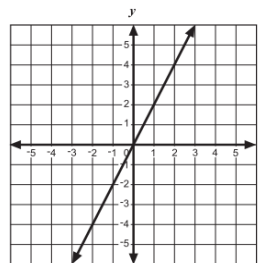


**Arkansas Council of Teachers of Mathematics  
2012 State Competition  
Algebra I Exam**

For questions 1 through 25, mark your answer choice on the answer sheet provided. After completing items 1 through 25, answer each of the tiebreaker items in sequential order (do #1 first, followed by #2, and then #3 last). Be sure that your name is printed on each of the tiebreaker pages. Congratulations for being selected to participate in the ACTM State Contest.

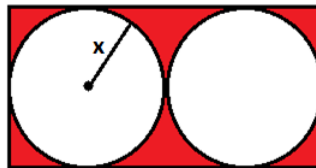
1. Which equation best represents the graph provided?
- $y = x$
  - $y = 2x$
  - $y = x + 2$
  - $y = 2x + 2$
  - None of the above.



2. The names of 17 high school students are written on paper and placed in a hat for a drawing. Eight of the students are seniors, four are juniors, and five are sophomores. What is the probability that the first slip of paper drawn from the hat does not have a name of a sophomore on it?
- $4/17$
  - $5/17$
  - $8/17$
  - $12/17$
  - $13/17$
3. Which statement *best* explains why there is no real solution to the quadratic equation  $2x^2 + x + 7 = 0$ ?
- The value of  $1^2 - 4(2)(7)$  is positive.
  - The value of  $1^2 - 4(2)(7)$  is equal to 0.
  - The value of  $1^2 - 4(2)(7)$  is negative.
  - The value of  $1^2 - 4(2)(7)$  is not a perfect square.
  - None of the above.

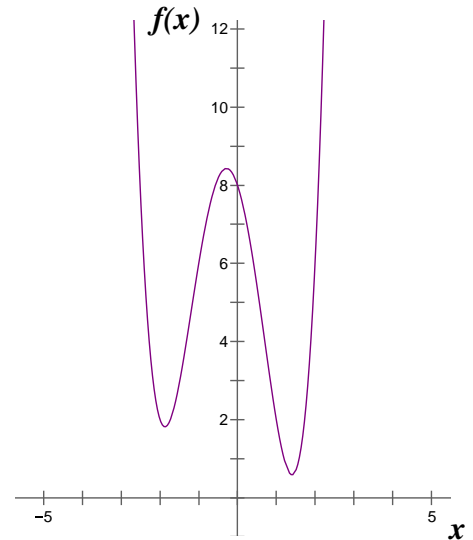
4. Write an expression in factored form for the area  $A$  of the shaded region. The area of a circle is  $\pi r^2$ .

- $A = 2x^2(4 - \pi)$
- $A = 4x^2(\pi^2 - 2)$
- $A = 4x^2(2 - \pi^2)$
- $A = 2\pi x(4 - x)$
- None of the above.



5. Approximate (to two decimal places) any real zeros and any local extrema of the function  $f(x) = x^4 + x^3 - 5x^2 - 3x + 8$ , given its graph.

- a) Minimums:  $(-1.87, 1.81)$  &  $(1.41, .59)$   
 Maximum:  $(-.28, 8.43)$   
 Zeros: None
- b) Minimums:  $(-1.87, 1.81)$  &  $(1.41, .59)$   
 Maximum:  $(-.28, 8.43)$   
 Zeros:  $(8, 0)$
- c) Minimum:  $(-.28, 8.43)$   
 Maximums:  $(-1.87, 1.81)$  &  $(1.41, .59)$   
 Zeros:  $(0, 8)$
- d) Minimums:  $(1.81, -1.87)$  &  $(.59, 1,41)$   
 Maximum:  $(8.43, -.28)$   
 Zeros: None
- e) None of the above.



6. Evaluate the function for the given domain value, and round to the nearest hundredth.

$$f(a) = \frac{4}{7}(-3a + 2)^2 - \frac{3}{4}(a - 7)^{-3} + \frac{5}{8}, \text{ when } a = 4.5$$

- a) 11  
 b) 74.22  
 c) 74.24  
 d) 76.24  
 e) 137.86
7. Solve for x:  $\frac{x}{2} - 4 = \frac{7}{4}(5x - 7) - 5$
- a)  $-\frac{85}{33}$   
 b)  $-\frac{55}{33}$   
 c)  $\frac{11}{3}$   
 d)  $\frac{53}{33}$   
 e)  $\frac{5}{3}$

8. Which quadratic function when graphed has a vertex at  $(-3, 4)$ ?

I.  $f(x) = (x - 3)^2 + 4$

II.  $f(x) = -3(x + 3)^2 + 4$

III.  $f(x) = (x - 3)^2 - 4$

IV.  $f(x) = -3x^2 - 18x - 23$

V.  $f(x) = (-x - 3)^2 + 4$

- a) I only  
 b) II only  
 c) Only I and V  
 d) Only II and IV  
 e) Only II, IV and V

9. Given  $x = \begin{bmatrix} 2 & 3 & 1 \\ -1 & 5 & 4 \end{bmatrix}$  and  $y = \begin{bmatrix} 6 & 0 & -2 \\ 4 & 1 & 5 \end{bmatrix}$ . What is  $2x - 3y$ ?

a)  $\begin{bmatrix} -14 & 6 & 8 \\ -14 & 7 & -7 \end{bmatrix}$

b)  $\begin{bmatrix} 22 & 6 & -4 \\ 10 & 13 & 23 \end{bmatrix}$

c)  $\begin{bmatrix} -14 & 3 & 8 \\ -14 & 7 & -7 \end{bmatrix}$

d)  $\begin{bmatrix} 22 & 9 & -4 \\ 10 & 13 & 23 \end{bmatrix}$

e)  $\begin{bmatrix} 14 & 6 & -4 \\ -14 & 7 & 7 \end{bmatrix}$

10. The graph of the equation  $Ax + 3y = -3$  is a line that passes through  $(2, -5)$ . What is the value of  $A$ ?

a) 3

b) 5

c) 6

d) 7

e) 8

11. The graph provided shows the distance of a boat from shore after  $t$  minutes. When is the **velocity** of the boat zero?

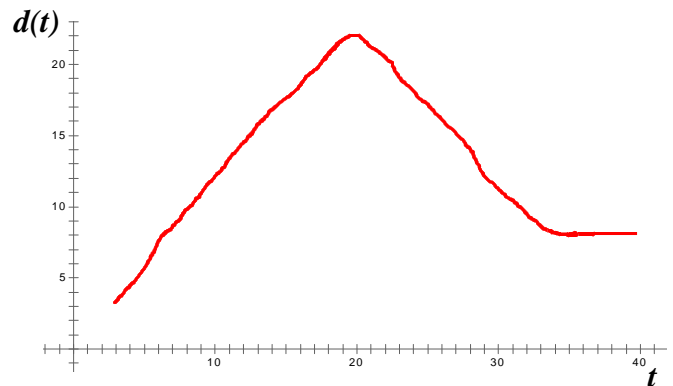
a)  $t = 1$

b)  $35 < t < 40$

c)  $t = 20$

d)  $t = 15$

e)  $35 < t < 40$  &  $t = 20$



12. For the data shown, which measure is greatest?

$\{5, 6, 6, 8, 9, 10\}$

a) Mode

b) Median

c) Range

d) Mean

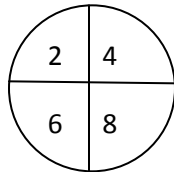
e) None of the above.

13. Determine the ordered pair that is a solution to the system.

$$\frac{2}{5}x - \frac{7}{3}y = 15$$

$$4x - 6y = 10$$

- a)  $(-\frac{125}{13}, -\frac{105}{13})$   
b)  $(\frac{250}{13}, -\frac{105}{13})$   
c)  $(-\frac{250}{13}, \frac{105}{13})$   
d)  $(\frac{125}{13}, \frac{105}{13})$   
e)  $(\frac{250}{13}, \frac{150}{13})$
14. Given  $a = 4$  and  $b = -5$ , determine the equation in point-slope form of the line passing through the points:  $(-\frac{3}{4}a + \frac{4}{5}, \frac{7}{10}b - \frac{5}{6})$ ,  $(a, b)$   
a)  $y + 5 = -\frac{10}{93}(x - 4)$   
b)  $y + 5 = -\frac{10}{61}(x - 4)$   
c)  $y - \frac{13}{3} = \frac{10}{61}(x + \frac{11}{5})$   
d)  $y - 5 = -\frac{10}{93}(x - 4)$   
e)  $y - 4 = -\frac{10}{93}(x + 5)$
15. Russ bought 3 medium and 2 large submarine sandwiches for a total of \$29.95. Stacy bought 4 medium and 1 large submarine sandwiches for a total of \$28.45. Which statement shows the cost of each medium and each large submarine sandwich?  
a) Each medium sandwich costs \$5.69 and each large sandwich costs \$6.89.  
b) Each medium sandwich costs \$5.69 and each large sandwich costs \$6.39.  
c) Each medium sandwich costs \$5.39 and each large sandwich costs \$6.89.  
d) Each medium sandwich costs \$5.39 and each large sandwich costs \$6.39.  
e) Each medium sandwich costs \$6.89 and each large sandwich costs \$5.39.
16. Pat spins the spinner below and rolls a numbered cube.



What is the probability that the spinner and the numbered cube both land on 2?

- a)  $1/24$   
b)  $1/6$   
c)  $5/12$   
d)  $2/3$   
e)  $22/24$

17.  $\frac{7z^2+7z}{4z+8} \div \frac{z^3+2z^2+z}{z^2-4} =$

a)  $\frac{7(z-2)}{4(z+1)}$

b)  $\frac{7(z+2)}{4(z-1)}$

c)  $\frac{7z(z+1)}{4(z+2)}$

d)  $\frac{7z(z-1)}{4(z+2)}$

e) None of the above.

18. The length of the hypotenuse of a right triangle is 1 cm more than the length of the longer leg. The length of the shorter leg is 7cm less than the length of the longer leg. What is the length of the shorter leg of the triangle?

a) 4

b) 5

c) 12

d) 13

e) None of the above.

19. Which of the following statements is true of the given lines?

Line 1:  $2x + y = -4$

Line 2:  $x + 2y = -10$

Line 3:  $-2x + 4y = -12$

a) Lines 1 and 2 are parallel.

b) Lines 2 and 3 are parallel.

c) Lines 1 and 3 are perpendicular.

d) Lines 2 and 3 are perpendicular.

e) None of the above.

20. A wooden garden tray with a base area of  $xy$  square inches is filled with soil up to a certain height. The tray contains  $(x^2y+ 10xy^2+ 12xy)$  cubic inches of soil. The height of the tray is represented by the expression  $\frac{x^2y+10xy^2+12xy}{xy}$ .

Which expression below is the correct simplified form of the height of the tray?

a)  $11xy + 12$

b)  $x + 10y + 12$

c)  $x + 10xy^2 + 12xy$

d)  $x^2y + 10xy^2 + 12$

e) None of the above.

21. Solve the following inequality:  $|\frac{7}{5}x - 8| \leq 12$
- $-20 \leq x \leq 100$
  - $-\frac{100}{7} \leq x \leq \frac{100}{7}$
  - $-\frac{20}{7} \leq x \leq \frac{100}{7}$
  - $-\frac{20}{7} \geq x \geq \frac{80}{7}$
  - $-\frac{7}{20} \leq x \leq \frac{7}{100}$
22. Assuming  $x > 0$ , which of these expressions is equivalent to  $9\sqrt{605x^3} - 11\sqrt{405x^3}$ ?
- 0
  - $-2\sqrt{200}$
  - $-2x\sqrt{200x}$
  - $104x\sqrt{5x}$
  - None of the above.
23. You roll a numbered cube. What is the probability that you will roll an even number or a number greater than 4 (round to the nearest hundredth)?
- 0.15
  - 0.16
  - 0.50
  - 0.67
  - 0.83
24. The balance,  $B_{n+1}$  in Mr. Smith's savings account at the end of a year is calculated by the equation  $B_{n+1} = 1.065 \cdot B_n$  where  $B_n$  is the balance at the end of the previous year. Mr. Smith made a deposit to open the account 4 years ago. He has not made any additional deposits or withdrawals since. The balance at the end of 2 years was \$1,701.34. What is the balance at the end of 4 years?
- \$1,922.51
  - \$1,929.70
  - \$2,143.69
  - \$2,188.72
  - \$2,272.53
25. Solve the following expression:  $\frac{5}{4}\left(14x - \frac{7}{8}\right) = \frac{5}{7}\left(-11x - \frac{2}{5}\right)$
- $-\frac{181}{5680}$
  - $\frac{181}{5680}$
  - $\frac{309}{5680}$
  - $\frac{181}{2160}$
  - $\frac{5680}{181}$

Name \_\_\_\_\_  
[Please Print Clearly]

School \_\_\_\_\_  
[Please Print Clearly]

## Tiebreaker Questions

Your solutions should be written clearly. All work leading to your final answer must be included. The questions will be used in sequential order to resolve ties for first, second, and/or third place.

### Tiebreaker #1.

A square has side length  $(k - x)$ . A smaller square with side length  $x$  is cut from one corner.

In terms of  $k$  and  $x$ , find the area of the left-over shape in simplest form.

Name \_\_\_\_\_  
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**Tiebreaker #2.**

In one English dictionary, there are the following number of words:

2 - One-letter words

96 - Two-letter words

1238 - Three-letter words

3391 - Four-letter words

a) If you write a letter from the English alphabet at random, what is the probability that the letter is one of the one-letter words found in the English dictionary? Write your answer as a percent and round to two decimal places.

b) If you write two letters from the English alphabet at random, what is the probability that the letters form one of the two-letter words found in the English dictionary? Write your answer as a percent and round to two decimal places.



Name \_\_\_\_\_

School \_\_\_\_\_

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**Tiebreaker #3.**

An atom consists of a combination of protons, electrons and neutrons.

The mass of a proton is  $1.67 \times 10^{-27}$  kg.

The mass of an electron is  $9.11 \times 10^{-31}$  kg.

The mass of a neutron is approximately the same as a proton,  $1.67 \times 10^{-27}$  kg.

A single atom of Fermium (#259) consists of 100 protons, 100 electrons and 159 neutrons.

a) What is the mass of one atom of Fermium?

b) What is the mass of one mole ( $6.02 \times 10^{23}$  atoms) of Fermium?

## *State Algebra I Contest Key*

### Multiple Choice

1. B
2. D
3. C
4. A
5. A
6. D
7. D
8. E
9. A
10. C
11. E
12. D
13. A
14. A
15. C
16. A
17. A
18. B
19. C
20. B
21. C
22. A
23. D
24. B
25. B

### Solutions to Tie-Breaker Questions

#### **Tie-Breaker #1:**

$$A_{shape} = (k^2 - 2kx)units^2$$

#### **Tie-Breaker #2:**

a) Probability = 7.69%  $\left(\frac{2}{26}\right) = 7.69\%$

b) Probability = 14.20%  $\left(\frac{96}{676}\right) = 14.20\%$

#### **Tie-Breaker #3:**

a) Mass of one atom =  $4.33 \times 10^{-25}$  kg

b) Mass of one mole =  $2.60 \times 10^{-1}$  kg