ACTM State Pre-Calculus/Trigonometry Exam
April 23, 2016

Mark your answer choice on the answer sheet provided. If you have time, answer each of the tiebreaker items in sequential order (do #1 first, followed by #2, and then #3 last). Be sure that your name is printed on each of the tiebreaker pages.

1. Evaluate $\log_4 8 \cdot \log_8 4 \cdot \log 2 \cdot \log_2 10$.

a) -1 b) 0 c) 1 d) 2

2. Determine the polar coordinates of the point in rectangular coordinates $(1, \sqrt{3})$.

a) $(2,30°)$ b) $(-2,60°)$ c) $(-2,30°)$ d) $(2,60°)$

3. Solve for x: $3^{6-x} = 9^x$.

a) 3/5 b) 2 c) 3 d) 3/2

4. Find the angle between vectors $u = i + j$ and $v = i - j$.

a) 0° b) 180° c) 90° d) 45°

5. Determine if the following function has a horizontal asymptote, and if so find it: $f(x) = \frac{6e^{-x}+4}{2-3x}$.

a) $y = 0$ b) $y = 2$ c) $y = 3$ d) no horizontal asymptote.

6. Solve the triangle with sides $a = 2, b = 3$, and angle $A = 30°$ (opposite to side a). How many solutions are there?

a) infinitely many b) none c) one d) two

7. Evaluate $\cot\left[\sin^{-1}\left(-\frac{3}{5}\right)\right]$.

a) $-\frac{3}{4}$ b) $-\frac{4}{3}$ c) $\frac{3}{4}$ d) $\frac{4}{3}$

8. A Ferris wheel has a diameter of 100 feet and a rider travels at a speed of 3 feet per second. If a rider is at the top, approximately how long does it take them to reach the ground?

a) 52 sec. b) 33 sec. c) 17 sec. d) 105 sec.

9. If $f(x)$ is an even function, choose the odd function:

a) $g(x) = f(\sin(x))$ b) $g(x) = \sin(f(x))$

c) $g(x) = \cos(f(x))$ d) none of these

10. If $\sin\theta = x$ and θ is in quadrant II, find $\sin(2\theta)$.

a) $-2x$ b) $-2x\sqrt{1-x^2}$ c) $2x\sqrt{1-x^2}$ d) $2\sqrt{1-x^2}$

11. Determine the fundamental period of the function $f(x) = \tan(3x-1)+2$.

a) π b) 3π c) $\pi/3$ d) none of these

12. Simplify the expression $\frac{1+2i}{3-4i} + \frac{2-i}{5i}$.

a.) -2/5 b.) $\frac{12i+4}{25}$ c.) $\frac{12i-4}{25}$ d.) 1

13. What is the rectangular equation of the polar equation $r = 3\cos\theta$?

a.) $x^2+y^2 = 9y$ b.) $x^2+y^2 = 3x$

c.) $x^2+y^2 = 3$ d.) $x^2+y^2 = 3y$

14. Find the asymptotes of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$.

a.) $y = \pm\frac{4}{3}x$ b.) $y = \pm\frac{16}{9}x$ c.) $y = \pm\frac{3}{4}x$ d.) $y = \pm\frac{9}{16}x$

15. Find the domain of $f(x) = \ln\left(\frac{x^2-9}{x}\right)$.

a) $(-\infty,-3) \cup (3,\infty)$ b) $(3,\infty)$ c) $(-\infty,-3] \cup [3,\infty)$ d) $(-3,0) \cup (3,\infty)$

16. Choose the interval on which the sine function is invertible.

a) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ b) $[0, \pi]$ c) $[\pi, 2\pi]$ d) $[0, 2\pi]$

17. A satellite dish is 12 feet in diameter at its opening and 4 feet deep at its center. A receiver must be placed at the focus in order for the dish to work properly. How far should the receiver be placed from the vertex of the dish?

a.) 4 feet b.) 3.5 feet c.) 2.25 feet d.) 1.75 feet

18. Find the center of the ellipse $4x^2 + 9y^2 - 8x - 36y + 4 = 0$.

a.) $(2, 1)$ b.) $(1, 2)$ c.) $(-1, -2)$ d.) $(-2, -1)$

19. A triangular piece of glass is to be cut to fit in a side window of a small boat. The dimensions of the glass should be 14 inches, 30 inches, and 36 inches. In order to mark off the triangle, the glass cutter needs to know the angle opposite the longest side. Find this angle.

a.) $98°$ b.) $85°$ c.) $95°$ d.) $104°$

20. Write the standard form of the ellipse satisfying the given conditions: center: (2,3); vertices: (2,6) and (2,0); endpoints of minor axis: (0,3) and (4,3).

a.) $\frac{(x-3)^2}{4} + \frac{(y-2)^2}{9} = 1$ b.) $\frac{(x-2)^2}{9} + \frac{(y-3)^2}{4} = 1$

c.) $\frac{(x-3)^2}{9} + \frac{(y-2)^2}{4} = 1$ d.) $\frac{(x-2)^2}{4} + \frac{(y-3)^2}{9} = 1$

21. Find the sum: $\sum_{n=2}^{\infty} \left(\frac{3}{4}\right)^{n-1}$

a) 3 b) 4 c) $4/3$ d) $1/4$

22. Find the inverse function of $f(x) = \sin\left(\frac{x}{x+1}\right)$.

a) $f^{-1}(x) = \frac{\sin^{-1} x}{1+\sin^{-1} x}$

b) $f^{-1}(x) = \frac{1}{1-\sin^{-1} x}$

c) $f^{-1}(x) = \frac{\sin^{-1} x}{1-\sin^{-1} x}$

d) $f^{-1}(x) = \frac{1}{\sin^{-1} x-1}$

23. Find the solution set of $\cos(2\theta) + \cos(\theta) = 0$ for $0 \leq \theta < 2\pi$.

a) $\left\{\frac{\pi}{3}, \pi, \frac{5\pi}{3}\right\}$

b) $\left\{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\right\}$

c) $\left\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

d) $\left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\right\}$

24. Solve for x: $\log_3(1 + x) = 1 + \log_3(x)$.

a) 1

b) $1/2$

c) 3

d) 2

25. Find the modulus of the complex number: $\frac{5i}{2+i}$.

a) $\sqrt{2}$

b) 1

c) $\frac{5}{3}\sqrt{5}$

d) $\sqrt{5}$

TIEBREAKERS

Name: ________________________________

Show all your work to receive maximum credit.

1. A vertical tower that is 123 feet high stands on a hill. The angle of elevation of the top of the tower from a point 296 feet down the hill from the base is 26.2 degrees. Find the angle of inclination of the hill with the horizontal.

2. A population of bacteria in a petri dish is growing according to the function $N(t) = N_0 e^{rt}$ where $N(t)$ is the population size at time t in hours, N_0 is the initial population size, and r is the growth rate. If the population was 100 after 1 hour and 250 after 2 hours, what was the initial population size?

3. Given that angle θ is in the first quadrant and that $\sin\theta = \frac{3}{5}$, evaluate the sum: $\sum_{n=0}^{\infty} \sin^n(3\theta)$

TEST ANSWER KEY:

1) C
2) D
3) B
4) C
5) A
6) D
7) B
8) A
9) D
10) B
11) C
12) A
13) B
14) C
15) D
16) A
17) C
18) B
19) D
20) D
21) A
22) C
23) A
24) B
25) D

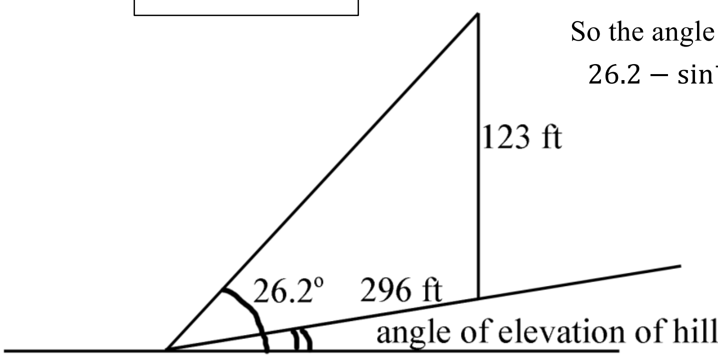
TIEBREAKER SOLUTIONS:

1) Answer: 4.3°

Angle from top of tower is 90-26.2 = 63.2°

So the angle of elevation of the hill is

$$26.2 - \sin^{-1}\left(123\,\frac{\sin 63.2°}{296}\right) = 4.3°$$



2) $r = \ln\frac{N(2)}{N(1)} = \ln 2.5$ so $N(0) = \frac{N(1)}{e^r} = 40$

Answer:
$N_0 = 40.$

3) $\sin 3\theta = \sin(2\theta + \theta) = \sin 2\theta \cos\theta + \cos 2\theta \sin\theta = 2\sin\theta\cos^2\theta + (1 - 2\sin^2\theta)\sin\theta$

$$= 2\frac{3}{5}\left(\frac{4}{5}\right)^2 + \left(1 - 2\left(\frac{3}{5}\right)^2\right)\frac{3}{5} = \frac{117}{125}$$

Thus the series sums to $\frac{1}{1-\frac{117}{125}} = \frac{125}{8}$

Answer:
$$\frac{125}{8}$$