Arkansas Council of Teachers of Mathematics
2012 State Competition
Algebra II Exam

For questions 1 through 25, mark your answer choice on the answer sheet provided. After completing items 1 through 25, answer each of the tiebreaker items in sequential order (do #1 first, followed by #2, and then #3 last). Be sure that your name is printed on each of the tiebreaker pages. Congratulations for being selected to participate in the ACTM State Contest!

1. For b ≠ 0, find x so that $\frac{b^3 b^x}{b^4 b^{-x}} = b^5$.

A. All real numbers B. 3 C. $\frac{3}{2}$ D. 2 E. None of these

2. Ty bought grass seed at the store. He bought a total of 9.9 lbs of seed for $42.84 (without tax included). The Bermuda grass seed costs $5.20 per lb and the Fescue seed costs $3.85 per pound. How many pounds of Bermuda did Ty buy?

A. 6.4 lbs
B. 3.5 lbs
C. 8.2 lbs
D. 11.1 lbs
E. None of these

3. Find the solution for the inequality $\frac{|2x-4|}{x+3} \le 0$

A. $(-\infty, 2]$ B. $(-\infty, -3) \cup (-3, 2]$ C. $(-\infty, -3) \cup (-3, \infty)$ D. $(-\infty, -3)$ E. None of these

4. Use a system of equations to solve for a, b and c in the following equations:

2a – 3b – 4c = 9
a + b – 2c = – 3
3a – 5 = 2c – b

A. $a = -2, b = -3, c = -1$
B. $a = -1, b = -3, c = -0.5$
C. $a = 4, b = 13, c = 10$
D. $a = 4, b = -3, c = 2$
E. None of these

5. Find the domain of $f(x)=\dfrac{3x}{\sqrt{x^3-4x^2}}$

A. $(-\infty,0)\cup(0,\infty)$

B. $(-\infty,4)\cup(4,\infty)$

C. $(-\infty,0)\cup(0,4)\cup(4,\infty)$

D. $(4,\infty)$

E. None of these

6. Compute the following product: $\left(2-\sqrt{3}\right)\left(4+2\sqrt{3}\right)$

A. 2 B. – 10 C. $2+2\sqrt{3}$ D. $6+\sqrt{3}$ E. None of these

7. Find x in the following equation: $0=\sqrt{2}x^2+\sqrt{3}x-\sqrt{2}$

A. $\dfrac{-\sqrt{3}\pm\sqrt{11}}{4}$ B. $\dfrac{-\sqrt{3}\pm\sqrt{\sqrt{3}-8}}{2\sqrt{2}}$ C. $\dfrac{-\sqrt{6}\pm\sqrt{22}}{4}$ D. $\dfrac{-\sqrt{6}\pm i\sqrt{10}}{4}$ E. None of these

8. Simplify: $\sqrt[3]{2a^2}\cdot\sqrt[3]{16a^4}$

A. $\sqrt[3]{32a^8}$ B. $2a^2\sqrt[3]{4}$ C. $2a\sqrt[3]{4}$ D. $8a^2\sqrt[3]{4a^2}$ E. None of these

9. Simplify: (2 + 3*i*) / (4 – 2*i*)

A. $\dfrac{1}{10}+\dfrac{4}{5}i$ B. $\dfrac{2+16i}{10}$ C. $\dfrac{7}{10}+\dfrac{4}{5}i$ D. $\dfrac{7}{6}+\dfrac{2}{3}i$ E. None of these

10. Solve for x: $3+\sqrt{3x+1}=x$

A. 1 B. 8 C. 1 and 8 D. $\varnothing$ E. None of these

11. Find x: $\frac{x^2}{4}=\frac{x}{2}-\frac{1}{6}$

A. $\frac{3\pm\sqrt{15}}{3}$ B. $\frac{-3\pm\sqrt{15}}{3}$ C. $2\sqrt{3}$ D. $\varnothing$ E. None of these

12. Find a function with integer coefficients that has the zeros: 2 and 1 – 3*i*

A. $y=x^2+6x-16$

B. $y=x^2-x+2+3xi-6i$

C. $y=x^3-4x^2+4x+32$

D. $y=x^3-4x^2+14x-20$

E. None of these

13. A soccer player kicks a soccer ball that is sitting on the ground. The ball travels 50 m before hitting the ground again. An observer measures the height of the ball to be 20 m at its highest point. Find the value of ***a*** for a quadratic equation describing the path of this ball in the form $y=ax^2+bx+c$.

A. -1 B. $-\frac{1}{20}$ C. $-\frac{4}{125}$ D. $\frac{1}{20}$ E. None of these

14. Given that (x-1) is a factor of $f(x)=x^4-x^3+8x-8$, which of the following is not a factor.

A. (x^2-2x+4)

B. $\left(x+2\right)^3$

C. $(x-1-i\sqrt{3})$

D. $\left(x^3+8\right)$

E. $(x+2)$

15. Find x in $\dfrac{b^{2y}\sqrt{a^3}}{a^{4x}b^{1/3}} = a^{7/10}b^{1/15}$

A. $x=\dfrac{2}{5}$ B. $x=\dfrac{1}{10}$ C. $x=\dfrac{1}{5}$ D. $x=5$ E. None of these

16. Solve for x: $2\log_3 x+\log_3\dfrac{1}{10}=\log_3 5+\log_3 2$

A. $x=\pm 10$ B. $x=50$ C. $x=1$ D. $x=10$ E. None of these

17. Find the inverse of: $f(x)=4x^3-8$

A. $f^{-1}(x)=\sqrt[3]{x+2}$

B. $f^{-1}(x)=\sqrt[3]{\dfrac{x-8}{4}}$

C. $f^{-1}(x)=\sqrt[3]{\dfrac{x+8}{4}}$

D. The inverse does not exist

E. None of these

18. Find x if A = $\begin{bmatrix} 1 & 3 & 4 \\ 2 & 2 & -1 \\ x-4 & x-2 & -2 \end{bmatrix}$ and $\det(A)=28$

A. x = 3 B. x = – 42 C. x = – 1 D. x = 31 E. None of these

19. Solve for x: $(\log(x))^3 = \log(x^{16})$

I. 10^4 II. 10^0 III. $10^{\frac{16}{3}}$ IV. 13 V. $\frac{1}{10{,}000}$

A. I only B. III only C. IV only D. Only I, II and III E. Only I, II and V

20. Let $f^{-1}(x) = x^2 + 3$. Find the domain of $f(x)$.

A. $(-\infty, \infty)$ B. $(-\infty, 3) \cup (3, \infty)$ C. $[0, \infty)$ D. $[3, \infty)$ E. None of these

21. Give the ordered pair (in terms of *a*) that satisfies the system of equations $\begin{cases} ax + 3y = 6 \\ -2x - y = 4 \end{cases}$

A. $\left(\frac{18}{a-6}, \frac{-4(a+3)}{a-2}\right)$

B. $\left(\frac{18}{a+6}, \frac{a+3}{a-2}\right)$

C. $\left(\frac{a-6}{18}, \frac{6-a}{4(a+3)}\right)$

D. No Solution

E. None of these

22. Evaluate: $\left(\sqrt[3]{\sqrt{6}}\right)^{12}$

A. 216 B. $\sqrt{6}$ C. 36 D. $\sqrt[4]{6}$ E. None of these

23. A soccer field, measuring 50 yards by 100 yards, is built in a very wet location. In order to drain the field, a crown is built in the field that resembles a parabola with the equation $f(x) = -0.00062x^2 + .093x$ where x is the distance (in feet) from one sideline of the field and y is the height of the field (in feet) from the horizontal. Find the height of the crown at its maximum point.

A. 0 feet
B. 3.5 feet
C. 3.1 feet
D. 6.9 feet
E. None of these

24. Describe what happens to the graph of $f(x) = -a^x + b$, where a > 0, as x goes to positive infinity.

I. y goes to infinity II. y goes to negative infinity III. y goes to –b IV. y goes to b V. y = b-1

A. I only
B. II only
C. Only II or III
D. Only II or IV
E. Only II, IV, or V

25. Which of the following are factors of $f(x) = 3x^5 - 2x^3 - 24x^2 + 16$

I. x – 2 II. x + 2 III. $x^2 + 2x + 4$ IV. $\left(\sqrt{3}x - \sqrt{2}\right)$

A. I only
B. Only I and II
C. Only I, II, and III
D. III only
E. Only I, III and IV

Name_____ **School**_____

[Please Print Clearly] **[Please Print Clearly]**

Tiebreaker Questions

Your solutions should be written clearly. All work leading to your final answer must be included. The questions will be used in sequential order to resolve ties for first, second, and/or third place.

Tiebreaker #1.

If an object is given an initial velocity of V_0 feet per second from a height of s_0 feet, then its height S after t seconds is given by the formula:

$$S = -16t^2 + v_0 t + s_0$$

A rock is thrown into the air with an initial velocity of 42 feet per second from a height of 21 feet. How long does it take to reach the ground? Round to two decimals.

Tiebreaker #2.

The number of cell phone users in a city in Arkansas numbered 250 in 1996 and increased at a rate of 35% per year after that date. Approximately how many cell phone users are in that city today in 2012?

In what year will the number of users reach 100,000?

Name_____ **School**_____

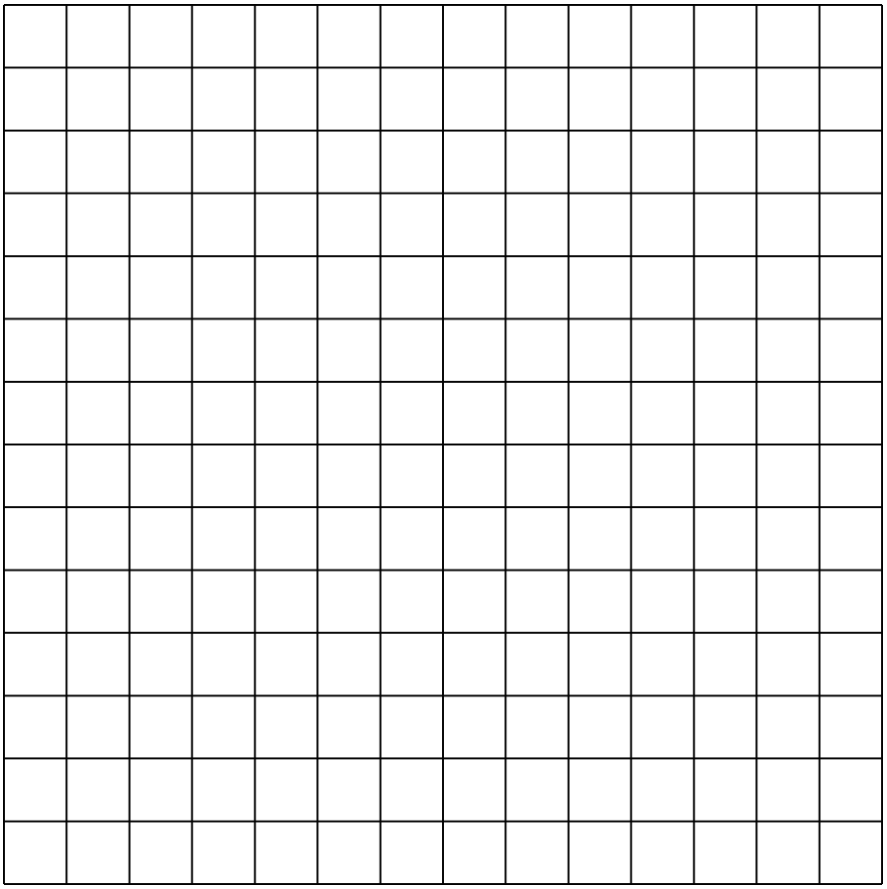
[Please Print Clearly] **[Please Print Clearly]**

Tiebreaker #3.

If you purchase car insurance from the company Grand Arkansan, then your cost will be $225 per month as long as you do not have an accident.

If you purchase car insurance from the company Mutual of Ouachita, your cost is $245 per month. However, Mutual of Ouachita will reduce your monthly payment by $10 per month after every 6 months that you drive without an accident.

Graph the cost of both insurance companies and find the number of months that you must drive accident free before the total cost that you have paid to Mutual of Ouachita is equal to what you paid to Grand Arkansan over the same period of time.



Arkansas Council of Teachers of Mathematics
2012 State Competition
Algebra II Exam Key

Answer Sheet

1. B
2. B
3. E
4. D
5. D
6. A
7. C
8. B
9. A
10. B
11. E
12. D
13. C
14. B
15. C
16. D
17. C
18. A
19. E
20. D
21. E
22. C
23. B
24. E
25. E

Arkansas Council of Teachers of Mathematics
2012 State Competition
Algebra II Exam
Tiebreaker Key

Tiebreaker #1

$S = -16t^2 + 42t + 21$

Let S = 0 and solve for t: $t = \dfrac{-42 \pm \sqrt{42^2 - 4(-16)(21)}}{2(-16)}$

This gives: t = -.43seconds and t = 3.05 seconds
Since a negative time does not make sense, the answer is 3.05 seconds.

Tiebreaker #2

a) Use the equation: $y = a(1+r)^t = 250(1+.35)^{16} \approx 30{,}428$ cell phone users

b) $100{,}000 = 250(1.35)^t$

$\ln(400) = t * \ln(1.35)$

$t = 19.96 years$

The year 2016

Tiebreaker #3: 30 months