ACTM REGIONAL CALCULUS COMPETITION
MARCH 7, 2015

Instructions: Select the best choice for each question. Afterward, attempt the tie-breaker questions in sequential order (Do #1 first, followed by #2, and then #3 last). Unless otherwise stated, assume all variables are real and all functions are continuous over relevant domains.

1. Evaluate the following limit: $\lim\limits_{x \to 0} \dfrac{x}{|x|} =$

   A. 0
   B. 1
   C. -1
   D. does not exist
   E. None of these

2. $\displaystyle\int_0^{15} \dfrac{dx}{\sqrt{1+x}} =$

   A. 1.5

   B. 6

   C. 14

   D. 42

   E. None of these

3. Determine $\dfrac{dy}{dx}$ when $x = \tan(xy)$.

   A. $\dfrac{\cos^2(xy)}{x}$

   B. $\dfrac{1 - y\tan(xy)\sec^2(xy)}{x\tan(xy)\sec^2(xy)}$

   C. $\dfrac{\sec^2(xy) - y}{x}$

   D. $\cos^2(xy)$
   E. None of these

4. Determine the derivative of $f(x) = \ln(\ln(\ln(x)))$.

A. $f'(x) = \dfrac{1}{\ln(\ln(x))}$

B. $f'(x) = \dfrac{1}{x\ln(\ln(x))}$

C. $f'(x) = \dfrac{1}{x\ln x\ln(\ln(x))}$

D. $f'(x) = \dfrac{1}{\ln x\ln(\ln(x))}$

E. None of these

5. Determine $-\int \csc^2(5x)\,dx$.

A. $5\cot(5x) + C$

B. $-5\cot(5x) + C$

C. $-\dfrac{1}{5}\cot(5x) + C$

D. $\dfrac{1}{5}\cot(5x) + C$

E. None of these

6. Suppose that f and g are differentiable, and that the following are true:

- $f(2) = 4$
- $f'(2) = 7$
- $g(2) = 6$
- $g'(2) = -4$

What is $\dfrac{d}{dx}(f \cdot g)$ at $x = 2$?

A. -28

B. 26

C. 3

D. 6

E. None of these

7. The volume of a cone of radius r and height h is given by $V = \dfrac{1}{3}\pi r^2 h$. If the radius and height both increase at a constant rate of 2 inches per second, at what rate is the volume increasing when the height is 6 inches and the radius is 3 inches?

A. 2π cubic inches
B. 12π cubic inches
C. 24π cubic inches
D. 30π cubic inches
E. None of these

8. If $f(x) = \begin{cases} \dfrac{\sqrt{2x+5} - \sqrt{x+7}}{x-2} & x \neq 2 \\ \\ k & x = 2 \end{cases}$ and $f(x)$ is continuous everywhere, then $k =$ _____.

A. 0
B. $\dfrac{1}{6}$
C. $\dfrac{1}{3}$
D. $\dfrac{7}{5}$
E. None of these

9. The function $f(x) = x^2 e^x$ is increasing on
A. $(-\infty, 2)$
B. $(0, 2)$
C. $(-2, 0)$
D. $(-2, \infty)$
E. None of these

10. The average value of $\cos x$ on the interval from $[-2, 5]$ is

A. $\dfrac{\sin 5 - \sin 2}{3}$
B. $\dfrac{\sin 5 - \sin 2}{7}$
C. $\dfrac{\sin 5 + \sin 2}{3}$
D. $\dfrac{\sin 5 + \sin 2}{7}$
E. None of these

11. Determine the minimum product xy whenever $y = 5x - 8$.

A. $-\dfrac{32}{5}$

B. $-\dfrac{16}{5}$

C. 0

D. -13

E. None of these

12. The graph of $y = ln\ x$ has a tangent line with slope 1 at the point

A. $(1, 1)$
B. $(e, 1)$
C. $(1, 0)$
D. $(1, e)$
E. None of these

13. The cost of producing x washing machines is given by the equation $c(x) = 2000 + 100x - 0.1x^2$. The marginal cost of producing 100 washing machines is

A. $80
B. $90
C. $110
D. $11,000
E. None of these

14. $\dfrac{d^{2015}}{dx^{2015}} \cos x =$

A. $\sin x$
B. $-\sin x$
C. $\cos x$
D. $-\cos x$
E. None of these

15. A water balloon is tossed upwards off the top of a tall building. The object's distance above the ground after t seconds is $s(t) = -32t^2 + 32t + 192\ ft$. What is the object's speed at the moment of impact with the ground?

A. 32 ft/sec
B. 96 ft/sec
C. 160 ft/sec
D. 224 ft/sec
E. None of these

16. The equation of the line that represents the linear approximation of
$f(x) = x^4 - 3x^2 + 5 + \sqrt{x+2}$ at the point a = 2 is

A. $y = \dfrac{81}{4}x + \dfrac{59}{2}$

B. $y = \dfrac{101}{4}x - \dfrac{81}{2}$

C. $y = \dfrac{79}{4}x - \dfrac{57}{2}$

D. $y = \dfrac{81}{4}x - \dfrac{59}{2}$

E. None of these

17. Find the value for c such that $f(x) = \dfrac{cx+3}{2x^2+1}$ has a horizontal tangent line at x = 1.

A. -3

B. 3

C. 4

D. 12

E. None of these

18. $\int x \sin x \, dx =$

A. $\sin x + x \cos x + C$

B. $\sin x - x \cos x + C$

C. $-\sin x + x \cos x + C$

D. $-\dfrac{1}{2} x^2 \cos x + C$

E. None of these

19. If $x^3 + xy = 33$, then when x = 3, $\dfrac{dy}{dx} =$

A. $\dfrac{3}{29}$

B. $\dfrac{29}{3}$

C. $-\dfrac{29}{3}$

D. 3

E. None of these

20. Evaluate the derivative $\dfrac{d}{dx}\left[\dfrac{f(x)g(x)}{x}\right]$ at x = 3, given the table below.

x =	1	2	3	4	5
f(x)	5	4	1	2	3
f '(x)	4	2	3	1	5
g(x)	2	3	5	1	4
g'(x)	1	4	2	5	3

A. $\dfrac{4}{3}$

B. $\dfrac{5}{3}$

C. $\dfrac{46}{3}$

D. $\dfrac{46}{9}$

E. None of these

21. The function $f(x) = 4x^3 - 7x^2 + 28$ changes concavity at $x =$

A. $\dfrac{7}{12}$

B. $\dfrac{7}{6}$

C. 0

D. $-\dfrac{7}{12}$

E. None of these

22. If $f(x) = \dfrac{e^{4x}}{4x}$, then $f''(x) =$

A. $\dfrac{e^{4x}(4x+1)}{4x^2}$

B. $\dfrac{e^{4x}(8x^2 + 4x + 1)}{2x^2}$

C. $\dfrac{e^{4x}(8x^2 - 4x + 1)}{2x^3}$

D. $\dfrac{e^{4x}(4x+1)}{4x^4}$

E. None of these

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23. Which of the following are hypotheses of the Mean Value Theorem for Derivatives?

I. f is continuous on [a, b]
II. $f(a) = f(b)$
III. f is differentiable on (a,b)

A. I and II only
B. II and III only
C. I and III only
D. All of these
E. None of these

24. The arc length of the parabola $y = x^3$ on the interval [-1, 1] is given by

A. $\displaystyle\int_{-1}^{1} \sqrt{1+3x^2}\, dx$

B. $\displaystyle\int_{-1}^{1} \sqrt{1+9x^4}\, dx$

C. $\displaystyle\int_{-1}^{1} \sqrt{1+x^6}\, dx$

D. $\displaystyle\int_{-1}^{1} \sqrt{1+x^3}\, dx$

E. None of these

25. If a population $P(t)$ decreases at an annual rate of 5% per year from its initial value P_0, then the equation for $P(t)$ is

A. $P(t) = P_0 e^{-0.05t}$

B. $P(t) = P_0 e^{\ln(0.95)t}$

C. $P(t) = P_0 e^{-0.95t}$

D. $P(t) = P_0 e^{-\ln(0.95)t}$

E. None of these

Reminder: Attempt the tie-breaker questions in sequential order (Do #1 first, followed by #2, and then #3 last).

1. Air is being pumped into a spherical balloon so that its volume increases at a rate of 100 cm³/sec. How fast is the radius of the balloon increasing when the diameter is 50 cm? (Recall the volume of a sphere is $\frac{4}{3}\pi r^3$) Leave your answer in terms of pi.

2. Determine the 99th derivative of $f(x) = 5xe^{-x}$.

3. Determine the area of the region enclosed by the line $y = x - 1$ and the curve $y^2 = 2x + 6$.

Calculus Key

1. D
2. B
3. E
4. C
5. D
6. B
7. D
8. B
9. E
10. D
11. B
12. C
13. A
14. A
15. C
16. D
17. E
18. B
19. C
20. D
21. A
22. C
23. C
24. B
25. B

1. $\dfrac{1}{25\pi} cm/\sec$

2. $495e^{-x} - 5xe^{-x}$

3. 18