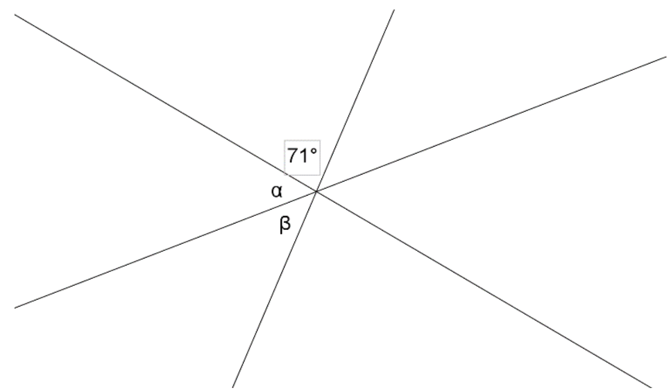
2016 State Mathematics Contest
Geometry Test

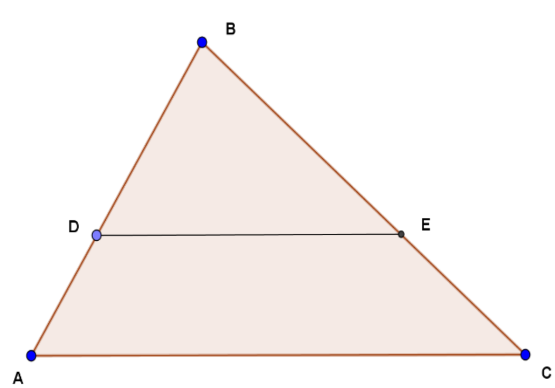
In each of the following, choose the BEST answer and record your choice on the answer sheet provided. To ensure correct scoring, be sure to make all erasures completely. The tiebreaker questions at the end of the exam will be used to resolve ties in first, second, and/or third place. They will be used in the order given. Complete the 25 multiple choice questions before attempting the tiebreaker questions. Figures are not necessarily drawn to scale.

1. In the figure to the right, if $\alpha = 2\beta - 26$, the measure or angle α is
   a. $45°$
   b. $64°$
   c. $50°$
   d. $109°$
   e. None of these



2. In a $30° - 60° - 90°$ triangle, if the length of the hypotenuse is 11, what is the length of the side opposite the side opposite the $60°$ angle?
   a. $\frac{11}{2}$
   b. $11\sqrt{3}$
   c. $\frac{11\sqrt{3}}{2}$
   d. $11\sqrt{2}$
   e. $\frac{11\sqrt{3}}{3}$

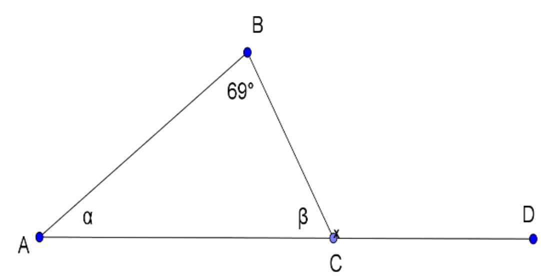
3. In the figure below, $5AC = 8DE$. If $AD = 24$, then $AB =$
   a. 64
   b. 40
   c. 88
   d. 32
   e. 56



4. To circumscribe a triangle, which of the following must be constructed?
   a. The perpendicular bisectors of two sides.
   b. The bisectors of two angles.
   c. The altitudes of the triangle.
   d. The diameter of the circle.
   e. None of these.

5. The base angles of an isosceles triangle are $30°$. If the length of the base is 15, find its area. Round to the nearest tenth.
   a. 194.9
   b. 97.4
   c. 110.9
   d. 77.9
   e. None of these

6. In the figure below, find α if $m\angle BCD = 3\beta$.
   a. 45°
   b. 55°
   c. 66°
   d. 37°
   e. Not enough information



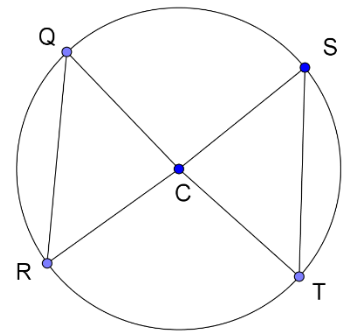
7. Write the equation of the circle with the segment whose endpoints are $(-2, 5)$ and $(4, -7)$ as a diameter.
   a. $(x+2)^2 + (y-5)^2 = 180$
   b. $(x+1)^2 + (y-1)^2 = 45$
   c. $(x-4)^2 + (y+7)^2 = 180$
   d. $(x-1)^2 + (y+1)^2 = 180$
   e. $(x-1)^2 + (y+1)^2 = 45$

8. If a regular pentagon is circumscribed by a circle of diameter 20, what is the length of a side? Round to the nearest hundredth.
   a. 19.4
   b. 10.51
   c. 14.14
   d. 8.66
   e. 11.76

For problems 9 – 11, consider the following proof.

Given: A circle with center C with chords $\overline{QR} \cong \overline{ST}$

Prove: $\widehat{QR} \cong \widehat{ST}$



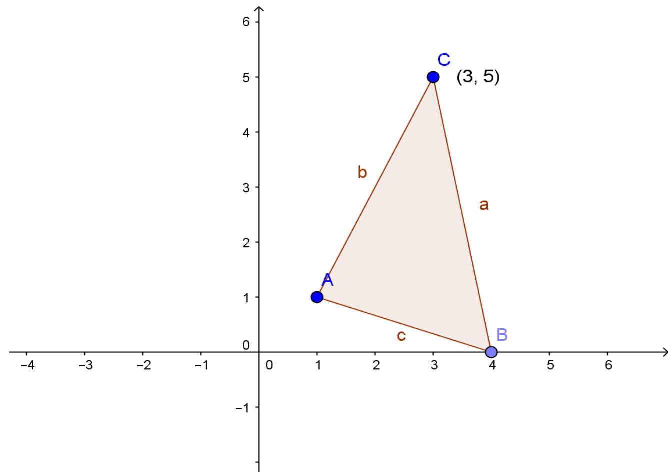
Statements	Reasons
1. $\overline{QR} \cong \overline{ST}$	1. Given
2. $\overline{CQ} \cong \overline{CS}, \overline{CR} \cong \overline{CT}$	2. ?
3. $\Delta CQR \cong \Delta CST$	3. ?
4. $\angle QCR \cong \angle SCT$	4. Corresponding angles of congruent triangles are congruent.
5. $\widehat{QR} \cong \widehat{ST}$	5. ?

9. Which reason justifies the statement in step 2 of the proof?
   a. Definition of midpoint.
   b. In a given circle, all radii are congruent.
   c. Corresponding sides of congruent triangles are congruent.
   d. Definition of distance.
   e. None of these.

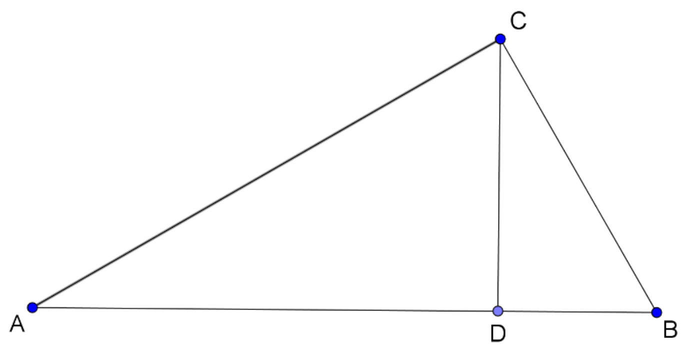
10. Which reason justifies the statement in step 3 of the proof?
   a. Side-Angle-Side
   b. Angle-Side-Angle
   c. Angle-Angle-Side
   d. Side-Side-Side
   e. None of these.

11. Which reason justifies the statement in step 5 of the proof?
   a. Congruent chords have congruent corresponding arcs.
   b. If the radii are congruent, then the arcs are congruent.
   c. Congruent central angles cut off congruent arcs.
   d. Vertical angles cut off congruent arcs.
   e. None of these.

12. If the triangle in the figure is rotated counterclockwise about the origin through an angle of $90°$, what will be the coordinates of the image of $C(3, 5)$?
   a. $(-3, 5)$
   b. $(-5, 3)$
   c. $(5, 3)$
   d. $(-3, -5)$
   e. None of these

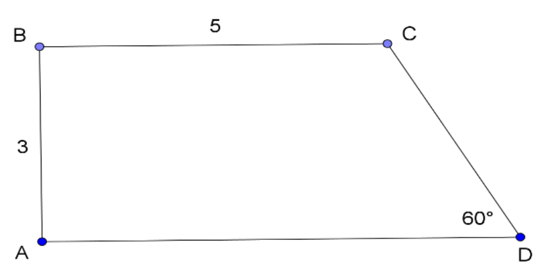


13. In the right triangle below, if $AB = 35$ and $AC = 28$, then $BD =$
   a. 14.8
   b. 15.2
   c. 10.8
   d. 12.6
   e. 9.7



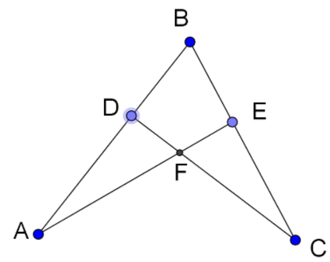
14. Which of the following points is on the perpendicular bisector of the segment with endpoints $(-4, 1)$ and $(8, -3)$?
   a. $(-4, 1)$
   b. $(8, -3)$
   c. $(-1, 0)$
   d. $(-2, 1)$
   e. $(3, 2)$

15. Assuming that $\overleftrightarrow{AB} \perp \overleftrightarrow{AD}$, Find the area of the trapezoid in the figure.
   a. $15 + \frac{3\sqrt{3}}{2}$
   b. $15 + 3\sqrt{3}$
   c. $30 + 3\sqrt{3}$
   d. $\frac{45}{\sqrt{3}}$
   e. None of these



For problems 16 and 17, consider the following proof.

Given: $\overline{AB} \cong \overline{CB}$, $\overline{AF} \cong \overline{CF}$,
$\overline{AE}$ and $\overline{CD}$ intersect at F
Prove: $\overline{AD} \cong \overline{EC}$.



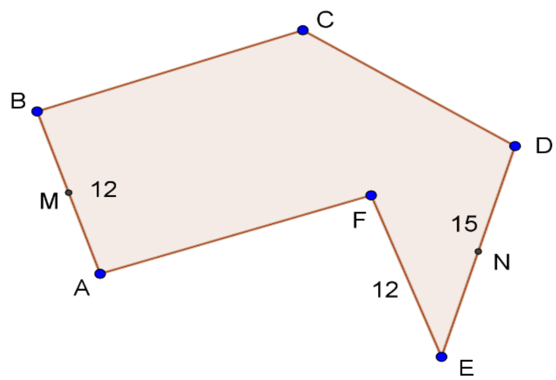
Statements	Reasons
1. $\overline{AB} \cong \overline{CB}$, $\overline{AF} \cong \overline{CF}$, $\overline{AE}$ and $\overline{CD}$ intersect at F.	1. Given
2. $\angle BCA \cong \angle BAC$, $\angle FCA \cong \angle FAC$	2. ?
3. $m\angle BCA = m\angle BCD + m\angle ECA$, $m\angle BAC = m\angle BAE + m\angle EAC$	3. Angle Addition Postulate
4. $m\angle DAF = m\angle ECF$	4. Subtraction Property of Equality
5. $\angle DAF \cong \angle ECF$	5. Definition of Congruence
4. $\angle DFA \cong \angle EFC$	4. Vertical angles are congruent.
6. $\Delta DAF \cong \Delta ECF$	6. ?
7. $\overline{AD} \cong \overline{EC}$	7. Corresponding sides of congruent triangles are congruent.

16. Which reason justifies the statement in step 2 of the proof?
    a. Corresponding angles of congruent triangles are congruent.
    b. Sides opposite congruent angles are congruent.
    c. Angles opposite congruent sides are congruent.
    d. Angle Addition Postulate
    e. None of these

17. Which reason justifies the statement in step 6 of the proof?
    a. Side-Angle-Side
    b. Angle-Side-Angle
    c. Angle-Angle-Side
    d. Side-Side-Side
    e. None of these.

18. If a sphere has radius 6, find the ratio of the volume of the sphere to its surface area.
    a. 6:1
    b. 3:1
    c. 4:3
    d. 12:1
    e. 2:1

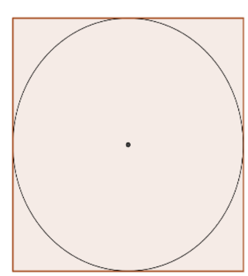
For problems 19 and 20, refer to the figure below. Assume that $\overleftrightarrow{AB} \parallel \overleftrightarrow{EF}$, D is on $\overleftrightarrow{AF}$, $\overline{FN}$ is an altitude of ΔDEF, $m\angle DFE = 90°$, and $AD = 25$.



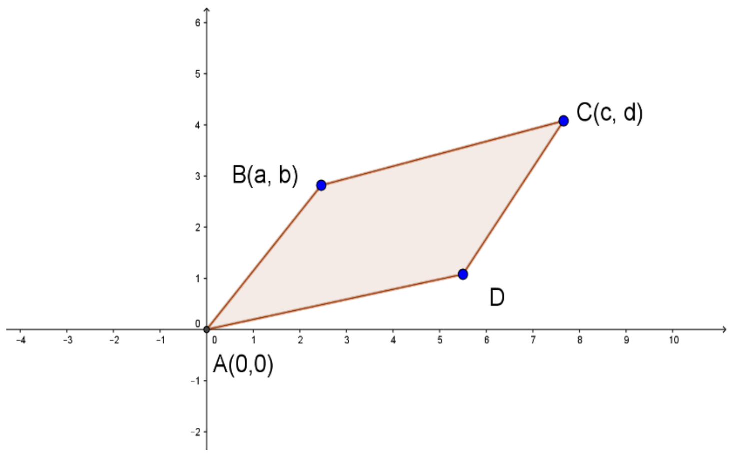
19. If M is the midpoint of $\overline{AB}$, what is the length of the shortest path from M to N entirely within the polygon?
    a. 55
    b. 41.5
    c. $12 + 6\sqrt{2}$
    d. 24.3
    e. $6\sqrt{3} + 6\sqrt{2}$

20. If $BC = AF$, find the radius of a circle whose area is equal to the area of the figure.
    a. 9.77
    b. 10
    c. 17.32
    d. 8.66
    e. None of these

21. In the figure to the right, the inscribed circle has area 200π. Find the area of the square.
    a. 628.3
    b. 200
    c. 800
    d. 401.1
    e. None of these



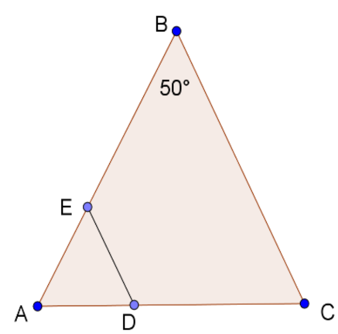
22. In the figure to the right, the coordinates D are
    a. $(a + c, b + d)$
    b. $(a - c, b - d)$
    c. $(c - a, d - b)$
    d. (ac, bd)
    e. None of these



23. If two angles of a triangle are $52°$ and $25°$, which of the following would be an exterior angle of the triangle?
    a. $103°$
    b. $61°$
    c. $33°$
    d. $155°$
    e. None of these

In the isosceles triangle to the right, $\overleftrightarrow{ED} \parallel \overleftrightarrow{BC}$, the ratio of BE to EA is 3:2, and $AC = 12$.

24. What is the area of the trapezoid $EDCB$?
    a. 46.32
    b. 64.85
    c. 51.47
    d. 34.31
    e. None of these

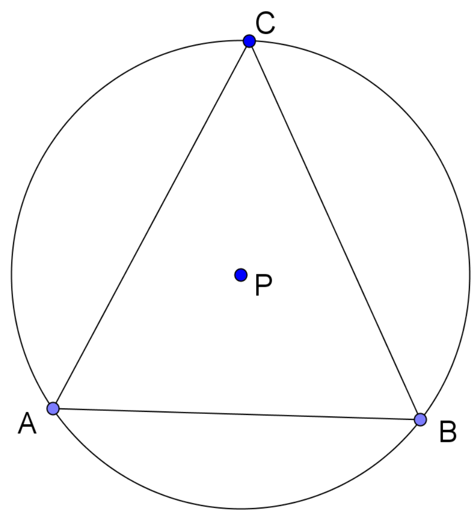


25. What is the length of $\overline{BE}$?
    a. 8.52
    b. 9.51
    c. 5.68
    d. 7.43
    e. None of these

Tie Breaker #1

Name: ___________________________________

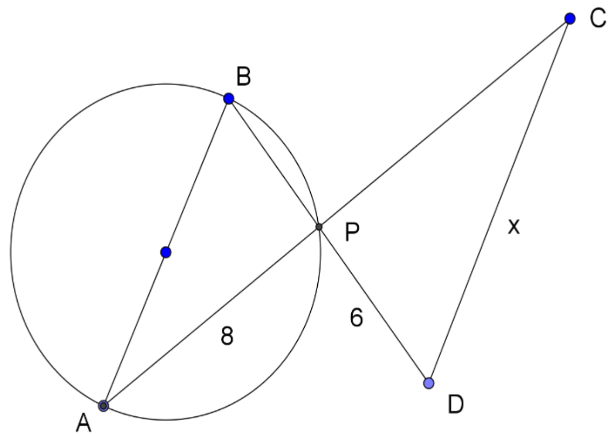
In circle P, $AC = BC$ and $m\angle ACB = \frac{2}{9} m\angle ABC$. Find the measure of $\angle BPC$.



Tie Breaker #2

Name: ____

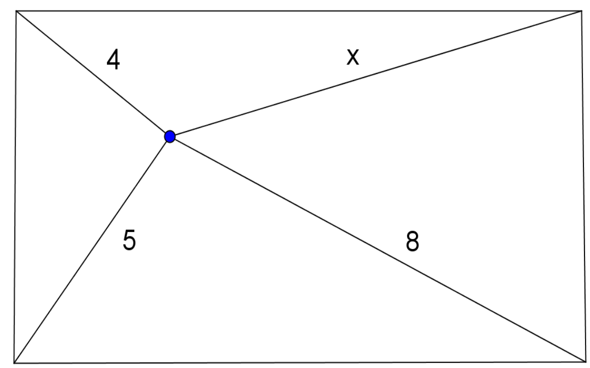
In the figure, $\overline{AB}$ is a diameter of the circle and $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$. Express the area of the circle in terms of x.



Tie Breaker #3

Name: ____

The figure below is a rectangle. Find the value of *x*.



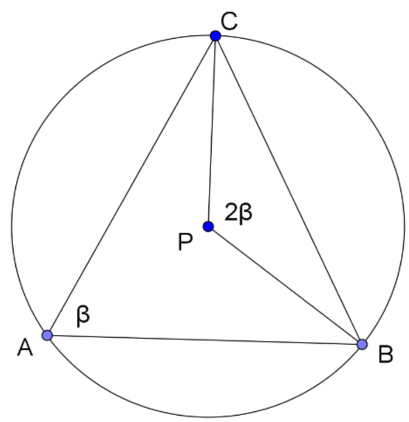
Answer Key

State Geometry Exam

1. A
2. C
3. A
4. A
5. E
6. C
7. D
8. E
9. B
10. D
11. C
12. B
13. D
14. E
15. A
16. C
17. B
18. E
19. D
20. A
21. C
22. C
23. D
24. B
25. A

Tie Breaker #1

In circle P, $AC = BC$ and $m\angle ACB = \frac{2}{9} m\angle ABC$. Find the measure of $\angle BPC$.



Since $AC = BC$, $m\angle ABC = m\angle BAC$. Let $\beta = m\angle ABC$. Then $m\angle ACB = \frac{2}{9}\beta$.

Therefore,

$$\beta + \beta + \frac{2}{9}\beta = 180°$$

$$\frac{20}{9}\beta = 180°$$

$$\beta = 81°$$

Since $\angle BAC$ is an inscribed angle,

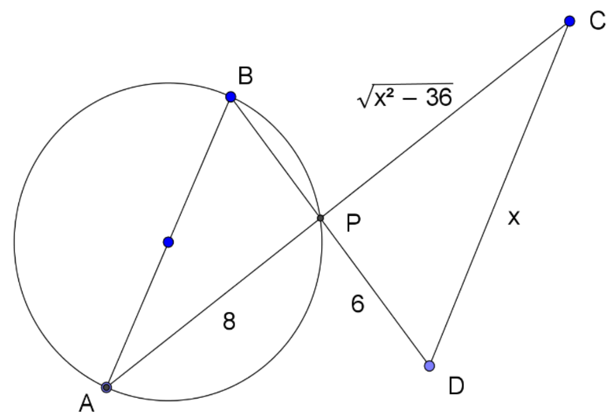
$$m\widehat{BC} = 2\beta = 162°$$

Since $\angle BPC$ is the central angle of $\widehat{BC}$,

$$m\angle BPC = 162°$$

Tie Breaker #2

In the figure, $\overline{AB}$ is a diameter of the circle and $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$. Express the area of the circle in terms of x.



Since $\overline{AB}$ is a diameter of the circle, ΔAPB is inscribed in a semicircle and is therefore a right triangle with $\angle APB$ being a right angle. By vertical angles, $\angle CPD$ is also a right angle so $\angle APB \cong \angle CPD$. Also, since $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$, $\angle BAP \cong \angle DCP$ by alternate interior angles. Therefore $\Delta APB \sim \Delta CPD$.

Hence we have the proportion

$$\frac{AP}{AB} = \frac{CP}{CD}$$

By Pythagorean Theorem,

$$CP = \sqrt{x^2 - 36}$$

Therefore, we have

$$\frac{8}{AB} = \frac{\sqrt{x^2 - 36}}{x}$$

$$8x = AB\sqrt{x^2 - 36}$$

$$AB = \frac{8x}{\sqrt{x^2 - 36}}$$

Therefore the radius of the circle is

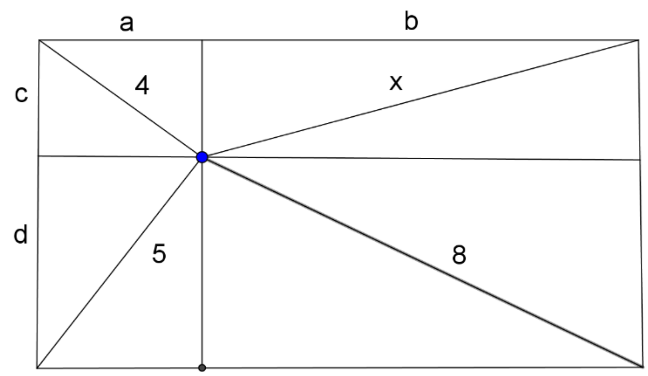
$$r = \frac{1}{2}AB = \frac{4x}{\sqrt{x^2 - 36}}$$

Therefore the area of the circle is

$$A = \pi r^2 = \frac{16\pi x^2}{\sqrt{x^2 - 36}}$$

Tie Breaker #3

The figure below is a rectangle. Find the value of x.



By Pythagorean Theorem, we have

1. $a^2 + c^2 = 4^2$
2. $a^2 + d^2 = 5^2$
3. $b^2 + c^2 = x^2$
4. $b^2 + d^2 = 8^2$

Subtracting equation (1) from equation (2) and equation (3) from equation (4), we get

5. $d^2 - c^2 = 5^2 - 4^2$
6. $d^2 - c^2 = 8^2 - x^2$

Combining equations (5) and (6), we get

$$8^2 - x^2 = 5^2 - 4^2$$

$$64 - x^2 = 9$$

$$x^2 = 55$$

$$x = \sqrt{55}$$