

2015 – Regional Mathematics Contest

Geometry Test

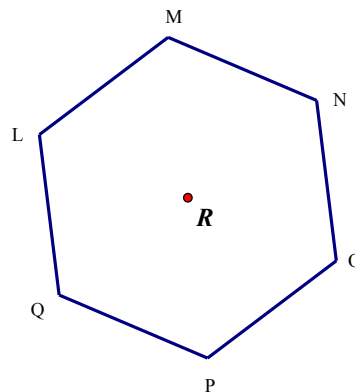
In each of the following choose the BEST answer and record your choice on the answer sheet provided. To insure correct scoring, be sure to make all erasures completely. The tie-breaker questions at the end will only be used to resolve ties in first, second, and/or third place. They will be used in the order given. Complete the first 25 questions before attempting the tie-breaker questions. Figures are not necessarily drawn to scale.

1 .Which of the following is/are **always** true for a circle

- I. A chord is a diameter
 - II. A line segment with one endpoint at the center and the other endpoint on the circle is the radius
 - III. A secant is a line that passes through the circle
- a. I only
 - b. I and II only
 - c. II only
 - d. III only
 - e. II and III only

2. Given LMNOPQ is a regular hexagon. Which of the following rotations about the center R does the figure have in which it maps to itself?

- a. 30°
- b. 45°
- c. 90°
- d. 120°
- e. None of these



3. A person who is 1000 feet from the base of the Washington Monument finds the angle the ground makes with line sight to the top of the monument to be approximately 29° . How tall is the monument?

- a. 554.3 feet
- b. 1804.0 feet
- c. 484.8 feet
- d. 874.6 feet
- e. Not enough information

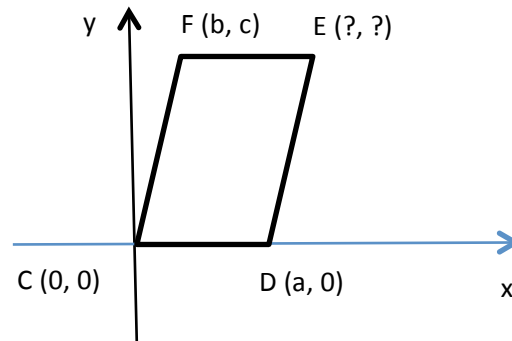
4. Write the equation of a circle that has the same center as the circle $(x + 4)^2 + (y - 3)^2 = 9$ and is tangent to the y-axis.

- a. $(x - 4)^2 + (y - 3)^2 = 9$
- b. $(x + 3)^2 + (y - 4)^2 = 9$
- c. $(x + 4)^2 + (y - 3)^2 = 16$
- d. $(x + 4)^2 + (y - 3)^2 = -16$
- e. None of these

5. Supply the missing coordinates without introducing new letters to make the given shape a parallelogram.

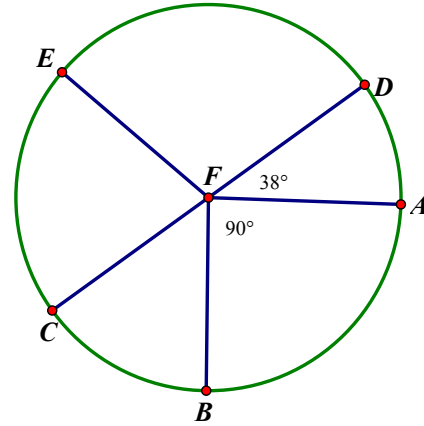
- a. (b, c)
- b. (a, c)
- c. (a + b, c)
- d. (a, b + c)
- e. None of these

CDEF is a
Parallelogram



6. The circle with center F is divided into sectors. In circle F , \overline{CD} is a diameter. What is the angle measure of arc \overline{AEC} ?

- a. 218°
- b. 180°
- c. 128°
- d. 322°
- e. Not enough information

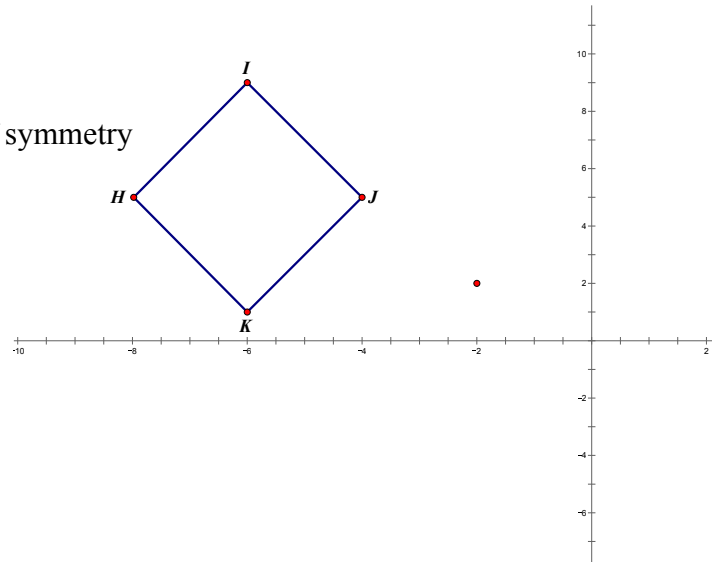


7. Using the diagram above. The length of \overline{FD} is 5 units. Find the measure of the arc length of arc \overline{CBA} .

- a. 10π
- b. $\frac{71}{18}\pi$
- c. $\frac{71}{180}\pi$
- d. $\frac{1}{36}\pi$
- e. Not enough information

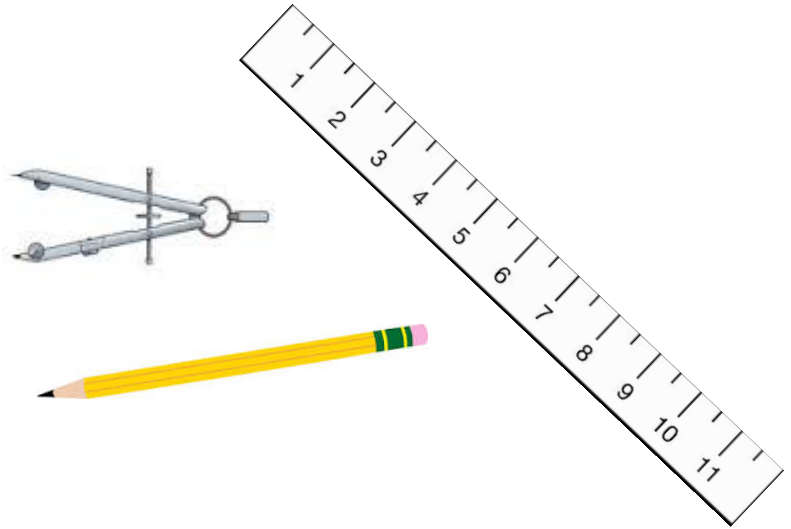
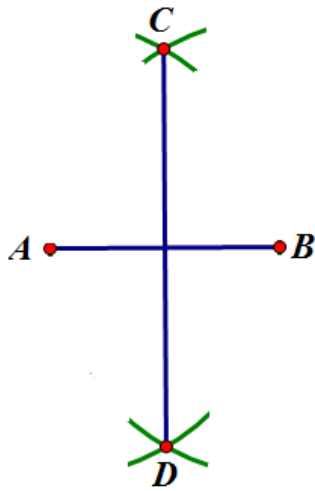
8. Rhombus HJK is shown in the xy -coordinate plane. The rhombus will be rotated 180° degrees about the point $(-2, 2)$. Which characteristics of Rhombus $H'I'J'K'$ will be the same for the corresponding characteristics of Rhombus HJK ?

- I. The coordinate value of H'
 - II. The measure of $\angle J'$
 - III. K' and I' are on a vertical line of symmetry
- a. I only
 - b. II only
 - c. I, II, and III
 - d. II and III
 - e. I and II



Use the information provided to answer Questions 9 - 11

The figure shows the result of a geometric construction.



9. The first step of the construction is to draw an arc centered at A passing through B and an arc centered at B passing through A. What is the purpose for this first step?

- a. To create two congruent circles
- b. $\overline{DC} \cong \overline{AB}$
- c. Practice drawing arcs
- d. Not enough information
- e. None of the above

10. Congruent triangles are created by this construction. $\triangle ACD \cong \triangle BCD$ (not shown in picture). Which congruency theorem can you use to prove this?

- I. SAS
 - II. SSS
 - III. ASA
 - IV. AAA
-
- a. I only
 - b. II only
 - c. I and II
 - d. I, II, III, IV
 - e. None of these.

11. $\triangle ACB$ is what type of triangle.

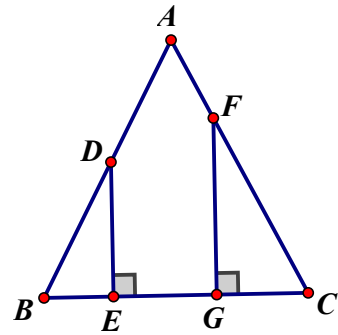
- I. Isosceles
 - II. Equilateral
 - III. Scalene
 - IV. Acute
 - V. Obtuse
 - VI. Right
- a. I and V
 - b. III and VI
 - c. I and VI
 - d. II and IV
 - e. II and V

Use the information provided to answer questions 12-15.

Given: $\triangle ABC$ is isosceles with base \overline{BC} , $\overline{DE} \perp \overline{BC}$, $\overline{FG} \perp \overline{BC}$

Prove: $\frac{DE}{FG} = \frac{BE}{CG}$

Proof:



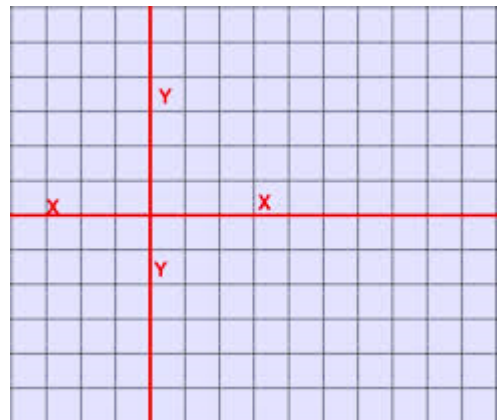
Statements	Reasons
1. $\triangle ABC$ is isosceles	1. Given
2. $\angle B \cong \angle C$	2. Base Angles of an isosceles triangle are congruent
3. $\overline{DE} \perp \overline{BC}$, $\overline{FG} \perp \overline{BC}$	3. Given
4. $\angle BED$ and $\angle CGF$ are right angles.	4. ?
5. ?	5. Definition of Right Triangle
6. $\triangle DEB \sim \triangle FGC$	6. ?
7. $\frac{DE}{FG} = \frac{BE}{CG}$	7. ?

12. Which reason justifies the statement for step 4 in the proof?
- Right angles form perpendicular line segments
 - The angles are marked right angles in the diagram
 - Right Triangles have right angles.
 - Line segments that intersect at right angles form perpendicular lines.
 - Two perpendicular line segments intersect at right angles.
13. Which statement is justified by the reason for step 5 in the proof?
- $\triangle ABC$ is a right triangle
 - $\triangle ABC$ and $\triangle DEB$ are right triangles
 - $\triangle FGC$ and $\triangle ABC$ are right triangles
 - $\triangle FGC$ and $\triangle DEB$ are right triangles
 - None of the above
14. Which reason justifies the statement for step 6 in the proof?
- If two angles and the included side of one triangle are congruent respectively to two angles and the included side of another triangle, then the two triangles are congruent.
 - If an acute angle of one right triangle is congruent to an acute angle of another right triangle, then the triangles are similar.
 - All right triangles are similar
 - Both A and B
 - None of the above

15. Which reason justifies the statement for step 7 in the proof?
- Ratios of corresponding sides of similar triangles are equal.
 - Any two pairs of sides in similar triangles form a proportion.
 - Lengths of altitudes from corresponding vertices of similar triangles are in the same ratio as the lengths of corresponding sides.
 - Both A and C
 - None of the above

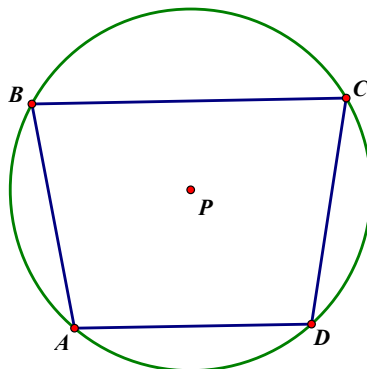
16. In the xy -coordinate plane, Hexagon $ABCDEF$ has vertices at $A(-2, 2)$, $B(0, 4)$, $C(2, 4)$, $D(4, 2)$, $E(2, 0)$ and $F(0,0)$, and Hexagon $A'B'C'D'E'F'$ has vertices at $A'(-4, 1)$, $B'(0,5)$, $C'(4, 5)$, $D'(8,1)$, $E'(4, -3)$. And $F'(0,-3)$. What is the center and scale factor for this dilation?

- The Center is $(3,0)$
 - The Center is $(0,3)$
 - The Center is $(-3,0)$
 - The scale factor is 2
 - The scale factor is $\frac{1}{3}$
 - The scale factor is $\frac{1}{2}$
- II and IV
 - I and V
 - II and VI
 - II and V
 - III and IV



17. In the figure, **isosceles trapezoid** $ABCD$ is inscribed in Circle P . If the $m\angle ADC = 104^\circ$, find the $m\widehat{BAD}$.

- 76°
- 180°
- 152°
- Not enough information
- None of these



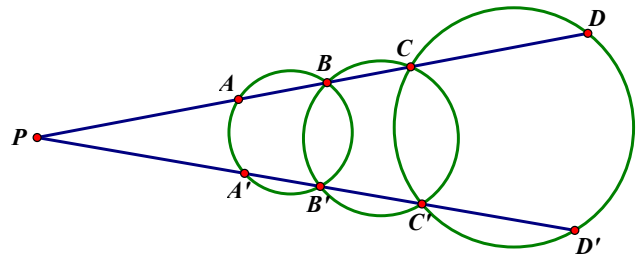
18. If the ratio of the diameters of two similar cylinders is 3 to 5, what is the ratio of their volumes?

- a. 3 to 5
- b. 9 to 25
- c. 6 to 10
- d. 9 to 15
- e. None of these

Suppose that \overline{PD} and $\overline{PD'}$ are secants of all three circles. Answer the following questions.

19. $PA \times PB = PA' \times PB'$

- a. All of the time
- b. Some of the time
- c. None of the time
- d. Not enough information
- e. None of these



20. $PA \times PC = PA' \times PC'$

- a. All of the time
- b. Some of the time
- c. None of the time
- d. Not enough information
- e. None of these

21. The length of a side of a rhombus is 24 cm. The smaller angle of the rhombus has measure 64° . Find the length of the shorter diagonal.

- a. 20.35 cm
- b. 25.44 cm
- c. 12.72 cm
- d. 24.44 cm
- e. None of these

22. Which 3-dimensional shape do you create by rotating a right triangle about its longest leg?

- a. Right Triangular Prism
- b. Cylinder
- c. Right Triangular Pyramid
- d. Right Cone
- e. None of these

23. Given a line segment, \overline{AB} , with endpoints located at A (-3, 5) and B(6, 1). How far is the point that partitions the line into a 2:3 ratio from Point A.

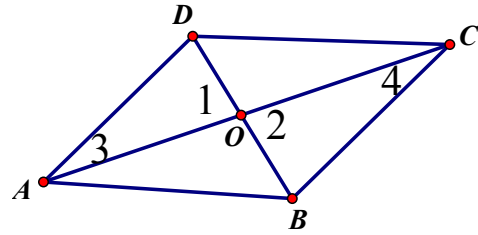
- a. 9.84 units
- b. 3.94 units
- c. 5.90 units
- d. 6.56 units
- e. None of these

24. Complete the following proof by putting the justifications in the correct order.

Given: \overline{AC} and \overline{BD} bisect each other

Prove: $\angle 3 \cong \angle 4$

Proof:



Statements	Reasons
1. \overline{AC} and \overline{BD} bisect each other	1.
2. $\overline{AO} \cong \overline{CO}$, $\overline{BO} \cong \overline{DO}$	2.
3. $\angle 1 \cong \angle 2$	3.
4. $\triangle AOD \cong \triangle COB$	4.
5. $\angle 3 \cong \angle 4$	5.

- I. Vertical Angles are congruent
- II. Given
- III. SAS
- IV. Corresponding parts of congruent triangles are congruent
- V. A segment bisector forms two congruent segments.

- a. II, III, I, V, IV
- b. II, V, III, IV, I
- c. II, V, I, III, IV
- d. II, I, III, V, IV

25. Determine the equation of a parabola with the following directrix and focus.

$$y = -5 \text{ and } (2, 7)$$

- a. $y = \frac{-x^2}{4} + x - \frac{81}{4}$
- b. $y = \frac{-x^2}{4} + x + \frac{81}{4}$
- c. $y = \frac{-x^2}{4} + x - 7$
- d. $y = \frac{x^2}{4} + x + 7$
- e. None of these

Tie-Breaker Question 1

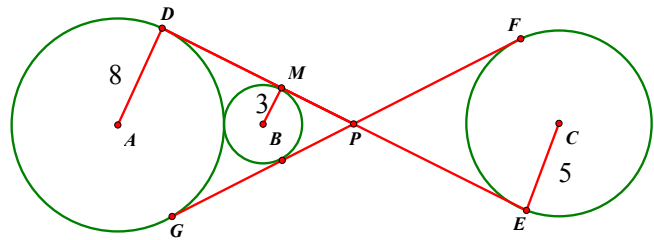
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School: _____

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The following three questions will be used to break ties for first, second and/or third place. They will be used in the order they are given. Be sure you have answered the previous 25 questions before proceeding to the tie-breaker questions. Your answers should be written clearly with reasonable justification of your work.

\overline{DE} and \overline{FG} are common tangents for the three circles. If the radii of the three circles are 8, 3, and 5 units as shown, find BC .



Tie-Breaker Question 2

Name: _____

School: _____

Please Print

Answer both parts of the question below for full credit.

Round solutions to the nearest tenth.

- A. Phoenix is 140.0 miles due south of Flagstaff, and Winslow is 60.0 miles due east of Flagstaff. If a pilot were to fly from Phoenix to Winslow, how many degrees north of due east should she head?
- B. Air traffic control in Winslow spots the plane on radar at 40° south of west. Another air traffic control tower 32 miles due south from Winslow spotted the plane due West of its position. How far is the plane from Winslow?

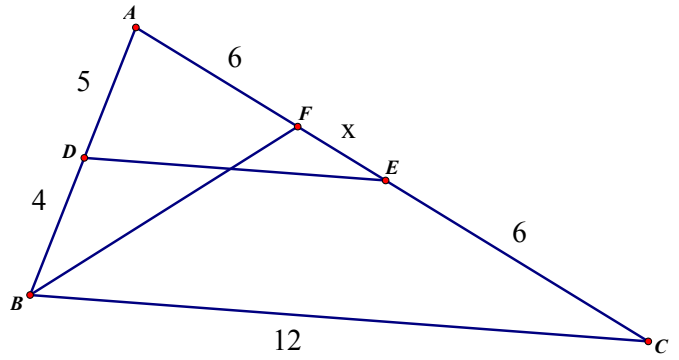
Tie-Breaker Question 3

Name: _____

School: _____

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If $\overline{DE} \parallel \overline{BC}$, show algebraically that \overline{BF} cannot be the angle bisector of $\angle B$ in the figure below.



Solutions to Multiple Choice.

1. E
2. D
3. A
4. C
5. C
6. A
7. B
8. D
9. A
10. B
11. D
12. E
13. D
14. B
15. D
16. A
17. C
18. E
19. A
20. B
21. B
22. D
23. B
24. C
25. E

Tie-Breaker Question 1 -- Solution

Name: _____

School: _____

Please Print

The following three questions will be used to break ties for first, second and/or third place. They will be used in the order they are given. Be sure you have answered the previous 25 questions before proceeding to the tie-breaker questions. Your answers should be written clearly with reasonable justification of your work.

\overline{DE} and \overline{FG} are common tangents for the three circles. If the radii of the three circles are 8, 3, and 5 units as shown, find BC.

Facts

1. $\angle D$ is right, $\angle E$ is right, $\angle M$ is right
2. Draw auxiliary line \overline{AC} ,

This gives us similar triangles

$\triangle BMP \sim \triangle CPE$, $\triangle CPE \sim \triangle ADP$ and

$\triangle BMP \sim \triangle ADP$ (all by AA similarity theorem)

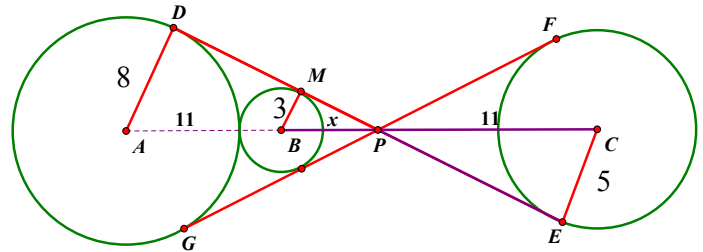
3. $AB = 11$ (it is the sum of the radii of Circle A and Circle B)
4. Let $BP = x$

We can write proportions

$$\frac{3}{8} = \frac{x}{x+11}, \text{ then } x = 6.6 = BP$$

$$\frac{3}{5} = \frac{6.6}{CP}, \text{ then } CP = 11$$

Therefore we can conclude that $BC = 11 + 6.6 = 17.6$ units



Tie-Breaker Question 2 – Solution

Name: _____

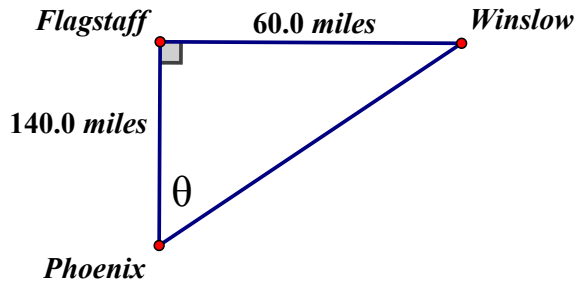
School: _____

Please Print

Answer both parts of the question below for full credit.

Round solutions to the nearest tenth.

- A. Phoenix is 140.0 miles due south of Flagstaff, and Winslow is 60.0 miles due east of Flagstaff. If a pilot were to fly from Phoenix to Winslow, how many degrees north of due east should she head?



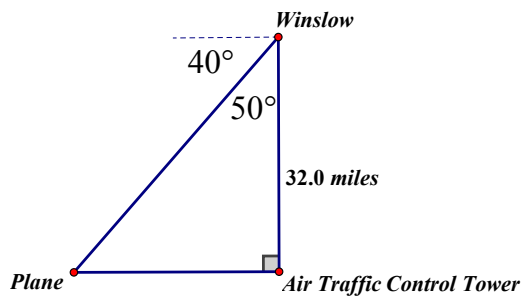
$$\tan \theta = \frac{60}{140}$$

$$\theta = \tan^{-1} \left(\frac{60}{140} \right) = 23.2^\circ \text{ (This is the angle east of north)}$$

Angle north of east

$$90^\circ - 23.2^\circ = 66.8^\circ$$

- B. Air traffic control in Winslow spots the plane on radar at 40° south of west. Another air traffic control tower 32 miles due south from Winslow spotted the plane due West of its position. How far is the plane from Winslow?



$$\cos 50^\circ = \frac{32}{x}$$

$$x = 49.8 \text{ miles}$$

Tie-Breaker Question 3 – Solution

Name: _____

School: _____

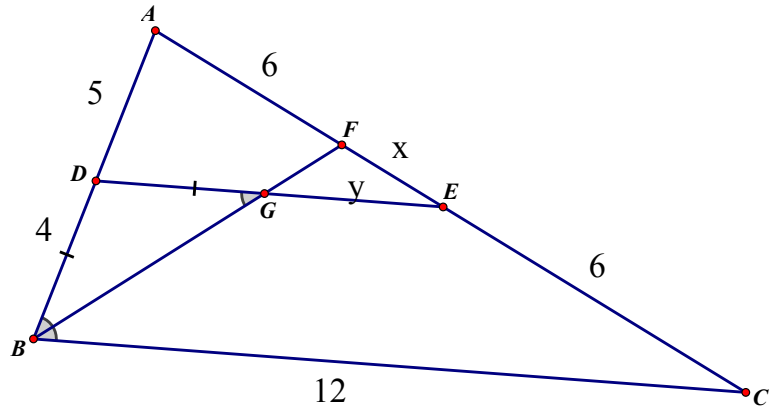
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If $\overline{DE} \parallel \overline{BC}$, show algebraically that \overline{BF} cannot be the angle bisector of $\angle B$ in the figure below.

If a line is parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

$$\frac{5}{4} = \frac{6+x}{6} \text{ and } \triangle ADE \sim \triangle ABC$$

so $x = 1.5$



Assume that \overline{BF} is the angle bisector, then $\angle DGB \cong \angle GBC$ using parallel line theorems. This makes $\triangle DBG$ an isosceles triangle. So $DG = 4$
 We can find GE (y) using $\triangle ADE \sim \triangle ABC$

$$\frac{9}{12} = \frac{5}{4+y}$$

$$y = \frac{8}{3}$$

We know that $\triangle FGE \sim \triangle FBC$, so we can write the following proportion:

$$\frac{\frac{8}{3}}{12} = \frac{x}{x+6}$$

so $x = 1.714$

This shows that x would need to equal two different values, so \overline{BF} cannot be an angle bisector