ACTM STATE CALCULUS COMPETITION
April 25, 2015

Instructions: Select the best choice for each question. Afterward, attempt the tie-breaker questions in sequential order (Do #1 first, followed by #2, and then #3 last). Unless otherwise stated, assume all variables are real and all functions are continuous over relevant domains.

1. If $\int_0^c 6cx - 3x^2 dx = 16$, then c =

A. -2
B. 2
C. -4
D. 4
E. None of these

2. Suppose that $f''(x) = 2x - \frac{2}{x^2}$. On which of the following intervals is f decreasing?

A. $(-\infty, 0)$ only

B. $(-\infty, -\sqrt[3]{2})$ only

C. $(-\sqrt[3]{2}, 0)$ only

D. $(-\infty, -\sqrt[3]{2})$ and $(0, \infty)$ only

E. None of these

3. Let f be a differentiable function with $f(2) = 3$ and $f'(2)=4$. Let g be defined by $g(x) = x^2 f(x)$. Which of the following is an equation of the tangent line for g at the point $x = 2$?

A. $y - 12 = 4(x - 2)$
B. $y - 3 = -28(x - 2)$
C. $y - 3 = 4(x - 2)$
D. $y - 12 = 28(x - 2)$
E. None of these

4. $\lim_{h \to 0} \dfrac{\dfrac{1}{x} - \dfrac{1}{x+h}}{h} =$

A. 0
B. 1
C. DNE
D. $+\infty$
E. None of these

5. A farmer wants to create a rectangular pen using 1000 yards of fencing. The pen will be bounded on one side by a barn, so fencing is only needed on the other three sides of the pen. What is the maximum size that the pen can be?

A. 62,500 square yards
B. 111,088.89 square yards
C. 125,000 square yards
D. 250,000 square yards
E. None of these

6. Suppose that the width of a rectangle is increasing at rate of 3 inches/sec and its length is increasing at a rate of 4 inches/sec. At what rate is the area of the rectangle increasing when its width is 5 inches and its length is 6 inches?

A. 30 sq inches/sec
B. 38 sq inches/sec
C. 39 sq inches/sec
D. 7 sq inches/sec
E. None of these

7. A science class decides to drop a rock from the top of a 400 ft building. The rock falls to the ground in a straight line, and the rock has fallen a distance of $s(t) = 16t^2$ feet after t seconds. What is the speed of the rock when it hits the ground?

A. 32 feet/sec
B. 160 feet/sec
C. 320 feet/sec
D. 640 feet/sec
E. None of these

8. $\lim_{x \to -\infty} \frac{\sqrt{4x^2 - 7}}{3x - 8} =$

A. 0
B. $-\frac{2}{3}$
C. -1
D. $-\frac{4}{3}$
E. None of these

9. Determine the slope of the line tangent to the curve $x^2y - y^2x = x^2 + 3$ at the point (-3, 1).

A. $\frac{1}{15}$
B. $\frac{2}{3}$
C. $\frac{1}{3}$
D. $-\frac{2}{3}$
E. None of these

10. Let $f(x) = \sqrt{x - 1}$ on the interval from [1, 5]. What is the value of *c* guaranteed to exist by the Mean Value Theorem?

A. 1
B. 2
C. 3
D. 4
E. None of these

11. If $f'(x)$ is an increasing and negative function, then *f* is

A. increasing and concave up
B. increasing and concave down
C. decreasing and concave up
D. decreasing and concave down
E. None of these

12. Calculate the left Riemann sum for the function $f(x) = -x$ on the interval [-2, 2] when $n = 2$.

A. 3
B. 4
C. 2
D. -2
E. None of these

13. If $\int_1^7 f(x)dx = 13$ and $\int_1^5 f(x)dx = 6$, then $\int_7^5 f(x)dx =$

A. 7
B. 19
C. -7
D. -19
E. None of these

14. Suppose that $f(5) = 2$, $f'(5) = 6$, $g(5) = 3$, and $g'(5) = 1$. What is $(g/f)'(5)$?

A. -4
B. $\frac{16}{9}$
C. 4
D. $-\frac{16}{9}$
E. None of these

15. A company determines that the profit *P* (in dollars) made from selling *x* pencils is given by the function $P(x) = 0.0003x^3 + 6x$. What is the marginal profit when 76 pencils are sold?

A. \$11.20
B. \$11.40
C. \$12.00
D. \$11.05
E. None of these

16. Determine the point(s) at which $f(x) = 3x^5 - 5x^3 + 15$ has a local maximum.

A. 0 only
B. 1 only
C. -1, 0, 1
D. -1 only
E. None of these

17. Determine $\frac{dy}{dx}$ when $y = 10^{x^2-1}$.

A. $(\ln 10)10^{x^2-1}$
B. $(2x)10^{x^2-1}$
C. $(x^2-1)10^{x^2-1}$
D. $x^2(\ln 10)10^{x^2-1}$
E. None of these

18. If the function $f(x) = ax^3 + 4x^2 + cx + d$ has an inflection point at (1,0), then a = ____.

A. $\frac{2}{3}$
B. $\frac{4}{3}$
C. $\frac{1}{2}$
D. $-\frac{4}{3}$
E. None of these

19. If the function $f(x) = x^3$ has an average value of 9 on the closed interval [0, k], then k =

A. 3
B. $3^{\frac{1}{2}}$
C. $36^{\frac{1}{3}}$
D. $36^{\frac{1}{4}}$
E. None of these

20. Suppose that f is continuous on [-3, 3], and $f(-3) = 5$, $f(0) = -3$, and $f(3) = 17$. Using the Intermediate Value Theorem, how many roots are guaranteed on the closed interval [-3, 3]?

A. 0
B. 1
C. 2
D. 3
E. None of these

21. If $f(x) = (x-1)(x^2+2)^3$, then $f'(1) =$

A. 54
B. 27
C. 0
D. -27
E. None of these

22. $\int \frac{x}{x^2-4} dx =$

A. $\frac{1}{2(x^2-4)} + C$
B. $\frac{1}{2}\ln|x^2-4| + C$
C. $2\ln|x^2-4| + C$
D. $\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$
E. None of these

23. Determine the value of x such that $f(x) = \frac{8x^2}{x^2+1}$ has a horizontal tangent.

A. $x = 0$
B. $x = \frac{1}{8}$
C. $x = -\frac{1}{8}$
D. $x = -8$
E. None of these

24. $\int_1^2 \left(p + \frac{1}{p} \right)^2 dp =$

A. $\frac{15}{2}$
B. $\frac{37}{6}$
C. $\frac{5}{6}$
D. $\frac{29}{6}$
E. None of these

25. At what point does the function $f(x) = x^4 - 2x^2$ attain its average value on the interval from [0, 1]?

A. $x = .13$
B. $x = .24$
C. $x = .37$
D. $x = .53$
E. None of these

ACTM Regional Calculus Competition **Name:_____**
Tie Breaker Questions
April 25, 2015 **School/Teacher:_____**

Reminder: Attempt the tie-breaker questions in sequential order (Do #1 first, followed by #2, and then #3 last).

1. The function $f(x) = x^3 + ax^2 + bx + c$ has a relative maximum at (-3, 25) and an inflection point at $x = -1$. Determine a, b, and c.

2. If *a* and *b* are positive numbers, find the maximum value of $f(x) = x^a(1-x)^b$ on the closed interval [0, 1].

3. Determine the point on the line $y = 2x + 3$ that is closest to the origin.

Calculus Key

1. B
2. C
3. D
4. E
5. C
6. B
7. B
8. B
9. A
10. B
11. C
12. B
13. C
14. A
15. A
16. D
17. E
18. D
19. C
20. C
21. B
22. B
23. A
24. D
25. E

1. $a = 3, b = -9, c = -2$

2. $f\left(\frac{a}{a+b}\right) = \frac{a^a b^b}{(a+b)^{a+b}}$

3. $\left(-\frac{6}{5}, \frac{3}{5}\right)$