Arkansas Council of Teachers of Mathematics

2014 Regional Exam

Calculus

For questions 1 through 25, mark your answer choice on the answer sheet provided. After completing items 1 through 25, answer each of the tiebreaker items in sequential order (do #1 first, followed by #2, and then #3 last). Be sure that your name is printed on each of the tiebreakers.

1. Evaluate the following limit: $\lim_{x\to\infty} x\sin\left(\frac{1}{x}\right) =$

   A. 0
   B. 1
   C. e
   D. $+\infty$
   E. None of these

2. If sin x = sin y, then $\frac{d^2y}{dx^2} =$

   A. $\frac{\tan y\cos^2 x - \sin x\cos y}{\cos^2 x}$

   B. $\frac{\tan y\cos^2 x - \sin x\cos y}{\cos^2 y}$

   C. $\frac{\cos y(\sin x - \sin y)}{\cos^2 y}$

   D. $\frac{\cos y(\sin x - \sin y)}{\cos^2 x}$

   E. None of these

3. Evaluating the integral $\int \frac{x}{x^2+1}dx$ yields the result

   A. $x\tan^{-1}x + C$
   B. $\frac{x+3}{2x} + C$
   C. $\frac{1}{2}\ln(x^2+1) + C$
   D. $\frac{1}{2}\ln|x| + C$
   E. None of these

4. A rock is dropped off a bridge and its distance s (in ft) from the bridge after t seconds is $s = 15t^2$. What is the velocity of the rock at $t = 3$ seconds?

   A. 15 ft/sec
   B. 45 ft/sec
   C. 90 ft/sec
   D. 135 ft/sec
   E. None of these

5. Determine the value of a so that the function $f(x) = \begin{cases} \dfrac{5\sin 3x}{x} & x < 0 \\ 3a - 4x & x \geq 0 \end{cases}$ is continuous at $x = 0$.

   A. 1
   B. 3
   C. 6
   D. 15
   E. None of these

6. The graph of a function f that satisfies $f'(x) < 0$ and $f''(x) > 0$ on (0, 10) means that f

   A. is increasing at an increasing rate on (0, 10)
   B. is increasing at a decreasing rate on (0, 10)
   C. is decreasing at an increasing rate on (0, 10)
   D. is decreasing at a decreasing rate on (0, 10)
   E. None of these

7. Which of the following functions has vertical asymptotes at $x = 0$, $x = 4$, and $x = -3$, and a horizontal asymptote at $y = 2$?

   A. $f(x) = \dfrac{2x^3}{x(x-4)(x+3)}$
   B. $f(x) = \dfrac{2x^3+3}{x(x-4)(x-3)}$
   C. $f(x) = \dfrac{8x^3}{4x(x+4)(x+3)}$
   D. $f(x) = \dfrac{4x^2+10}{2x(x-4)(x+3)}$
   E. None of these

8. The point *c* guaranteed to exist by Rolle's Theorem for the function $f(x) = 2x^2 - 8x$ on the interval [0, 4] is

A. 0
B. 1
C. 2
D. 3
E. None of these

9. The sides of a cube are increasing at a rate of *R* cm/sec. When the sides have a length of 4 cm, the rate of change of the volume of the cube is

A. $4R$
B. $64R$
C. $16R$
D. $48R$
E. None of these

10. The doubling time for a bank account whose balance is given by $A(t) = 500e^{.042t}$ is

A. 0.042 (ln 2)
B. 0.042 / ln 2
C. ln 2 / 0.042
D. 1 / 0.042
E. None of these

11. Determine the slope of the tangent line to the graph of $g(x) = \dfrac{3x^2 + 2x - 4}{3x - 5}$ at the point $x = 2$.

A. – 14
B. – 22
C. 50
D. 12
E. None of these

12. Compute the derivative of $h(x) = 3\csc x - 4\cot x + 5\sec x$

A. $-3\csc x \cot x - 4\csc^2 x + 5\sec x \tan x$
B. $-3\csc^2 x - 4\cot^2 x + 5\sec^2 x$
C. $3\csc x \cot x + 4\csc^2 x + 5\sec x \tan x$
D. $3\tan^2 x - 4\csc^2 x + 5\csc\cot x$
E. None of these

13. The area of the region between the curves $y = x$ and $y = 2 - x^2$ is

A. $-\frac{1}{2}$
B. $\frac{21}{6}$
C. $-\frac{3}{2}$
D. $\frac{27}{6}$
E. None of these

14. If $5x^2 + 2xy + y^3 = 25$, then the value of $\frac{dy}{dx}$ at the point (2, 1) is

A. $\frac{11}{3}$
B. $\frac{22}{7}$
C. $-\frac{20}{7}$
D. $\frac{20}{3}$
E. None of these

15. Let $g(x) = 2x^3 + Ax^2 + Bx - 3$ with g(2) = 13 and g ' (2) = 30. What is A+B?

A. -5
B. -3
C. 1
D. 3
E. None of these

16. Suppose $f(x) = e^{nx}$. What is $f^n(x)$?

A. $n^n e^{nx}$
B. ne^{nx}
C. $n!e^{nx}$
D. $n^n e^x$
E. None of these

17. Compute $\frac{dy}{dx}$ for $f(x) = \tan^{-1}\left(\frac{1}{x}\right)$.

A. $\frac{1}{x^2+1}$
B. $\frac{-1}{x^2+1}$
C. $\frac{-1}{x^2\left(x^2+1\right)}$
D. $\frac{1}{x^2\left(x^2+1\right)}$
E. None of these

18. On the interval [-2, 2], the function $f(x) = x^4$

A. has an absolute maximum but no local maxima
B. has an absolute maximum at an interior point of the interval
C. has no local or absolute extrema
D. has a local minimum but no absolute minimum
E. None of these

19. How many points *c* satisfy the conclusion of the Mean Value Theorem for the function $f(x) = x^3$ on the interval from [-1, 1] ?

A. 0
B. 1
C. 2
D. 3
E. None of these

20. Let $f(x) = 5x - 7$ and note that $\lim_{x \to 2} f(x) = 3$. For ε = 0.5, determine the value for δ > 0 so that $|f(x) - 3| < \varepsilon$ whenever $|x - 2| < \delta$.

A. 0.1
B. 0.2
C. 0.25
D. 0.5
E. None of these

21. $\int \frac{x^4}{e^{x^5}} =$

A. $-\frac{1}{5}\ln e^{x^5} + C$

B. $\frac{1}{5}\ln e^{x^5} + C$

C. $\frac{1}{5e^{x^5}} + C$

D. $-\frac{1}{5e^{x^5}} + C$

E. None of these

22. Determine the average value of the function $f(x) = x^2 - 5$ on the interval [0, 3].

A. -6
B. -2
C. 3
D. 5
E. None of these

23. Calculate the derivative for the function $f(x) = \tan^2(x^2 + 3x - 5)$.

A. $(2x + 3)\cdot \sec^4(x^2 + 3x - 5)$
B. $(2x + 3)\cdot \tan(x^2 + 3x - 5)\cdot \sec^2(x^2 + 3x - 5)$
C. $(4x + 6)\cdot \tan^2(x^2 + 3x - 5)\cdot \sec^2(x^2 + 3x - 5)$
D. $(4x + 6)\cdot \tan(x^2 + 3x - 5)\cdot \sec^2(x^2 + 3x - 5)$
E. None of these

24. The graph of $f(x) = \frac{4}{x - 3}$ is concave down whenever

A. x < 0
B. x < 2
C. x < 3
D. x < 4
E. None of these

25. The area of the region in the first quadrant bounded by the graph of $f(x) = x(4 - x)$ and the x-axis is

A. $9\frac{1}{2}$
B. $10\frac{2}{3}$
C. $11\frac{1}{3}$
D. $12\frac{1}{2}$
E. None of these

Name:_____

Reminder: Attempt the tie-breaker questions in sequential order (Do #1 first, followed by #2, and then #3 last).

Tie-Breaker 1

Two nonnegative numbers are such that the first plus the square of the second is 10. Find the numbers if their sum is as large as possible.

Name:_____

Tie-Breaker 2

Once Jimmy's kite reaches a height of 50 feet directly above his hands, it rises no higher but drifts due east in a wind blowing 5 feet per second. How fast is the string running through Jimmy's hands at the moment he has released 120 feet of string? Round your answer to 2 decimal places.

Name:_____

Tie-Breaker 3

Determine the volume of the solid obtained by revolving the region bounded by the curves $x = -y^2 + 2y$ and $x = -2y^2 + 4y$ about the y-axis.

Calculus Key

1. B
2. B
3. C
4. C
5. E
6. C
7. E
8. C
9. D
10. C
11. B
12. E
13. D
14. E
15. B
16. A
17. B
18. A
19. C
20. A
21. D
22. B
23. D
24. C
25. B

1. $9\frac{3}{4}$ and $\frac{1}{2}$

2. 4.54 feet per second

3. $\frac{16\pi}{5} units^3$