ACTM Regional Pre-Calculus/Trig Exam
April 25, 2015

Mark your answer choice on the answer sheet provided. If you have time, answer each of the tiebreaker items in sequential order (do #1 first, followed by #2, and then #3 last). Be sure that your name is printed on each of the tiebreaker pages.

1. Which function below is an even function?

   a. $\sin x + 2\tan x$ b.) $\cot x \csc^2 x$ c.) $\sec x \sin^3 x$ d.) $\sin^2 x \cos^2 x$

2. Find the inverse function of $f(x) = 1 - x^5$.

   a.) $f^{-1}(x) = \sqrt[5]{x-1}$ b.) $f^{-1}(x) = \sqrt[5]{1-x}$

   c.) $f^{-1}(x) = -\sqrt[5]{x-1}$ d.) $f^{-1}(x) = -\sqrt[5]{1-x}$

3. Determine the sum $\sum_{n=1}^{\infty} \cos^n\left(\frac{\pi}{3}\right)$.

   a.) 1 b.) 0 c.) $\frac{\sqrt{3}}{2}$ d.) $\frac{1}{2}$

4. Let vectors $u = i - j$ and $v = i + j$. Find $\|2v - u\|$.

   a.) 2 b.) $\sqrt{10}$ c.) $2\sqrt{5}$ d.) $\sqrt{5}$

5. What is the rectangular equation corresponding to the polar equation $r = 1 - 2r\sin\theta$?

   a.) $x^2 + 3y^2 + 4y = 1$ b.) $x^2 + 3y^2 - 4y = 1$

   c.) $x^2 - 3y^2 + 4y = 1$ d.) $x^2 - 3y^2 - 4y = 1$

6. Determine the modulus of the complex number $z = \dfrac{4}{1+i}$.

   a.) $2\sqrt{2}$ b.) $\sqrt{2}$ c.) 2 d.) $4\sqrt{2}$

7. Determine the solution set of the equation $\log_2(3 - x) + \log_2(x) = 1$.

   a.) $\{1\}$ b.) $\{1,2\}$ c.) $\{-1,-2\}$ d.) $\{2\}$

8. Find the distance between the point in polar coordinates $\left(\sqrt{2}, \frac{3\pi}{4}\right)$ and the point in rectangular coordinates $(1,-1)$.

   a.) 2 b.) $\sqrt{2}$ c.) $2\sqrt{2}$ d.) $\sqrt{3}$

9. Express $\sin(2\cos^{-1} x)$ in terms of x.

a.) $2\sqrt{1-x^2}$ b.) $x\sqrt{1-x^2}$ c.) $2x\sqrt{1+x^2}$ d.) $2x\sqrt{1-x^2}$

10. Find the exact value of $\csc\left[\cos^{-1}\left(-\frac{1}{2}\right)\right]$.

a.) $\frac{2}{\sqrt{3}}$ b.) $\sqrt{3}$ c.) $-\sqrt{3}$ d.) $-\frac{2}{\sqrt{3}}$

11. Which of the following is a focus of the ellipse $x^2 - 2x + 4y^2 + 16y + 13 = 0$.

a.) $(1, -2+\sqrt{3})$ b.) $(1, -2-\sqrt{3})$

c.) $(1-\sqrt{3}, 2)$ d.) $(1+\sqrt{3}, -2)$

12. Evaluate $\sin 1° + \sin 2° + \cdots + \sin 360°$.

a) 1 b) 0 c) -1 d) $\sqrt{2}$

13. Find the phase shift of $y = \cos\left(-2x - \frac{\pi}{4}\right) + 7$.

a) $\frac{\pi}{4}$ b) $-\frac{\pi}{4}$ c) $-\frac{\pi}{8}$ d) $\frac{\pi}{8}$

14. Determine the solution of the inequality $|x+1| \leq 5$.

a.) $(-6, 4)$ b.) $[-5, 5]$

c.) $[-3,6]$ d.) none of these

15. Find the solution set for the equation $1 - \sin\theta = 2\cos^2\theta$ where $0 \leq \theta \leq 2\pi$.

a) $\left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ b) $\left\{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ c) $\left\{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$ d) $\left\{\frac{\pi}{2}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

16. Given the function $f(x) = \frac{x+1}{x-1}$, find the domain of $(f \circ f)(x)$.

a) $(-\infty, \infty)$ b) $(-\infty, -1) \cup (-1, \infty)$

c) $(-\infty, 1) \cup (1, \infty)$ d) $(-\infty, -1) \cup (1, \infty)$

17. Solve the triangle with sides $a = 4$, $b = 3$, $c = 6$. Which of the following is true for C, the angle opposite to side c?

a) $110° < C < 120°$ b) $20° < C < 30°$ c) $30° < C < 40°$ d) none of these

18. Convert $\frac{-\pi}{12}$ to degrees and write as the least possible positive coterminal angle.

a.) $-15°$ b.) $-195°$ c.) $165°$ d.) $345°$

19. Find the solution set of the inequality $x^2 + 2x - 3 \leq 0$.

a.) $(-\infty, -3)$ b.) $(-\infty, -3]$ c.) $[-3,1]$ d.) $[1, \infty)$

20. Which of the following is the best approximation to the area of the triangle with sides measuring a = 42 feet, b = 17 feet, and c = 35 feet?

a.) $290ft^2$ b.) $300ft^2$ c.) $280ft^2$ d.) $295ft^2$

21. Find the period of $y = \frac{1}{2}\sin\left(3x - \frac{\pi}{2}\right)$.

a.) 2π b.) $\frac{2\pi}{3}$ c.) $\frac{\pi}{2}$ d.) π

22. One side of a right triangle is 3 meters longer than the shortest side, and the hypotenuse is 15 meters in length. Find the length of the longest side.

a.) 9 m b.) 6m c.) 12 m d.) 8 m

23. Find the asymptotes of the function $f(x) = \dfrac{x^2 - 25}{x + 5}$.

a.) $y = x - 5$ b.) $x = -5$ c.) $y = x - 5 \text{ and } x = -5$ d.) No asymptotes

24. Evaluate $\dfrac{(n!)^2}{(n-1)!\,(n-1)!}$.

a.) $n(n-1)$ b.) n^2 c.) $n!$ d.) $\frac{n^2}{(n-1)^2}$

25. Which of the following is equivalent to $\cos(\theta + 60^\circ)$.

a) $\frac{1}{2}\left(\cos(\theta) - \sqrt{3}\sin(\theta)\right)$ b) $\frac{1}{2}\left(\sqrt{3}\sin(\theta) - \cos(\theta)\right)$

c) $\frac{1}{2}\left(\cos(\theta) + \sqrt{3}\sin(\theta)\right)$ d) none of these

Name: ____

Tie Breaker #1:

Find two positive numbers such that their sum is 50, and their product is a maximum, and determine their product.

Name: ____

Tie Breaker #2:

Evaluate exactly: $\sin\left[\sin^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(-\frac{5}{13}\right)\right]$.

Name: ____

Tie Breaker #3:

Find the exact value of x: $\tan(\tan^{-1} x + \tan^{-1} 2 + \tan^{-1} 3) = x - 1$.

ANSWERS

1) D
2) B
3) A
4) B
5) C
6) A
7) B
8) C
9) D
10) A
11) D
12) B
13) C
14) D
15) A
16) C
17) A
18) D
19) C
20) A
21) B
22) C
23) D
24) B
25) A

Tiebreaker solutions:

Tiebreaker 1: Find two positive numbers such that their sum is 50, and their product is a maximum, and determine their product.

X+Y=50 and P=X*Y is maximized

Find the minimum of P=X(50-X) which is an upside down parabola, so we find the vertex:
P=-X^2+50X-625+625 = -(X-25)^2+625

So X=Y=25 and their product is 625.

Tiebreaker 2: Evaluate exactly: $\sin\left[\sin^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(-\frac{5}{13}\right)\right]$.

Use angle sum formula for sine and Pythagorean theorem: sin(a+b)=sin(a)cos(b)+cos(a)sin(b).

Sin(a)=4/5 and cos(b)=-5/13 thus cos(a)=sqrt(1-16/25)=3/5 and sin(b)=sqrt(1-25/169)=12/13

So the answer is 4/5 * (-5/13) + 3/5 * 12/13 = 16/65.

Tiebreaker 3: Find the exact value of x: $\tan(\tan^{-1} x + \tan^{-1} 2 + \tan^{-1} 3) = x - 1$.

Using the triple sum formula for tangent:

$$\tan(a+b+c) = \frac{\tan(\mathrm{a}) + \tan(\mathrm{b}) + \tan(\mathrm{c}) - \tan(\mathrm{a})\tan(\mathrm{b})\tan(\mathrm{c})}{1 - \tan(\mathrm{a})\tan(\mathrm{b}) - \tan(\mathrm{a})\tan(\mathrm{c}) - \tan(\mathrm{b})\tan(\mathrm{c})}$$

so

$$\mathrm{x} - 1 = \tan(\tan^{-1} x + \tan^{-1} 2 + \tan^{-1} 3) = \frac{x + 2 + 3 - 6x}{1 - 2x - 3x - 6} = \frac{5 - 5x}{-5 - 5x}$$

$-1 + x = \frac{1-x}{-1-x}$ ➔ $1 - x^2 = 1 - x$ ➔ $x^2 = x$ ➔ $x = 0\ or\ 1$

One could also use the double sum formula for tangent or the double angle formulas for sine and cosine. The students should at least know the latter.