Arkansas Council of Teachers of Mathematics
2013 State Contest
Trigonometry/Pre-Calculus Exam

In each of the following choose the BEST answer and shade the corresponding letter on the Scantron Sheet. Answer all 25 multiple choice questions before attempting the tie-breaker questions. The tie-breaker questions at the end are to be used to resolve any ties between 1st, 2nd, and/or 3rd place. Be sure that your name is printed on each of the tiebreaker pages. The figures are not necessarily drawn to scale. Good Luck!

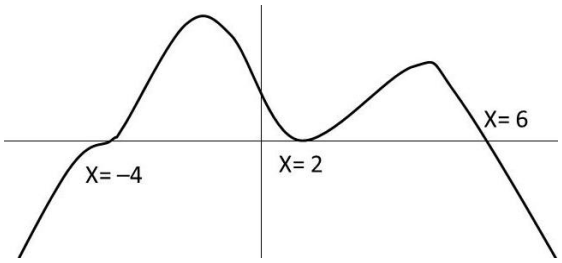
1. How many positive zeros (including multiplicities) could the following function have? $f(x) = x^4 - 5x^3 + 22x^2 - 80x + 96$

A) 0 B) 2 C) 4 D) A, B and C E) None of these

2. Identify all asymptotes in the graph of $f(x) = \dfrac{x^2 + x - 6}{x^3 - 8}$

A) x = 2 B) y = 0 C)Both A & B D)None of these

3. Which value of *x* on the graph below could have multiplicity of 2?



A) x = – 4 B) x = 0 C) x = 2 D) x = 6

4. Find the expression that is equivalent to $\frac{1}{2}\log_3(x-4) + 2\log_3(x)$

A) $\log_3\left(\sqrt{x^5 - 4x^4}\right)$ B) $\log_3\left[\frac{1}{2}\left(x^3 - 4x^2\right)\right]$

C) $\log_3\left(x^2 - 4x\right)$ D) $\log_3\left[(x-4)^2 x^2\right]$

5. Find the terminal point for the side of an angle, θ, if the $\cos\theta = -\frac{\sqrt{3}}{3}$ and the y-coordinate of the terminal point is 6.

A) $(-3,6)$ B) $\left(-\sqrt{3},6\right)$ C) $\left(-\frac{\sqrt{3}}{3},6\right)$ D) None of these

6. Which of the following statements is true:

A) Characteristics of the tangent function include: periodic with period π; an odd function; the domain is the set of all real numbers, except $\frac{k\pi}{2}$ where k is an even integer; the range is all real numbers.

B) Characteristics of the cosecant function include: periodic with period 2π; the domain is the set of all real numbers, except x = $k\pi$, *k* an integer; the range is all real numbers y such that $y \neq 1$ or $y \neq -1$

C) The inverse sine function is the inverse of the restricted sine function $y = \sin x,\ 0 \le x \le \pi$. Thus, y = arcsin x is equivalent to sin y = x where $0 \le y \le \pi$, $-1 \le x \le 1$

D) Characteristics of the cotangent function include: periodic with period $\frac{\pi}{2}$; the domain is the set of all real numbers, except $k\pi$, k an integer; the range is all real numbers.

E) Each of the other answers is incorrect.

7. Find the sum: $\sum_{k=148}^{643} (4k + 3j)$

A) 783,090 + 1485 j B) 1,017,226 + 1929j
C) 784,672 + 1488j D) 1,373,176

8. A researcher began a study with 200g of a substance that decays in an exponential manner. After 2 years, it was found that 140g of the substance remained. Approximately how much of the substance will remain in 6 more years?

A) 69g B) 3289g C) 18g D) 48g

9. An angle and its reference angle sum to be a multiple of π. Which of the following could be true about the angle?

A) It is in Quadrant II B)It is in Quadrant III C) It is in Quadrant IV

D) Both A and C E) Not enough information to determine

10. For $f(x) = -a^x + b$, where *a>0,* describe what may happen to f(x) as *x* goes to positive infinity.

I. f(x) goes to infinity II. f(x) goes to negative infinity III. f(x) goes to b
IV. f(x) goes to -b V. f(x) = b-1

A) I only
B) II only
C) Only II or IV
D) Only II, III, or V
E) Only II, IV or V

11. For what value of *n* is the following true? $\sum_{k=0}^{n} 64\left(-\frac{1}{2}\right)^k = 42$

A) n = 8 B) n = 5 C) n= 4 D) n=6

12. The point $\left(4\frac{1}{2}, -5\frac{1}{4}\right)$ is on the terminal side of an angle, θ, in standard position. Find the exact value of sec θ.

A) 1.54 B) $\frac{6\sqrt{85}}{85}$ C) $\frac{\sqrt{85}}{6}$ D) $\frac{\sqrt{85}}{7}$

13. Find the inverse of $f\left(x\right) = \ln\left(x^4\right)$

A) $f^{-1}\left(x\right) = \left(10\right)^{x/4}$ B) $f^{-1}\left(x\right) = \left(e\right)^{x/4}$ C) $f^{-1}\left(x\right) = \sqrt[x]{e^4}$ D) None of these

14. Which of the following is equivalent to $\left(\sin x - \cos x\right)^2$

A) 1 B) $1+\cos(2x)$ C) $1-\sin(2x)$ D) $1-2\cos^2 x$

15. Given the function $f(\theta)=4\sin(\theta-b)$, which value/s of b listed below correspond to the point $\left(\frac{\pi}{2},2\right)$.

A) $\frac{\pi}{3}$ B) $-\frac{\pi}{3}$ C) $\frac{\pi}{6}$ D) A and B E) A, B and C

16. A forest ranger is able to use a laser range finder to determine the distance from where she is standing to two very large trees on the opposite bank of a river. She is able to measure the angle formed between the ray from her to the first large tree and the ray formed from her to the second large tree to be 25°. If the distance from the ranger to the first tree is 75 feet and the distance from the ranger to the second tree is 100 feet, then what is the distance between the two trees? (Round to the nearest foot.)

A) 28 feet B) 99 feet C) 2030 feet D) 45 feet

17. The area of a triangle whose sides are 150 ft, 210 ft, and 164 ft is approximately

A) $149{,}537{,}024ft^2$ B) $12{,}229ft^2$ C) $148{,}839ft^2$ D) None of these

18. Find the focus of the parabola $x^2+4x+6y-2=0$.

A) x = – 2 B) $\left(-2,-\frac{1}{2}\right)$ C) $(-2,1)$ D) $y=\frac{5}{2}$

19. Johna bought 14 pounds of grass seed. There were 3 types of grass seed (Bermuda, Zoysia and Centipede). The Bermuda was only half the weight of the Zoysia. The Bermuda cost $5 per pound, the Zoysia cost $8 per pound, and the Centipede cost $10 per pound and the total cost of all of her grass seed was $104. Assume the grass seeds are purchased in multiple pounds. How much Centipede did Johna buy?

. A) 4 pounds B) 8 pounds C) 2 pounds D) None of these

20. Convert the rectangular equation to polar form. $x^2+y^2-5x=0$

A) $r=5\sin\theta$ B) $r=\frac{1}{5}\cos\theta$ C) $r=5\cos\theta$ D) None of these

21. Find the equation of an ellipse that has x-intercepts at ±4 and a minor axis of 5 units that is centered at the origin.

A) $\frac{x^2}{16}+\frac{4y^2}{25}=1$ B) $\frac{x^2}{16}+\frac{y^2}{25}=1$ C) $\frac{x^2}{64}+\frac{4y^2}{25}=1$ D) $\frac{4x^2}{25}+\frac{y^2}{16}=1$

22. Which of the following functions has a phase shift of $\frac{3\pi}{4}$ when compared to the function: $y=a\sin(bx)$.

A) $y=4\sin(2x-3\pi)$ B) $y=3\sin(8x-6\pi)$

C) $y=2\sin(3\pi x-4)$ D) $y=3\pi\sin(x-4)$

23. A 12 foot ladder leans against a wall. There is a 6 foot tall window in the wall that is 3 feet above the ground. What is the minimum angle that the ladder can make with the ground in order for the ladder to touch the wall above the window? (Round to the nearest degree.)

A) 49° B) 1° C) 30° D) 37°

24. An observer stands on a 250 foot tall cliff overlooking the ocean. She notices two ships headed directly toward her with one ship following directly behind the other. She finds that the angle of depression from her line of sight to the front of the first ship is 7° and the angle of depression to the front of the second ship is 4.5°. How far apart is the front of the first ship from the front of the second ship (assuming the height of the ship to be the same as the base of the cliff)? (Round to the nearest foot.)

A) 1140 ft B) 2051 ft C) 4087 ft D) None of these

25. Find the exact value of $cos\frac{5\pi}{12}$.

A) 0.259 B) $\frac{-\sqrt{2}+\sqrt{6}}{4}$ C) $\frac{-\sqrt{2}-\sqrt{6}}{4}$ D) $\frac{\sqrt{2}+\sqrt{6}}{14.9}$

Tie-Breaker Questions

Name_____ **School_____**
[Please Print] **[Please Print]**

In each of the following you must show supporting work for your answers to receive credit. The questions will be used in the order given to resolve ties for 1st, 2nd, and/or 3rd place. Be sure that your name is printed on each of the tiebreaker pages.

1) Find the exact value of x that satisfies the equation: $2^{x-1} = 3^x$

Tie-Breaker Questions

Name_____ **School_____**
[Please Print] **[Please Print]**

2a) A driver averages 60 miles per hour when driving round trip from Little Rock to El Dorado. The trip is 120 miles each way. If the average speed driving to El Dorado is x and the average speed driving back to Little Rock is y, then find an equation that expresses y in terms of x.

2b) If the driver finds that their average speed going one direction is twice the average speed driving the other direction, then what are the two speeds? (Assume that there is no maximum speed limit.)

Tie-Breaker Questions

Name_____ **School_____**
[Please Print] **[Please Print]**

3. Given the finite sum: $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\ldots+\frac{1}{n(n+1)}$

A) Find the formula for the finite sum above.

B) Assuming your answer to A is true for n=k, then find the finite sum for n = k+1 using your solution to part A.

Pre-Calculus/Trig 2010 Regional Test
Answers:

1. D
2. B
3. C
4. A
5. D
6. E
7. C
8. D
9. D
10. D
11. B
12. C
13. B
14. C
15. D
16. D
17. B
18. B
19. C
20. C
21. A
22. B
23. A
24. A
25. B

Tie Breaker 1:

$$\log\left(2^{x-1}\right) = \log\left(3^{x}\right)$$
$$x-1)\log(2) = x\log(3)$$
$$x\log(2) - \log(2) = x\log(3)$$
$$x\log(2) - x\log(3) = \log(2)$$
$$x\left[\log(2) - \log(3)\right] = \log(2)$$
$$x = \frac{\log(2)}{\log(2) - \log(3)}$$

Tie Breaker 2:

a) The time to travel from Little Rock to El Dorado is $\frac{120}{x}$ and the time to travel from El Dorado to Little Rock is $\frac{120}{y}$. Therefor the average speed for the entire trip can be found by: $\frac{240}{\frac{120}{x}+\frac{120}{y}}=60$. This becomes:

$$\frac{4}{\frac{120}{x}+\frac{120}{y}}=1$$

$$4=\frac{120}{x}+\frac{120}{y}$$

$$4xy=120y+120x$$

$$xy=30y+30x$$

$$xy-30y=30x$$

$$y\left(x-30\right)=30x$$

$$y=\frac{30x}{x-30}$$

b) Let $x=\frac{1}{2}y$. This gives $y=\frac{15y}{\frac{1}{2}y-30}$. Solving for y:

$$y\left(\frac{1}{2}y-30\right)=15y$$

$$\frac{1}{2}y^2-30y=15y$$

$$\frac{1}{2}y^2-45y=0$$

$$y^2-90y=0$$

$$y\left(y-90\right)=0$$

This means that y = 0mph or y = 90mph. Since y=0 does not make sense, then y = 90. If y = 90mph, then x = 45mph (Note: Students could let $y=\frac{1}{2}x$ in which case the answers are the same only the direction for each changes.)

Tie Breaker 3:

A) For the first finite sum: $\frac{1}{2} = \frac{n}{(n+1)}$

For the second finite sum: $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} = \frac{1}{2} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3} = \frac{2}{2+1} = \frac{n}{(n+1)}$

B) Assume for the kth finite sum: $\frac{k}{(k+1)}$

For k+1:

You take the sum for the first k terms and add $\frac{1}{(k+1)\cdot(k+2)}$

This gives

$$\frac{k}{(k+1)} + \frac{1}{(k+1)\cdot(k+2)} = \frac{k(k+2)+1}{(k+1)(k+2)} = \frac{k^2+2k+1}{(k+1)(k+2)} = \frac{(k+1)(k+1)}{(k+1)(k+2)} = \frac{(k+1)}{(k+1)+1}$$