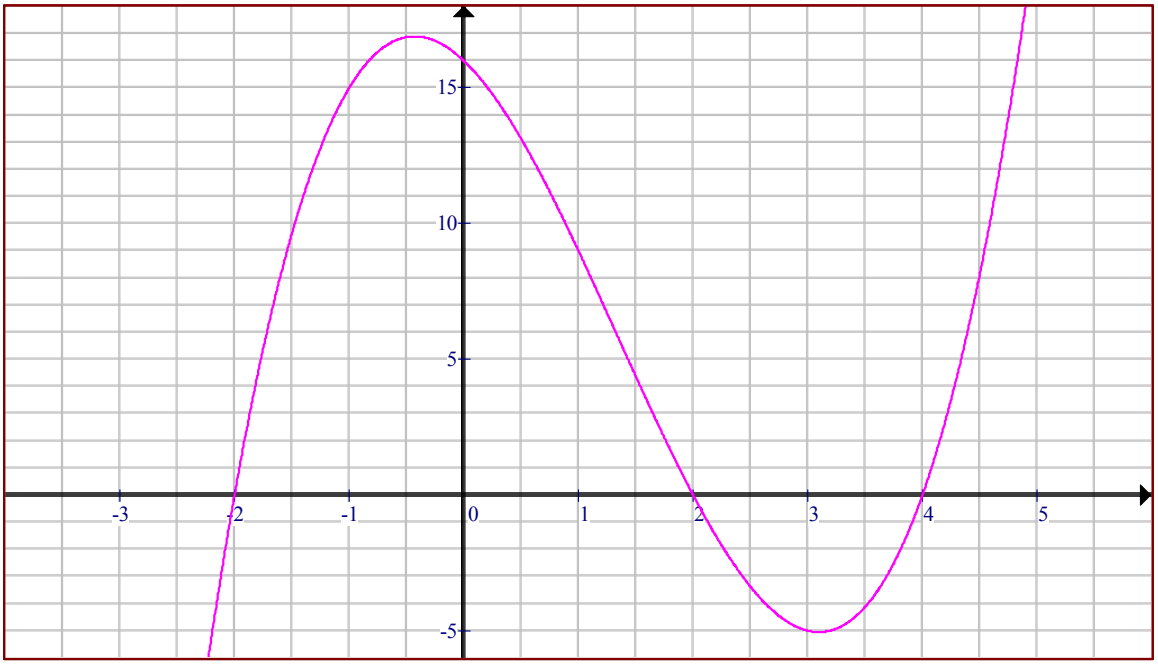
2015 ACTM Regional Algebra II Competition

Begin by removing the three tie breaker sheets at the end of the exam and writing your name on all three pages. Work the multiple choice questions first choosing the single best response from the choices available. Indicate your answer here and on your answer sheet. Make sure you attempt the tie-breaker questions at the end of the test starting with tie breaker 1, then 2, and then 3 if you have time. Turn in your answer sheet and the tie breaker pages when you are finished. You may keep the pages with the multiple choice questions.

1. $g(x) = 2x^2 - 3x + 4$. *f(x)* is given by the graph:



x	*h(x)*
-2	1
-1	0
0	2
1	-1
2	1

$g \circ f \circ h(0) =$

A. 409
B. 4
C. -1
D. 2
E. Each of the other answers is incorrect.

2. The sum of the solutions to $x^3 = 1$ is
   A. -1
   B. 0
   C. 1
   D. 3
   E. Each of the other answers is incorrect.

3. How many solutions does $5^x = x^5$ have?

   A. 0
   B. 1
   C. 2
   D. 3
   E. Each of the other answers is incorrect.

4. Solve the following system of equations:

$$\begin{aligned} 3x + 5y - 2z &= 14 \\ -1x + 4y - 3z &= 1 \\ 3x + y + z &= 10 \end{aligned}$$

   The *x* coordinate of the solution is:
   A. 0
   B. 1
   C. 2
   D. 3
   E. Each of the other answers is incorrect.

5. What are the vertical asymptotes of $f(x) = \dfrac{x-3}{x^2 - 4x + 3}$?
   A. $x = 1$
   B. $x = 3$
   C. $x = 1$ and $x = 3$
   D. $x = 0$
   E. Each of the other answers is incorrect.

6. The following gives temperatures and times at a certain city in Arkansas on a certain day.

Time	1:00 pm	2:00 pm	3:00 pm
Temperature	51^oF	40^oF	35^oF

   For the interval from 1:00 to 3:00 what is the average rate of change in the temperature?

   A. 42^oF
   B. -16^oF/hour
   C. -11^oF/hour
   D. -8^oF/hour
   E. Each of the other answers is incorrect.

7. We purchase a new automobile for $13,900. (That is the value of the automobile when it is driven off the lot is $13,900.) It depreciates 17.2% per year. What is the value of the car three years after it is purchased?
   A. $22,380
   B. $7,170
   C. $7,250
   D. $8,300
   E. $7,890

8. Which of the following is NOT an identity for positive values of the variables?
   A. $\ln(xy) = \ln(x) + \ln(y)$
   B. $\ln(x^n) = n\ln(x)$
   C. $\log_b(x) = \dfrac{\ln(x)}{\ln(b)}$
   D. $\ln\left(\frac{x}{y}\right) = \dfrac{\ln(x)}{\ln(y)}$
   E. $\log_b(1) = 0$

9. Simplify the following as far as possible using reduced radicals:
$$3\sqrt{28} - 2\sqrt{63} - \sqrt[4]{49} =$$
   A. $-\sqrt{7}$
   B. $14\sqrt{2} - 18\sqrt{3} - \sqrt{7}$
   C. $4\sqrt{7} - \sqrt[4]{49}$
   D. $-7\sqrt{7}$
   E. Each of the other answers is incorrect.

10. Completely simplify the following using reduced radicals and rationalized denominators:

$$\frac{3x^2y}{\sqrt[3]{16x^5y^4z^{15}}} =$$

A. $\dfrac{3x^2y\sqrt[3]{2xz}}{4z^5}$

B. $\dfrac{3\sqrt[3]{16x^5y^4z^{15}}}{16x^3y^3z^{15}}$

C. $\dfrac{3\sqrt[3]{4xy^2}}{4yz^5}$

D. $\dfrac{3\sqrt[3]{xy}}{4z^5}$

E. Each of the other answers is incorrect.

11. $i^{235,268,478,225,125,136,275} =$

A. 1

B. -1

C. i

D. $-i$

E. Each of the other answers is incorrect.

12. $\sqrt{x^2-10x+25} =$

A. $x-5$

B. $x+5$

C. $|x+5|$

D. $|x-5|$

E. Each of the other answers is incorrect.

13. A historian has a cotton uniform that he claims was made for General George Washington. A sample of the uniform is taken and dated using C-14 dating. It was determined that 98.55% of the original carbon 14 is still remaining. Carbon 14 decays exponentially with a half-life of 5730 years. What can we say about the historian's claim?

A. This is definitely not General Washington's uniform because it is made of cotton that was alive much after he was a general.

B. This is definitely not General Washington's uniform because it is made of cotton that died way earlier than he was a general.

C. This is definitely General Washington's uniform because it is made of cotton that was harvested during the Revolutionary War.

D. This could be General Washington's uniform because it is made of cotton that was harvested during the Revolutionary War.

E. None of the claims above can be made.

14. Which of the following is the correct process to simplify the division $\frac{2-4i}{3+2i}$?

A. $\frac{2-4i}{3+2i}=\frac{2-4i}{3+2i}\cdot\frac{2+4i}{3-2i}$

B. $\frac{2-4i}{3+2i}=\frac{2-4i}{3+2i}\cdot\frac{3+2i}{3+2i}$

C. $\frac{2-4i}{3+2i}=\frac{2-4i}{3+2i}\cdot\frac{3-2i}{3-2i}$

D. $\frac{2-4i}{3+2i}=\frac{2}{3}+\frac{-4}{2}i$

E. Each of the other answers is incorrect.

15. Describe the graph of $3x^2-4x+5y^2+2y-7=0$.

A. Circle
B. Ellipse
C. Parabola
D. Hyperbola
E. Each of the other answers is incorrect.

16. A couple is going to have exactly two children. If we assume that having a girl or boy are equally likely, what is the probability that they will end up with one boy and one girl?

A. 1/4
B. 1/3
C. 1/2
D. 0
E. 1

17. Louisa stores cans of soda in her cupboard. She has 4 cans of Coke, 3 cans of Sprite, and 8 cans of Root Beer. Without looking, she removes 2 cans at random (without replacement). Find the probability that both cans are Cokes.

A. 0.0571
B. 0.0711
C. 0.5333
D. 0.4810
E. 0.0533

18. If $\cos(\alpha)=\frac{3}{5}$ and $\tan(\alpha)<0$, then $\sin(\alpha)=$

A. $-\frac{4}{5}$
B. $\frac{4}{5}$
C. $-\frac{5}{3}$
D. $\frac{3}{4}$
E. Each of the other answers is incorrect.

For questions 19-21: In 1965, the composition of U.S. dimes, quarters, and half dollars was changed from silver to a copper clad. A woman has a bank in which she has accumulated several coins since childhood. The bank's contents are the following:

Date (Type of Coin)	**Dimes**	**Quarters**	**Half Dollars**	**Subtotals**
Before 1965 (Silver)	422	115	22	**559**
1965 and After (Copper Clad)	387	325	68	**780**
Subtotals	**809**	**440**	**90**	**1339**

19. Find the probability that a randomly selected coin from this collection is a silver coin.
    A. 42.35%
    B. 52.16%
    C. 41.75%
    D. 26.14%
    E. Each of the other answers is incorrect.

20. Given that you select a dime what is the probability that it is silver?
    A. 41.75%
    B. 31.52%
    C. 75.49%
    D. 52.16%
    E. All of the other answers are incorrect.

21. Find the probability that a randomly selected coin from this collection is a silver coin or a half-dollar.
    A. 48.47%
    B. 46.83%
    C. 1.64%
    D. 24.44%
    E. Each of the other answers is incorrect.

22. How does the graph of $y = 3\sin(x+2)$ compare to the graph of $y = \sin(x)$?
    A. It is shifted to the left 2 units and expanded vertically away from the *x*-axis by a factor of 3.
    B. It is shifted up 2 units and expanded horizontally away from the *y*-axis by a factor of 3.
    C. It is shifted down 2 units and expanded vertically away from the *y*-axis by a factor of 3.
    D. It is shifted to the right 2 units and expanded vertically away from the *x*-axis by a factor of 3.
    E. Each of the other answers is incorrect.

23. Which of the following describes the geometric relationship between the graph of a relation and the graph of its inverse relation?
    A. They are reflections of each other over the *x*-axis.
    B. They are reflections of each other over the *y*-axis.
    C. They are reflections of each other over the line $y = x$.
    D. They are rotations of each other by 180^{o} about the origin.
    E. Each of the other answers is incorrect.

24. $\sum_{k=1}^{7}\left(\frac{1}{3}\right)^k =$

A. $\frac{7}{2,187}$

B. $\frac{1,093}{2,187}$

C. $\frac{2,186}{2,187}$

D. $\frac{1}{2,186}$

E. Each of the other answers is incorrect.

25. Divide $\left(3x^4 - x^2 + 2x - 7\right)$ by $(x - 3)$. What is the remainder?

A. 233

B. 71

C. -103

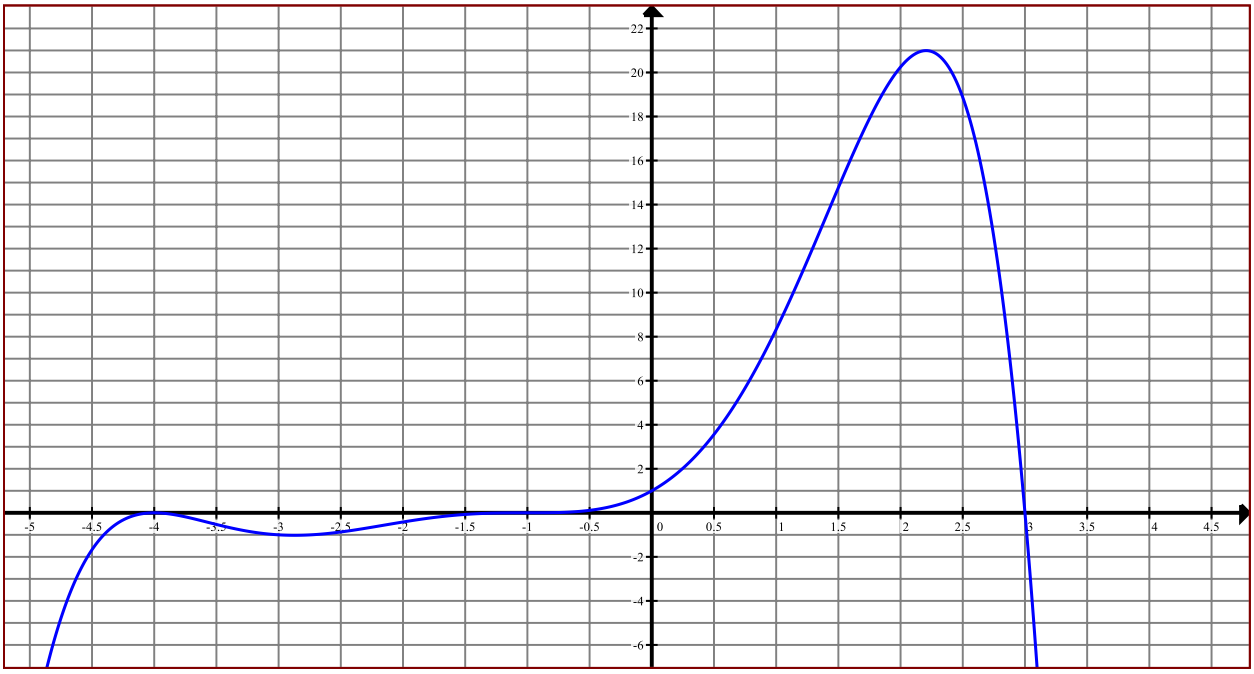
D. 0

E. Each of the other answers is incorrect.

ACTM Regional Algebra II Competition 2015
Tie Breaker 1

Name _____

Below is the graph of a polynomial function. All of the x and y intercepts have integer coordinates and are clearly visible in this window. What is the factored form of the formula for this function?



ACTM Regional Algebra II Competition 2015
Tie Breaker 2

Name _____

A manufacturer needs to make a box for her product so that its volume is 144 cm^3. The shape of the box must be a right square prism. Let s be the length of the side of the square.

A. Write the surface area A of the box as a function of s.

B. To the nearest hundredth of a centimeter what are the dimensions of such a box with a minimum surface area? What is this minimum surface area?

ACTM Regional Algebra II Competition 2015
Tie Breaker 3

Name _____

A. The population of a region is growing at a relative rate of 5% per year. This year its population is 350,000 people. Write a formula for the population P as a function of years from now t.

B. The food supply is growing at a rate of 25,000 units per year. Currently there is one unit of food per person. Write a formula for food supply f as a function of t.

C. If one unit of food per person is just adequate for sustaining the population when if ever will the region need to import food?

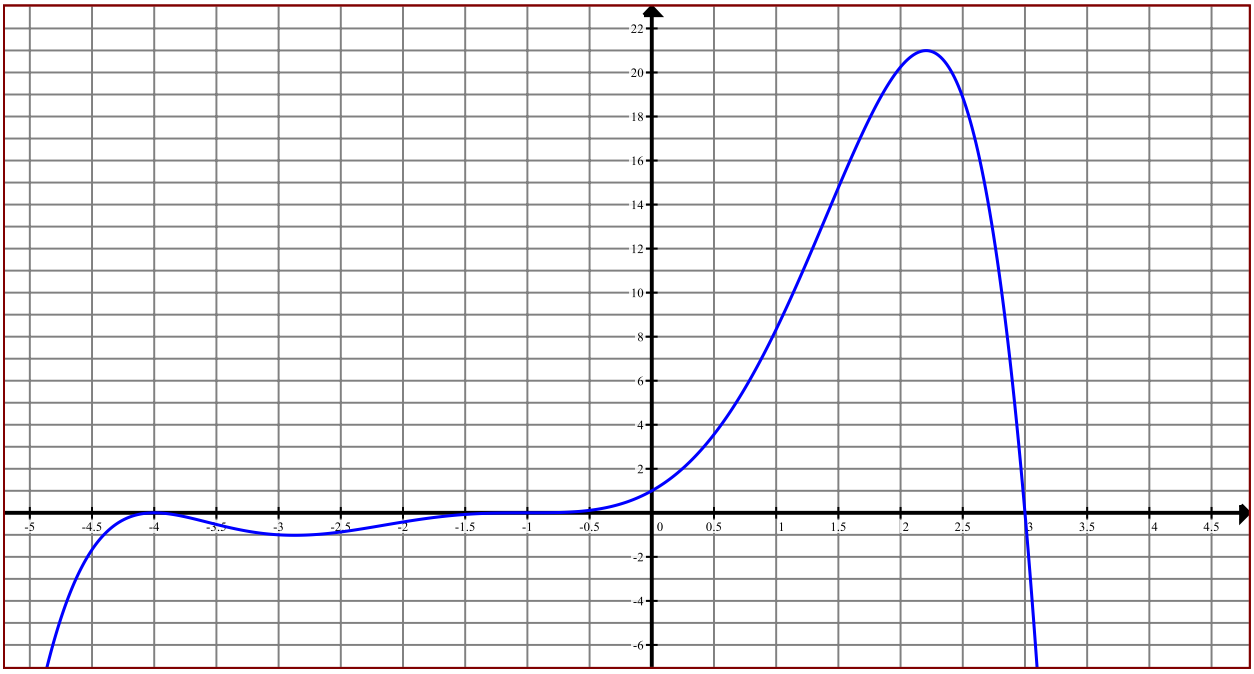
ANSWERS:

1. B
2. B
3. C
4. D
5. A
6. D
7. E
8. D
9. A
10. C
11. D
12. D
13. A
14. C
15. B
16. C
17. A
18. A
19. C
20. D
21. B
22. A
23. C
24. B
25. A

ACTM Regional Algebra II Competition 2015
Tie Breaker 1

Name **Solution**

Below is the graph of a polynomial function. All of the x and y intercepts have integer coordinates and are clearly visible in this window. What is the factored form of the formula for this function?



Working right to left, note that the function has an x-intercept of (3, 0) and at this point the graph looks like a non-horizontal line through the point if we zoom in close so this is a multiplicity 1 root. Therefore $(x - 3)$ is a factor of the polynomial but $(x - 3)^2$ is not a factor. There is another x-intercept at (-1, 0). For this intercept notice that if we zoom in close to the graph it is similar to a horizontal line, but the graph actually changes sides of the x-axis and has an inflection point there. Therefore, -1 is an odd multiplicity root with multiplicity at least 3 and $(x + 1)^3$ is a factor of the polynomial. The final x-intercept is at (-4, 0). At this point the graph touches the x-axis but turns around there so -4 is an even multiplicity root and $(x + 4)^2$ is a factor of the polynomial. A polynomial of smallest degree with these features will have the formula:

$$f(x) = a(x-3)(x+1)^3(x+4)^2$$

where a is a constant. Furthermore, the graph approaches negative infinity as x approaches infinity (i.e. the arrow points down on the right side) so we know that the value of a is negative. To find the appropriate value of a we note that the y-intercept is (0, 1). Therefore,

$$1 = a(0-3)(0+1)^3(0+4)^2$$
$$1 = a(-3)(1)^3(4)^2$$
$$1 = a(-3)(1)(16)$$
$$1 = -48a$$
$$-\tfrac{1}{48} = a$$

Therefore the formula is $\boxed{f(x) = -\tfrac{1}{48}(x-3)(x+1)^3(x+4)^2}$. Graphing with a calculator verifies that this is the correct formula.

ACTM Regional Algebra II Competition 2015
Tie Breaker 2

A manufacturer needs to make a box for her product so that its volume is 144 cm^3. The shape of the box must be a right square prism. Let s be the length of the side of the square.

A. Write the surface area A of the box as a function of s.

Let h = the height of the box and V = volume.

$V = s^2h$

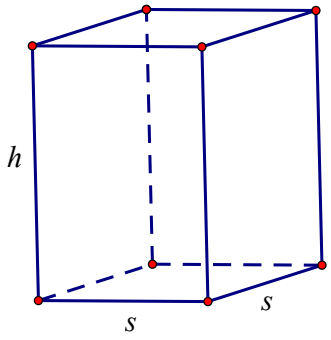
$144 = s^2h$

$h = \frac{144}{s^2}$

$A = 2s^2 + 4sh$

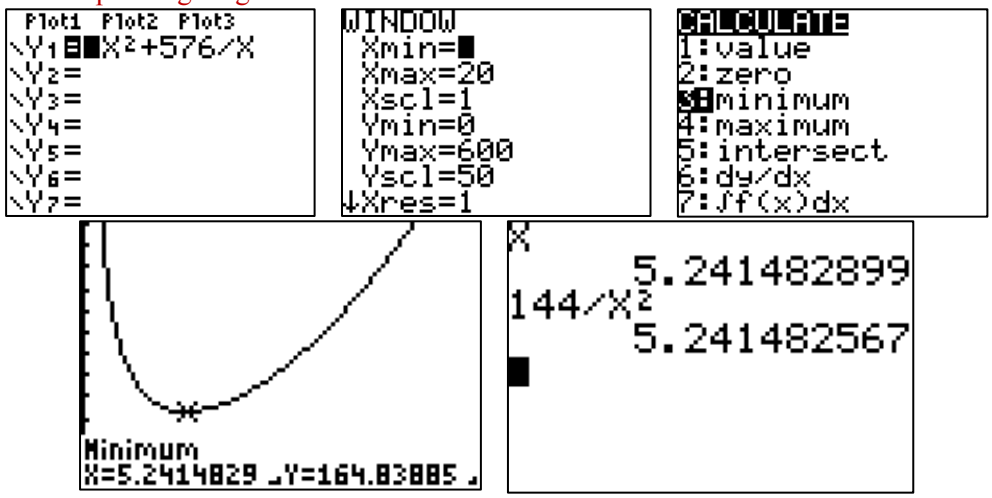
$A = 2s^2 + 4s\left(\frac{144}{s^2}\right)$

$\boxed{A(s) = 2s^2 + \frac{576}{s}}$



B. To the nearest hundredth of a centimeter what are the dimensions of such a box with a minimum surface area? What is this minimum surface area?

Since we have the surface area as a function of one variable we can graph this on the graphing calculator and use its ability to approximate a minimum value. We use the x value of the minimum point to compute the corresponding height.



The minimum surface area of 164.84 cm^2 is obtained when the box is a cube with edge length 5.24 cm.

ACTM Regional Algebra II Competition 2015
Tie Breaker 3

Name **Solution**

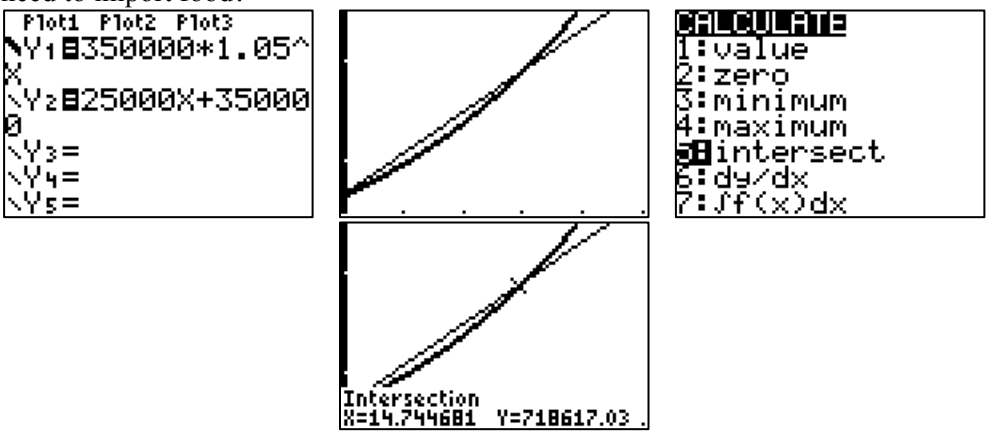
A. The population of a region is growing at a relative rate of 5% per year. This year its population is 350,000 people. Write a formula for the population P as a function of years from now t.

$$P(t) = (350{,}000)(1.05^t)$$

B. The food supply is growing at a rate of 25,000 units per year. Currently there is one unit of food per person. Write a formula for food supply *f* as a function of *t*.

$$f(t) = 25{,}000t + 350{,}000$$

C. If one unit of food per person is just adequate for sustaining the population when if ever will the region need to import food?



Notice that there is more than one unit of food per person when the linear graph is above the exponential graph. This happens for t values from t = 0 to t = 14.74 so they will have to start importing food in about 14.74 years.