ACTM State – Statistics

Work the multiple choice questions first, selecting the single best response from those provided and entering it on your scantron form. You may write on this test and keep the portion with the multiple-choice questions. After completing the multiple-choice section work the open response tie breakers in the order presented. Tie breaker 2 will be used only if there is still a tie after the first tie breaker and similarly the third tie breaker will be used only if there is a tie after reading the second tie breaker. You will turn in all three tie breakers with your name on the pages regardless of what you have or have not written there.

Multiple Choice Questions

1. Researchers wanted to determine whether there was an association between city driving and stomach ulcers. They selected a sample of 900 young adults and followed them for a twenty-year period. At the start of the study none of the participants was suffering from a stomach ulcer. Each person kept track of the number of hours per week they spent driving in city traffic. At the end of the study, each participant underwent tests to determine whether they were suffering from a stomach ulcer. The researchers analyzed the results to determine whether there was an association between city driving and stomach ulcers. Determine type of this observational study.
   a. Restrospective
   b. Cohort
   c. Cross-sectional
   d. None of the above
2. To determine if there are outliers in a least squares regression model's data set, we could construct a boxplot of the
   a. response variables.
   b. predictor variables.
   c. lurking variables.
   d. residuals.

3. The United States can be divided into four geographical regions: Northeast, South, Midwest, and West. The Northeast region consists of 9 states; the South region consists of 16 states; the Midwest consists of 12 states; and the West consists of 13 states. If a survey is to be administered to the governors of 10 of the states and we want equal representation for the states in each of the four regions, how many states from the South should be selected? Round to the nearest integer.
   a. 2
   b. 4
   c. 5
   d. 3

4. Find the standardized test statistic to test the hypothesis that $\mu_1 \neq \mu_2$. Two samples are randomly selected from each population. The sample statistics are given below. Use $\alpha = 0.02$.

$$n_1 = 51 \quad n_1 = 38$$
$$\bar{x}_1 = 6.3 \quad \bar{x}_1 = 6.7$$
$$s_1 = 0.76 \quad s_1 = 0.51$$

   a. -2.32
   b. -1.82
   c. -2.97
   d. -2.12

5. A two-pound bag of assorted candy contained 100 caramels, 83 mint patties, 93 chocolate squares, 80 nut clusters, and 79 peanut butter taffy pieces. To create a pie chart of this data, the angle for the slice representing each candy type must be computed. What is the degree measure of the slice representing the mint patties rounded to the nearest degree?
   a. 5°
   b. 69°
   c. 52°
   d. 19°

6. Parking at a large university has become a very big problem. University administrators are interested in determining the average parking time (e.g. the time it takes a student to find a parking spot) of its students. An administrator inconspicuously followed 120 students and carefully recorded their parking times. Identify the population of interest to the university administration.
   a. The parking times of the 120 students from whom the data were collected
   b. The entire set of faculty, staff, and students that park at the university
   c. The parking times of the entire set of students that park at the university
   d. The students that park at the university between 9 and 10 AM on weekdays.

7. A 1-pound bag of peanuts contained 430 peanuts. The distribution of weights (in grams) of the peanuts is given below. Estimate the sample standard deviation of the weight of a peanut.

Weights (grams)	Frequency
0.755 – 0.814	3
0.815 – 0.874	2
0.875 – 0.934	2
0.935 – 0.994	2
0.995 – 1.054	168
1.055 – 1.114	241
1.115 – 1.174	12

   a. 0.002 g
   b. 0.045 g
   c. 0.209 g
   d. 0.000004 g

8. Many people think that a national lobby's successful fight against gun control legislation is reflecting the will of a minority of Americans. A random sample of 4000 citizens yielded 2250 who are in favor of gun control legislation. Estimate the true proportion of all Americans who are in favor of gun control legislation using a 99% confidence interval. Express the answer in the form $\hat{p} \pm E$ and round to the nearest ten-thousandth.
   a. 0.4375 ± 0.0202
   b. 0.5625 ± 0.0202
   c. 0.4375 ± 0.6337
   d. 0.5625 ± 0.6337

9. Which of the following is not true about factors?
   a. Factors whose effect on the response variable is not of interest can be set after the experiment.
   b. One way to control factors is to fix their level at one predetermined value throughout the experiment.
   c. Any combination of the values of the factors is called a treatment.
   d. Factors whose effect on the response variable interests us should be set at predetermined levels.

10. The following is a sample of 19 November utility bills (in dollars) from a neighborhood. What is the largest bill in the sample that would not be considered an outlier?

    52, 62, 66, 68, 72, 74, 76, 76, 76, 78, 78, 82, 84, 84, 86, 88, 92, 96, 110

   a. 96
   b. 11
   c. 67
   d. 219

11. The following data represent the living situation of newlyweds in a large metropolitan area and their annual household income. Find the marginal frequency for newlyweds who rent their home

	<$20,000	$20,000 - $35,000	$35,000 - $50,000	$50,000 - $75,000	>$75,000
Own home	31	52	202	355	524
Rent home	67	66	52	23	11
Live w/ family	89	69	30	4	2

   a. 52
   b. 11
   c. 67

12. A bicycle manufacturer produces four different bicycle models. Information is summarized in the table below:

Model	Series Number	Weight	Style
Ascension	A120	33	Mountain
Road Runner	B640	21	Road
All Terrain	C300	27	Hybrid
Class Above	D90	17	Racing

Identify the variables and determine whether each variable is quantitative or qualitative.
   a. Series number: quantitative; weight: qualitative; style: qualitative
   b. Series number: qualitative; weight: qualitative; style: qualitative
   c. Series number: qualitative; weight: quantitative; style: qualitative
   d. Series number: quantitative; weight: quantitative; style: qualitative

13. A sports statistician is interested in determining if there is a relationship between the number of home team and visiting team losses and different sports. A random sample of 526 games is selected and the results are given below. Find the critical value χ_0^2 to test the claim that the number of home team and visiting team losses is independent of the sport. Use $\alpha = 0.01$.

	Football	Basketball	Soccer	Baseball
Home team losses	39	156	25	83
Visiting team losses	31	98	19	75

   a. 7.815
   b. 11.345
   c. 9.348
   d. 12.838

14. The data below are the final exam scores of 10 randomly selected chemistry students and the number of hours they slept the night before the exam. What is the best predicted value for y given $x = 6$?

Hours, x	3	5	2	8	2	4	4	5	6	3
Scores, y	65	80	60	88	66	78	85	90	90	71

   a. 87
   b. 86
   c. 84
   d. 85

15. In how many ways can a committee of three men and four women be formed from a group of 12 men and 12 women?
   a. 108,900
   b. 165
   c. 15,681,600
   d. 6.652,800

16. A farmer was interested in determining how many grasshoppers were in his field. He knows that the distribution of grasshoppers may not be normally distributed in his field due to growing conditions. As he drives his tractor down each row he counts how many grasshoppers he sees flying away. After several rows he figures the mean number of flights to be 57 with a standard deviation of 12. What is the probability of the farmer will count 52 or fewer flights on average in the next 40 rows down which he drives his tractor?
   a. 0.4959
   b. 0.9959
   c. 0.0410
   d. 0.0042

17. A teacher figures that final grades in the chemistry department are distributed as: A, 25%; B, 25%; C, 40%; D, 5%; F, 5%. At the end of a randomly selected semester, the following number of grades were recorded. Calculate the chi-square test statistics χ^2 to determine if the grade distribution for the department is different than expected. Use $\alpha = 0.01$.

Grade	A	B	C	D	F
Number	42	36	60	14	8

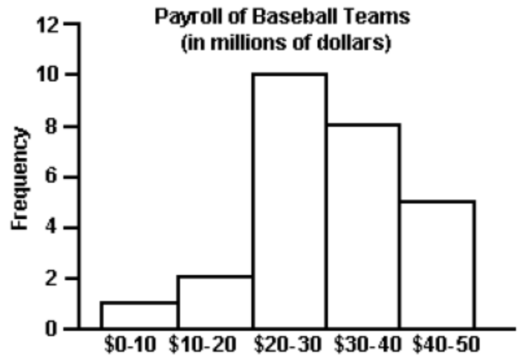
   a. 0.6375
   b. 5.25
   c. 6.87
   d. 4.82

18. A history professor decides to give a 12-question true-false quiz. She wants to choose the passing grade such that the probability of passing a student who guesses on every question is less than 0.10. What score should be set as the lowest passing grade?
   a. 8
   b. 9
   c. 10
   d. 7

19. The number of traffic accidents that occur on a particular stretch of road during a month follows a Poisson distribution with a mean of 6.3. Find the probability that the next two months will both result in five accidents each occurring on this stretch of road.
   a. 0.151868
   b. 0.023064
   c. 0.008924
   d. 0.303736

20. The length of time it takes college students to find a parking spot in the library parking lot follows a normal distribution with a mean of 5.5 minutes and a standard deviation of 1 minute. Find the cut-off time for which 75.8% of the college students exceed when trying to find a parking spot in the library parking lot.
   a. 5.8 min
   b. 6.3 min
   c. 6.0 min
   d. 6.2 min

21. The payroll amounts of 26 major-league baseball teams are shown below. Approximately what percentage of the payrolls were in the \$30-\$40 million range? Round to the nearest whole percent.



   a. 8%
   b. 19%
   c. 31%
   d. 42%

22. A doctor at a local hospital is interested in estimating the birth weight of infants. How large a sample must she select if she desires to be 90% confident that her estimate is within 2 ounces of the true mean? Assume that $s = 5$ ounces based on earlier studies.
   a. 16
   b. 4
   c. 5
   d. 17

23. The June precipitations (in inches) for 10 randomly selected cities are listed below. Construct a 90% confidence interval for the population standard deivation, σ. Assume that the data are normally distributed.

   2.0 3.2 1.8 2.9 0.9 4.0 3.3 2.9 3.6 0.8

   a. (0.32,0.85)
   b. (1.10,2.01)
   c. (0.81,1.83)
   d. (0.53,1.01)

24. A conservative estimate for the degrees of freedom used when testing two independent samples where the population standard deviation is unknown is
   a. $n_1 + n_2 - 1$.
   b. $n_1 + n_2 - 2$.
   c. the larger of $n_1 - 1$ or $n_2 - 1$.
   d. the smaller of $n_1 - 1$ or $n_2 - 1$.

25. A random sample of 100 students at a high school was asked whether they would ask their father or mother for help with a financial problem. A second sample of 100 different students was asked the same question regarding a dating problem. Forty-three students in the first sample and 47 students in the second sample replied that they turned to their mother rather than their father for help. Construct a 98% confidence interval for $p_1 - p_2$.
    a. (-1.113, 1.311)
    b. (-1.324, 1.521)
    c. (-0.591, 0.762)
    d. (-0.204, 0.124)

Name: ______________________________

Tie Breakers

1. **Is the Die Loaded?** A player in a craps game suspects that one of the dice being used in the game is loaded. A loaded die is one in which all the possibilities (1,2,3,4,5, and 6) are not equally likely. The player throws the die 400 times, records the outcome after each throw, and obtains the following results:

Outcome	**Frequency**
1	62
2	76
3	76
4	62
5	57
6	67

   a. Do you think that the die is loaded at the $\alpha = 0.01$ level of significance? Conduct an hypothesis test to answer this question. Write down the hypotheses, test statistics, P-value, decision, and conclusion. Assume that necessary conditions are satisfied to carry out the hypothesis test.
   b. Why do you think the player might conduct the test at the $\alpha = 0.01$ level of significance rather than, say, the $\alpha = 0.1$ level of significance?

Name: ______________________________

2. **Height versus Head Circumference** A pediatrician wants to determine the relation that may exist between a child's height and head circumference. She randomly selects 11 children from her practice, measures their heights and head circumferences, and obtains the following data:

Height (inches) x	**Head Circumference (inches) y**
27.75	17.5
24.50	17.1
25.50	17.1
26.00	17.3
25.00	16.9
27.75	17.6
26.50	17.3
27.00	17.5
26.75	17.3
26.75	17.5
27.50	17.5

a. Treating height as the explanatory variable, x, determine the estimates of β_0 and β_1 of a linear regression model.
b. Test whether a linear relation exists between height and head circumference at the $\alpha = 0.01$ level of significance. Report the P-value.
c. Compute the standard error of the estimate, s_e.
d. Construct a 95% confidence interval about the slope of the true least-squares regression line.
e. Use the fitted regression equation to obtain a good point estimate of the child's head circumference if the child's height is 26.5 inches.

Name: ____

3. **Three of a Kind** You are dealt 5 cards from a standard 52-card deck. Determine the probability of being dealt three of a kind (such as three aces or three kings) by answering the following questions:
   a. How many ways can 5 cards be selected from a 52-card deck?
   b. Each deck contains 4 twos, 4 threes, and so on. How many ways can three of the same card be selected from the deck?
   c. The remaining 2 cards must be different from the 3 chosen and different from each other. For example, if we drew three kings, the 4^{th} card cannot be a king. After selecting the three of a kind, there are 12 different ranks of card remaining in the deck that can be chosen. If we have three kings, then we can choose twos, threes, and so on. Of the 12 ranks remaining, we choose 2 of them and then select one of the 4 cards in each of the two chosen ranks. How many ways can we select the remaining 2 cards?
   d. Find the probability of obtaining three of a kind. That is, what is the probability of selecting three of a kind and two cards that are not like?

Answers to Multiple Choice Questions

1. B
2. D
3. D
4. C
5. B
6. C
7. B
8. B
9. A
10. A
11. D
12. C
13. B
14. B
15. A
16. D
17. B
18. B
19. B
20. D
21. C
22. D
23. C
24. D
25. D

Solutions of Tie Breaker Problems

1.
   a.
      i. Hypotheses: H_0 : All sides are equally likely to occur; H_1 : All sides are not equally likely to occur.
      ii. Test statistic: $\chi^2 = 4.67$.
      iii. P-value: 0.457
      iv. Decision: Failed to reject H_0.
      v. Conclusion: There is not enough evidence to indicate that the die is loaded at the $\alpha = 0.01$ level of significance.
   b. Since the accusation of a loaded die is very serious, the player would want very strong evidence to support this accusation.

2.
   a. $b_0 = 12.493\ inches; b_1 = 0.183$
   b. P-value=.000096. Reject H_0. Yes, there is an evidence that a linear relation exists.
   c. $s_e = 0.095$ inches.
   d. (0.12, 0.245).
   e. 17.34 inches.

3.

a. ${}_{52}C_5 = 2{,}598{,}960$.

b. ${}_{13}C_1 \cdot {}_4C_3 = 52$.

c. ${}_{12}C_2 \cdot {}_4C_1 \cdot {}_4C_1 = 66 \cdot 4 \cdot 4 = 1{,}056$.

d. $\dfrac{52 \cdot 1056}{2598960} = 0.0211$.