

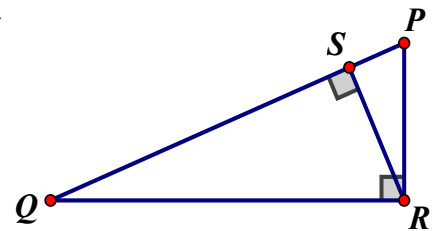
2015 – Arkansas Council of Teachers of Mathematics  
State Mathematics Contest  
Geometry Test

In each of the following choose the BEST answer and record your choice on the answer sheet provided. To insure correct scoring, be sure to make all erasures completely. The tie-breaker questions at the end will only be used to resolve ties in first, second, and/or third place. They will be used in the order given. Complete the first 25 questions before attempting the tie-breaker questions. Figures are not necessarily drawn to scale.

- Two rectangles are \_\_\_\_\_ similar.
  - Always
  - Sometimes
  - Never
  - Not enough information is provided
  
- If the diagonals of a quadrilateral are congruent and perpendicular, then the quadrilateral is \_\_\_\_\_ a square.
  - Always
  - Sometimes
  - Never
  - Not enough information is provided

- Find the length of  $\overline{PS}$  in the figure if  $QS = 6$  ft and  $PR = 4$  ft.

- 2
- 3
- 5
- 8
- Not enough information is provided



4. Which of the choices is the contrapositive of the following conditional statement. *If the fruit is picked, then it is ripe.*

- a. If the fruit is not ripe, then it is not picked.
- b. If the fruit is not picked, then it is not ripe.
- c. If the fruit is ripe, then it is picked.
- d. The fruit is not picked.
- e. None of the above

5.

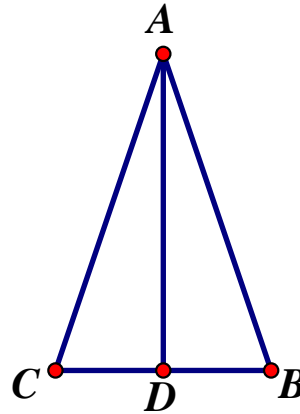
$\overline{DA}$  is the perpendicular bisector of  $\overline{CB}$ .

Which of the following is true?

P: AD is the angle bisector of  $\angle CAB$ .

Q:  $\triangle ADC \cong \triangle ADB$

R:  $\overline{CA} \cong \overline{AB}$

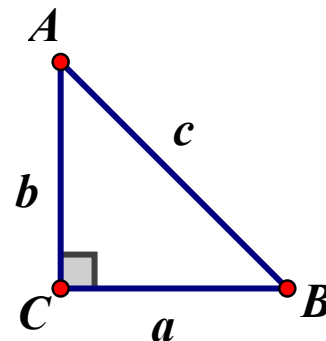


- a. P and Q
- b. P and R
- c. Q and R
- d. P, Q, and R
- e. None of the above

*Refer to the 45°-45°-90° triangle to answer questions 6.*

6. If  $a = 20$  ft., find  $c$ .

- a. 20 ft
- b.  $2\sqrt{10}$  ft
- c.  $20\sqrt{5}$  ft
- d.  $20\sqrt{2}$  ft
- e. None of the above

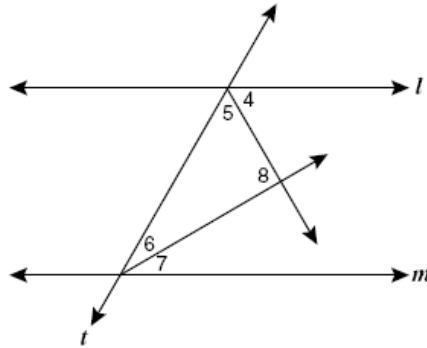


7. Parallel lines  $l$  and  $m$  are cut by transversal  $t$ ,  $m$ .

$\angle 8$  is a right angle because:

- a.  $m\angle 5 = m\angle 7$
- b.  $\angle 6$  and  $\angle 5$  are supplementary
- c.  $\angle 6$  and  $\angle 5$  are complementary
- d.  $m\angle 8 = m\angle 7$
- e. None of the above

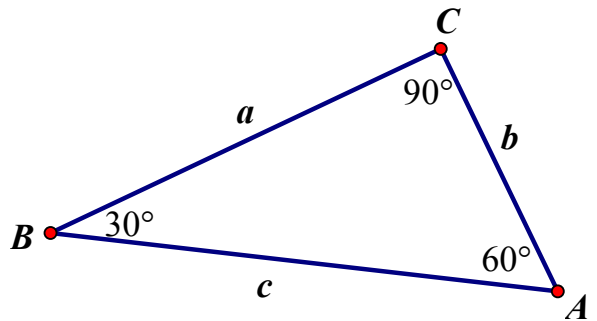
Parallel lines  $l$  and  $m$  are cut by transversal  $t$ ,  $m\angle 4 = m\angle 5$ , and  $m\angle 6 = m\angle 7$ .



Refer to the  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle to answer question 8.

8. If  $a = 20$  ft, find  $c$ .

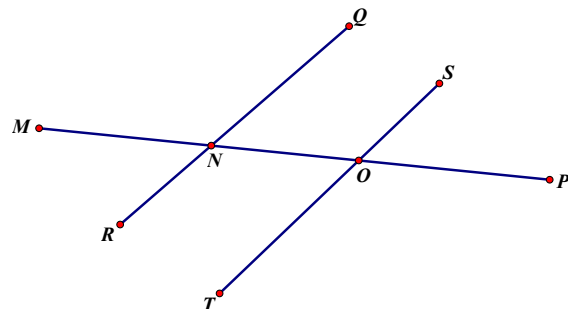
- a. 20 ft
- b. 40 ft
- c.  $10\sqrt{3}$  ft
- d.  $20\sqrt{3}$  ft
- e. None of the above



9. In the figure to the right, assume that  $\overline{RQ}$  is parallel to  $\overline{ST}$  and that  $\overline{MP}$  is a transversal. Let

$m\angle MNR = (4x + 35)^\circ$  and  $m\angle TOP = (73 - x)^\circ$ . Find the measure of  $m\angle RNO$  in degrees.

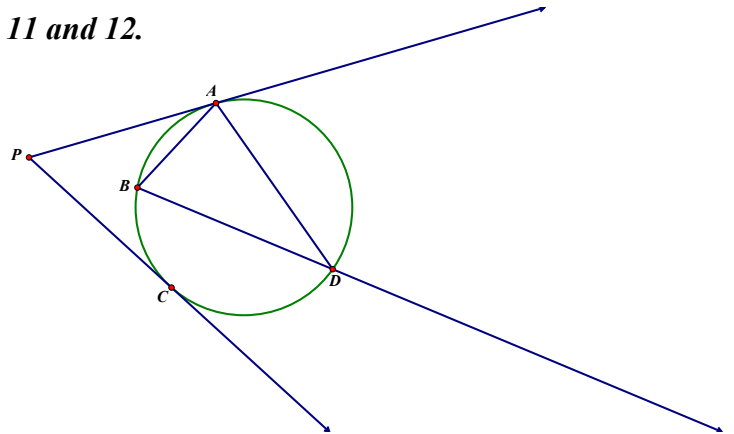
- a.  $24^\circ$
- b.  $49^\circ$
- c.  $131^\circ$
- d.  $82^\circ$
- e. None of the above



Use the diagram on the right to answer questions 11 and 12.

10. If  $m\angle ADB = 35^\circ$ , find  $m\angle PAB$ .

- a.  $140^\circ$
- b.  $70^\circ$
- c.  $35^\circ$
- d.  $17.5^\circ$
- e. None of the above

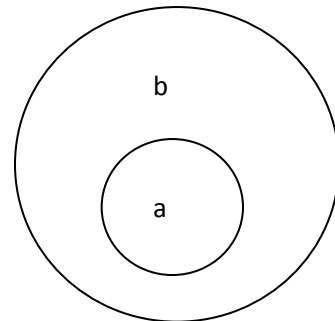


11. If  $AP = 20$  feet and  $DP = 30$  feet, find  $BP$ .

- a. 13.33 ft
- b. 30 ft
- c. 20 ft
- d. 25 ft
- e. None of the above

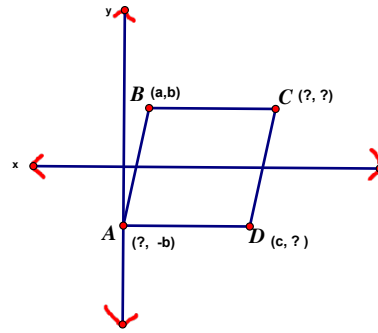
12. Which of the following statements does the Euler diagram represent?

- a.  $a$  if  $b$
- b.  $a$  only if  $b$
- c.  $b$  if and only if  $a$
- d.  $b$  only if  $a$
- e. None of the above



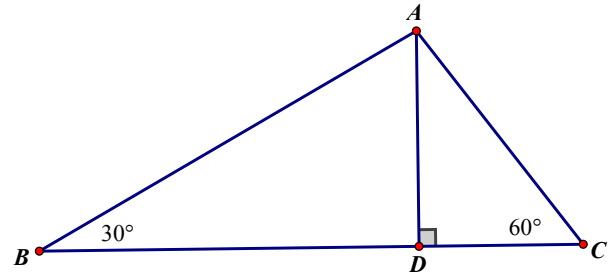
13. Given ABCD is a parallelogram. Find the missing coordinates using only the variable given and the properties of the figure.

- $A(-a, b), C(a, c), D(c, b)$
- $A(0, b), C(a + c, b), D(c, -b)$
- $A(-a, -b), C(a, c), D(c, -b)$
- $A(0, b), C(a + b, c), D(c, b)$
- Not enough information



14.  $\overline{AD}$  is an altitude for the  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle BCA. What is the relationship, if any, between BD and DC?

- $BD < DC$
- $BC = DC$
- $BD = 2(DC)$
- $BD = 3(DC)$
- No relationship exists



15. Write an equation of the line that is tangent to the circle  $x^2 + y^2 = 25$  at the point  $(-3, 4)$ .

- $y = \frac{4}{3}x + \frac{25}{4}$
- $y = \frac{3}{4}x + \frac{7}{4}$
- $y = \frac{-3}{4}x - \frac{25}{4}$
- $y = \frac{3}{4}x + \frac{25}{4}$
- None of the above

16. Find the coordinates of the point equidistant from the points  $(3, 11)$ ,  $(9, 5)$ , and  $(7, 1)$ .

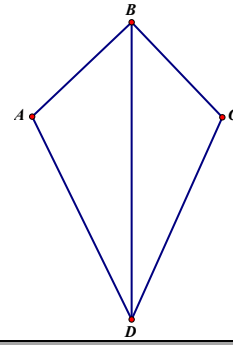
- $(-3.33, -5.33)$
- $(3.33, 5.33)$
- $(3, 5)$
- $(5.33, 3.33)$
- None of the above

Use the information provided to answer questions 17 - 19.

Given: Kite  $ABCD$  with diagonal  $\overline{BD}$

Prove:  $\overline{BD}$  bisects  $\angle ABC$  and  $\angle ADC$

Proof:



| Statements  | Reasons  |
|---|----------|
| 1. Kite $ABCD$ with diagonal $\overline{BD}$                              | 1. Given |
| 2. $\overline{AD} \cong \overline{CD}, \overline{AB} \cong \overline{CB}$ | 2. ?     |
| 3. ?  | 3. ?     |
| $\triangle BAD \cong \triangle BCD$                                       | 4. ?     |
| 4.  |          |
| 5. ?  | 5. CPCTC |
| $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$                     | 6. ?     |
| 6.  |          |

17. Which reason justifies the statement for step 2 in the proof?

- Definition of Rhombus
- Adjacent sides of a parallelogram are congruent
- Definition of Kite
- SSS
- None of the above

18. Which reason is justified by step 4 in the proof?

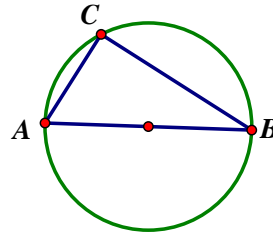
- SAS
- ASA
- SSS
- SSA
- None of the above

19. Which statement is justified by the reason for step 5 in the proof?

- $\angle ABD \cong \angle CBD, \angle ADB \cong \angle CDB$
- A kite has one pair of congruent angles
- One diagonal is the perpendicular bisector of the other  
 $\angle BAC \cong \angle BDC$
- None of the above.

20. The radius of the circle shown is 5. What are the possible lengths of the chords,  $\overline{AB}$  and  $\overline{BC}$ ?

- a.  $6, 4\sqrt{5}$
- b.  $5, 5\sqrt{3}$
- c.  $4, 3\sqrt{21}$
- d.  $3, 2\sqrt{26}$
- e. None of the above

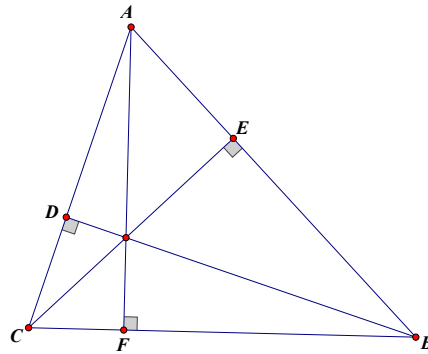


21. A quadrilateral ABCD is inscribed in a circle. What is always true about quadrilateral ABCD?

- a. It is a parallelogram
- b. It is a kite
- c. The opposite angles are supplementary
- d. The opposite sides are congruent.
- e. None of the above

22. Name the interior point in  $\triangle ABC$

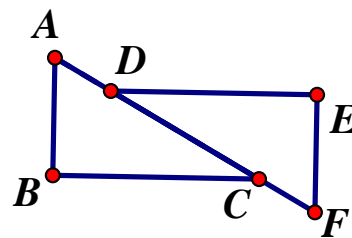
- a. Centriod
- b. Orthocenter
- c. Incenter
- d. Circumcenter
- e. None of the above



23. What additional information would you need to prove that  $\triangle ABC \cong \triangle FED$  if  $\overline{AD} \cong \overline{CF}$  by ASA?

- I.  $\overline{BC} \cong \overline{ED}$
- II.  $\overline{AB} \cong \overline{EF}$
- III.  $\angle BAC \cong \angle EFA$
- IV.  $\overline{BC} \parallel \overline{ED}$
- V.  $\overline{AB} \parallel \overline{EF}$

- a. I and III
- b. III only
- c. III and IV
- d. III and V
- e. None of the above



**24. An equilateral triangle and a regular hexagon are inscribed in the same circle. What is the ratio of the perimeter of the equilateral triangle to the perimeter of the hexagon?**

- a.  $1 : 2$**
- b.  $2 : 1$**
- c.  $\sqrt{3} : 2$**
- d.  $\sqrt{2} : 3$**
- e. Not enough information**

**25. A prism and a pyramid have the same number of edges. If the base of the pyramid is a dodecagon, then the number of vertices in the prism is:**

- a. 8**
- b. 10**
- c. 12**
- d. 16**
- e. None of the above**



### Tie-Breaker Question 1

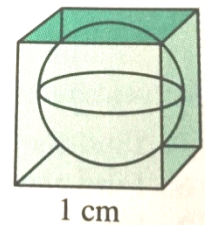
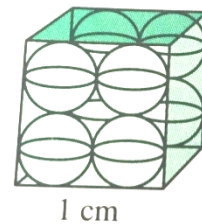
Name: \_\_\_\_\_

School: \_\_\_\_\_

Please Print

The following three questions will be used to break ties for first, second and/or third place. They will be used in the order they are given. Be sure you have answered the previous 25 questions before proceeding to the tie-breaker questions. Your answers should be written clearly with reasonable justification of your work.

A certain filter system is packed with spherical particles that absorb unwanted materials from liquids that are drained through the filter. The effectiveness of the filter depends on how much surface area of these particles is exposed. The particles come in two sizes – 1 cm in diameter (or 1 per cubic centimeter) and 0.5 cm in diameter (or 8 per cubic centimeter). Explain which size is more effective.



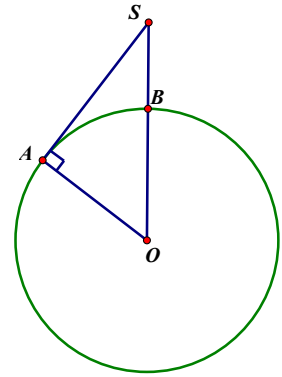
## Tie-Breaker Question 2

Name: \_\_\_\_\_

School: \_\_\_\_\_

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Assume that a cross section of the Earth is a circle with radius 4000 miles. If a communications satellite is in orbit 110 miles above the surface of the Earth, what is the approximate distance from the satellite to the horizon, the farthest point that can be seen on the surface of the Earth.



Tie-Breaker Question 3

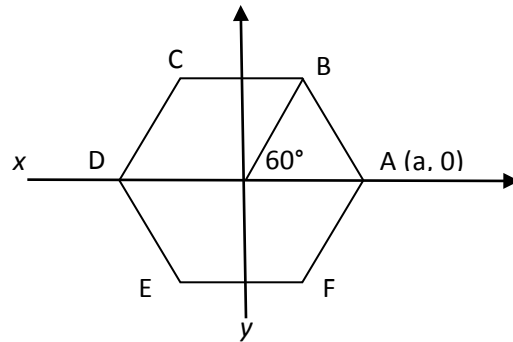
Name: \_\_\_\_\_

School: \_\_\_\_\_

Please Print

ABCDEF is a regular hexagon with coordinates for  $A$  as shown. Determine the coordinates for  $B$ ,

$D$ , and  $E$  and show that  $\overline{AB} \parallel \overline{DE}$ .



Solutions to Multiple Choice Geometry

1. B
2. B
3. A
4. A
5. D
6. D
7. C
8. E
9. A
10. C
11. A
12. B
13. B
14. D
15. D
16. B
17. C
18. C
19. A
20. B
21. C
22. B
23. C
24. C
25. D



Tie-Breaker Question 1—Solution

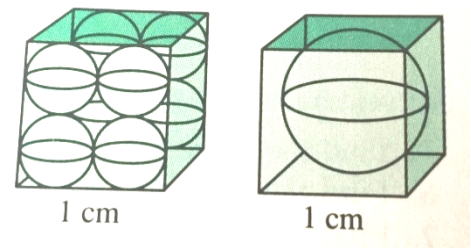
Name: \_\_\_\_\_

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Particles 0.5 cm in diameter are more effective. Larger particle has 4 times the surface area of one of the smaller, but there are 8 times as many of the smaller particles per unit volume.

Smaller size has twice the total surface area of the larger size particle.

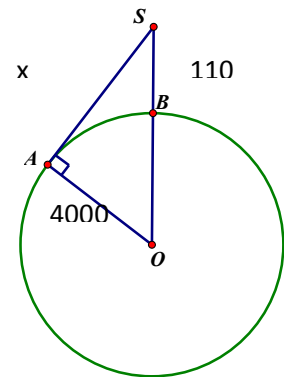
Tie-Breaker Question 2 – Solution

Name: \_\_\_\_\_

School: \_\_\_\_\_

Please Print

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$$x^2 + 4000^2 = 4110^2$$

$$x^2 + 16000000 = 16892100$$

$$x^2 = 892100$$

$$x = 944.5 \text{ miles away}$$

Tie-Breaker Question 3 – Solution

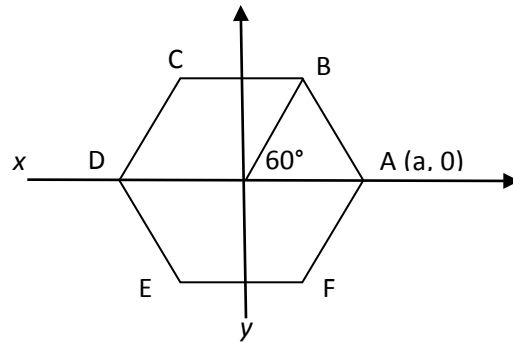
Name: \_\_\_\_\_

School: \_\_\_\_\_

Please Print

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$D$ , and  $E$  and show that  $\overline{AB} \parallel \overline{DE}$ .



$$B\left(\frac{a}{2}, \frac{a\sqrt{3}}{2}\right), D(-a, 0), E\left(-\frac{a}{2}, -\frac{a\sqrt{3}}{2}\right)$$

$$\text{Slope of } \overline{AB} = -\sqrt{3}, \text{Slope of } \overline{DE} = -\sqrt{3}$$