through 25, mark your answer choice on the answer sheet provided. (Figures may not be drawn to scale.) After completing items 1 through 25, answer each of the tiebreaker items in sequential order (do #1 first, followed by #2, and then #3 last). Be sure that your name is printed on each of the tiebreaker pages. Congratulations for being selected to participate in the ACTM State Contest!

1. If l and m are parallel lines and t is a transversal, what is the value of x in the given figure?

- A. 22
- B. 28
- C. $23\frac{3}{7}$
- D. $35\frac{1}{7}$
- E. None of these

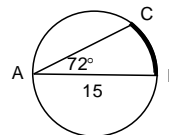


2. A triangle in which the medians and angle bisectors coincide is a(n)
- A. scalene triangle
 - B. isosceles triangle
 - C. right triangle
 - D. equilateral triangle
 - E. None of these
3. To construct a circle that is tangent to the three sides of a triangle one must first construct the
- A. altitudes of the triangle
 - B. angle bisectors of the triangle
 - C. perpendicular bisectors of the sides of the triangle
 - D. medians of the triangle
 - E. None of these
4. The bases of an isosceles trapezoid have lengths of 10 and 16. The median of the trapezoid has a length of
- A. 26
 - B. 12
 - C. 14
 - D. 11
 - E. None of these

5. Four angles of a pentagon have degree measures of 100, 108, 115, and 120. What is the measure of the fifth angle?

- A. 108° B. 97° C. 73° D. 78° E. None of these

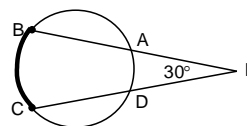
6. In the circle with diameter $AB = 15$, the inscribed angle $\angle BAC$ has a measure of 72° . The length of the minor arc BC is



Problem #6

- A. 6π B. 12π C. 3π D. 5π E. None of these

7. In the figure $m\angle AED = 30^\circ$ and the minor arc AB , BC and CD are all equal. The degree measure of the minor arc AD is



Problem #7

- A. Not enough information
 B. 30° C. 105° D. 45° E. None of these

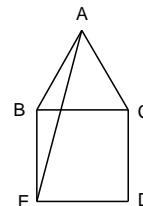
8. An interior angle of a regular polygon measures 140° . The polygon is a(n)

- A. octagon B. decagon C. heptagon
 D. hexagon E. None of these

9. A 20-foot rope and a 15-foot rope are tied to the top of a 12-foot wall. The ropes are then tied to the ground on opposite sides of the wall in a line that is perpendicular to the base of the wall. The distance between the points the ropes are tied to the ground is

- A. 25 ft B. 7 ft C. 24 ft D. 9 ft E. None of these

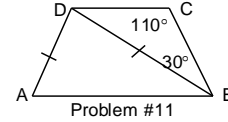
10. In the adjacent figure $\triangle ABC$ is an equilateral triangle and quadrilateral $BCDE$ is a square. The measure of $\angle AEB$ is



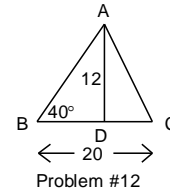
Problem #10

- A. 15° B. 30° C. 10° D. 25° E. None of these

11. In trapezoid ABCD, $\overline{AB} \parallel \overline{CD}$ and $\overline{BD} \cong \overline{AD}$. If $m\angle DCB = 110^\circ$ and $m\angle CBD = 30^\circ$ then $m\angle ADB$ is

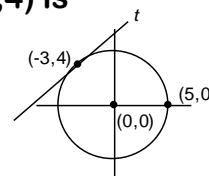


- A. 80°
 B. 40°
 C. 100°
 D. 110°
 E. None of these
12. In $\triangle ABC$, $m\angle ABC = 40^\circ$ and the altitude \overline{AD} is 12. If $BC = 20$, determine CD to the nearest hundredth.



- A. 14.30
 B. 5.70
 C. 15.66
 D. 4.34
 E. None of these
13. $\triangle ABC$ is a right triangle with right angle at C. D is the midpoint of \overline{AB} . If the length of the median from C is 6 then $AB =$
- A. 12
 B. 10
 C. 9
 D. Not enough information
 E. None of these
14. Let $A = (3, 5)$ and $B = (-2, 10)$ be points in the coordinate plane. The distance AB is
- A. $\sqrt{226}$
 B. $5\sqrt{2}$
 C. $2\sqrt{37}$
 D. $\sqrt{26}$
 E. None of these
15. The diameter of the circle whose equation is $x^2 + y^2 - 6x = 7$ is
- A. 10
 B. 5
 C. 8
 D. 4
 E. None of these

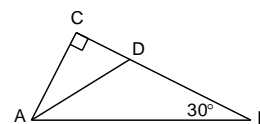
16. The slope of the line t that is tangent to the circle at the point $(-3,4)$ is



Problem #16

- A. $-\frac{3}{4}$ B. $\frac{3}{4}$ C. $-\frac{4}{3}$ D. $\frac{4}{3}$ E. None of these

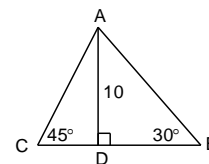
17. $\triangle ABC$ is a right triangle with right angle at C . If $AB = 12$, \overline{AD} bisects $\angle BAC$ and $m\angle ABC = 30^\circ$ then the area of $\triangle ABD$ is



Problem #17

- A. $12\sqrt{3}$
 B. $6\sqrt{3}$
 C. 36
 D. Not enough information
 E. None of these

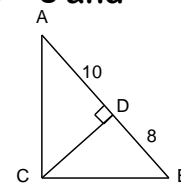
18. In $\triangle ABC$ if $m\angle ABC = 30^\circ$, $m\angle ACB = 45^\circ$ and the altitude $\overline{AD} = 10$ then the area of $\triangle ABC$ is



Problem #18

- A. $50 + 25\sqrt{3}$
 B. $100 + 100\sqrt{3}$
 C. $50 + 50\sqrt{3}$
 D. $50\sqrt{2} + 50\sqrt{3}$
 E. None of these

19. $\triangle ABC$ is a right triangle with right angle at C . If \overline{CD} is an altitude, $BD = 8$ and $AD = 10$ then $BC =$



Problem #19

- A. $4\sqrt{5}$
 B. $6\sqrt{5}$
 C. 6
 D. 12
 E. None of these

20. If $\triangle ABC \sim \triangle DEF$, $AB = 3DE$ and the area of $\triangle ABC$ is 90 square feet, then the area of $\triangle DEF$ is

- A. 270 sq. ft.
 B. 30 sq. ft.
 C. 810 sq. ft.
 D. 10 sq. ft.
 E. None of these

21. If the diameter of a circle is increased by π units then the circumference of the circle is increased by ? units.

- A. $\frac{\pi^2}{2}$ B. $\frac{1}{\pi}$ C. π D. π^2 E. None of these

22. If the diameter of a circle is 4 times the reciprocal of its circumference, then the area of the circle is

- A. π
B. 2
C. 1
D. $\frac{1}{\pi}$
E. None of these

23. The volume of a right circular cylinder is the same numerical value as the lateral surface area of the cylinder. The radius of the cylinder is

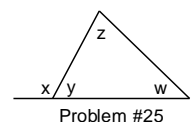
- A. Not enough information
B. 4
C. 1
D. 2
E. None of these

24. The perimeter of a semicircular region, measured in inches, is numerically equal to its area measured in square inches. The radius of the semicircle, measured in inches is

- A. π B. $\frac{2}{\pi}$ C. $2\pi + 2$ D. $2 + \frac{4}{\pi}$ E. None of these

25. Using the given diagram, which of the following statements is/are always true? The triangle is to be considered a general triangle.

- P: $y < x$ Q: $x < z$ R: $x < w$



- A. P only B. Q and R only C. Q only D. P, Q, and R E. None of these

Name _____
[Please Print Clearly]

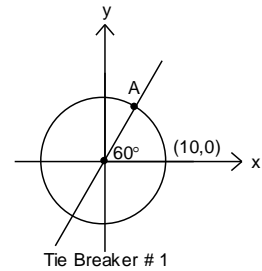
School _____
[Please Print Clearly]

Tiebreaker Questions

Your solutions should be written clearly. All work leading to your final answer must be included. The questions will be used in sequential order to resolve ties for first, second, and/or third place.

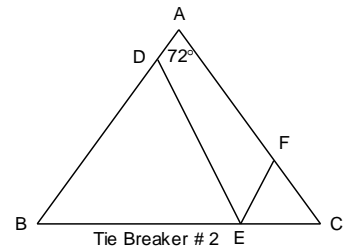
Tiebreaker #1.

In the figure the line makes a 60° angle with the positive x-axis. What are the exact coordinates of A, the point of intersection between the line and the circle in the first quadrant?



Tiebreaker #2.

2. In $\triangle ABC$, $AB = AC$, $BD = BE$ and $CE = CF$. If $m\angle BAC = 72^\circ$ find the measure of $\angle DEF$.

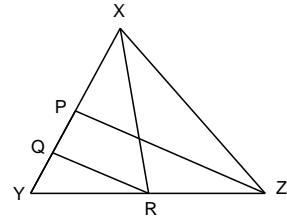


Name _____
[Please Print Clearly]

School _____
[Please Print Clearly]

Tiebreaker #3.

In $\triangle XYZ$, P is the midpoint of \overline{XY} , Q is the midpoint of \overline{PY} and R is the midpoint of \overline{YZ} . If the area of $\triangle QXR$ is 27 units squared, what is the area of $\triangle XYZ$?



Tie Breaker # 3

**Arkansas Council of Teachers of Mathematics
2012 State Competition
Geometry Exam Key**

- | | | | |
|-----|---|-----|---|
| 1. | B | 14. | B |
| 2. | D | 15. | C |
| 3. | B | 16. | B |
| 4. | E | 17. | A |
| 5. | B | 18. | C |
| 6. | A | 19. | D |
| 7. | D | 20. | D |
| 8. | E | 21. | D |
| 9. | A | 22. | C |
| 10. | A | 23. | D |
| 11. | C | 24. | D |
| 12. | B | 25. | E |
| 13. | A | | |

Arkansas Council of Teachers of Mathematics
 2012 State Competition
 Geometry Exam
 Tiebreaker Key

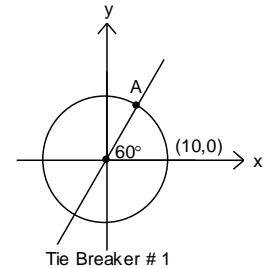
1. In the figure the line makes a 60° angle with the positive x-axis. What are the exact coordinates of the point A of intersection between the line and the circle in the first quadrant?

$$x = 10\cos(60^\circ)$$

$$x = 10\left(\frac{1}{2}\right) = 5$$

$$y = 10\sin(60^\circ)$$

$$y = 10\left(\frac{\sqrt{3}}{2}\right) = 5\sqrt{3}$$



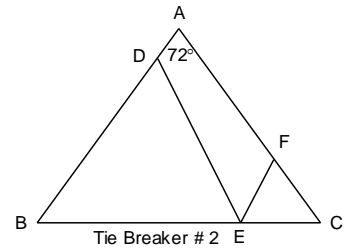
2. In $\triangle ABC$, $AB = AC$, $BD = BE$ and $CE = CF$. If $m\angle BAC = 72^\circ$ find the measure of $\angle DEF$.

Since $\triangle ABC$ is isosceles $m\angle ABC = m\angle ACB = 54^\circ$.

Since $\triangle BDE$ is isosceles $m\angle BDE = m\angle BED = 63^\circ$.

Since $\triangle FEC$ is isosceles $m\angle FEC = 63^\circ$.

Thus $m\angle DEF = 180^\circ - 63^\circ - 63^\circ = 54^\circ$.



3. In $\triangle XYZ$, P is the midpoint of \overline{XY} , Q is the midpoint of \overline{PY} and R is the midpoint of \overline{YZ} . If the area of $\triangle QXR$ is 27, what is the area of $\triangle XYZ$?

$$\text{Area}(\triangle PQR) = \text{Area}(\triangle RQY)$$

$$\text{Area}(\triangle PRY) = \text{Area}(\triangle PRX)$$

$$\text{Area}(\triangle XRY) = \text{Area}(\triangle XRZ)$$

$$\text{Let Area}(\triangle RQY) = a.$$

$$\text{Then Area}(\triangle QRX) = 3a = 27.$$

$$\text{So } a = 9.$$

$$\text{Thus, Area}(\triangle XRY) = 36. \text{ So Area}(\triangle XYZ) = 72.$$

