ACTM STATE CALCULUS COMPETITION
APRIL 26, 2014

Instructions: Select the best choice for each question. Afterward, attempt the tie-breaker questions in sequential order (Do #1 first, followed by #2, and then #3 last). Unless otherwise stated, assume all variables are real and all functions are continuous over relevant domains.

1. Evaluate the following integral: $\int \left(3e^{3x} - 4\sin(4x+3)\right)dx =$

A. $9e^{3x} + \cos(4x+3) + C$
B. $e^{3x} - 4\cos(4x+3) + C$
C. $9e^{3x} - \cos(4x+3) + C$
D. $e^{3x} + \cos(4x+3) + C$
E. None of these

2. The horizontal asymptote(s) for the function $f(x) = \dfrac{10x^3 - 3x^2 + 8}{\sqrt{25x^6 + x^4 + 2}}$ is/are:

A. $y = 2$
B. $y = -2$
C. $y = 2$ and $y = -2$
D. $y = 0$ and $y = 2$
E. None of these

3. Given the four functions $f(x) = x^2$, $g(x) = x^x$, $h(x) = e^x$, and $j(x) = x\ln x$, which function has the greatest growth rate?

A. *f(x)*
B. *g(x)*
C. *h(x)*
D. *j(x)*
E. All are the same

4. The function *f* that satisfies $f''(t) = 6t$ with $f'(0) = 1$ and $f(0) = 2$ is:

A. $f(t) = 3t^3 + 2$
B. $f(t) = t^3 + 2t + 1$
C. $f(t) = t^3 + t + 2$
D. $f(t) = t^3 + t + 1$
E. None of these

5. The function *f* has the property that $f'(c) = 0$ and $f''(c) < 0$. Which of the following must occur?

A. *f* has a local minimum at *x* = *c*.
B. *f* has a local maximum at *x* = *c*.
C. *f* is increasing on an interval containing *c*.
D. *f* has an absolute extrema at *x* = *c*.
E. None of these

6. The function $f(x) = \dfrac{x-2}{x^2+2}$ has

A. a zero, a vertical asymptote, and a horizontal asymptote
B. a zero and a vertical asymptote, but no horizontal asymptote
C. a vertical and horizontal asymptote, but no zero
D. a zero and a horizontal asymptote, but no vertical asymptote
E. None of these

7. Determine the point(s) guaranteed to exist by the Mean Value Theorem for the function $f(x) = x + 2 + \dfrac{3}{x-1}$ on the interval [2, 7].

A. *x* = 3.45
B. *x* = -1.45
C. *x* = 2.73
D. *x* = 2.22
E. None of these

8. At what point does the absolute minimum for the function $f(x) = -x^3 + 6x^2 - 9x - 2$ occur on the interval [0, 2]?

A. *x* = 0
B. *x* = 0.5
C. *x* = 1.25
D. *x* = 2
E. None of these

9. Evaluate $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\tan x}{\sec x}$

A. 0
B. 1
C. +∞
D. -∞
E. None of these

10. Calculate the derivative of $f(t) = \dfrac{e^t}{e^t + t}$.

A. $f'(t) = \dfrac{(t+1)e^t}{(e^t + t)^2}$

B. $f'(t) = \dfrac{-e^t}{(e^t + t)^2}$

C. $f'(t) = \dfrac{(t-1)e^t}{(e^t + t)^2}$

D. $f'(t) = \dfrac{e^{2t} + te^t + e^t}{(e^t + t)^2}$

E. None of these

11. Determine the equation of the tangent line to the curve $f(x) = \sqrt{x}$ at $x = 9$.

A. $y = 6x - 51$

B. $y = \dfrac{1}{6}x + \dfrac{3}{2}$

C. $y = \dfrac{3}{2}x - \dfrac{21}{2}$

D. $y = \dfrac{1}{6}x + \dfrac{9}{2}$

E. None of these

12. Suppose that *f* and *g* are differentiable at *x*, and that the following are true:

- $f(1) = 4$
- $f'(1) = 7$
- $g(1) = 6$
- $g'(1) = -4$

What is $\dfrac{d}{dx}(fg)$ at $x = 1$?

A. 58
B. 26
C. -58
D. -52
E. None of these

13. Calculate $f^{2014}(x)$ given that $f(x) = \sin x$.

A. $\sin x$
B. $\cos x$
C. $-\sin x$
D. $-\cos x$
E. None of these

14. If $x^3 + 3x^2y + y^3 = 14$, then $\frac{dy}{dx} =$

A. $-\frac{x^2 + 2xy}{x^2 + y^2}$

B. $\frac{x^2 + 2xy}{x^2 + y^2}$

C. $\frac{x^2 + 3xy}{x^2 + y^2}$

D. $-\frac{x^2 + 3xy}{x^2 + y^2}$

E. None of these

15. Evaluate $\int_0^8 \frac{dx}{\sqrt{1+x}}$

A. 1
B. 5
C. 5.66
D. 6
E. None of these

16. Determine the average value of the function $f(x) = 3x^3 - x^2$ on the interval [-1, 2].

A. $\frac{11}{4}$

B. $\frac{7}{2}$

C. 8

D. $\frac{33}{4}$

E. None of these

17. Determine the area of the region bounded by $y = x$ and $y = x^2 - 3x + 3$.

A. $\frac{2}{3}$

B. $\frac{4}{3}$

C. $\frac{8}{3}$

D. $\frac{14}{3}$

E. None of these

18. Calculate the derivative of $f(x) = \cot x - \tan x$.

A. $\sec x + \csc x$
B. $\sec^2 x - \csc^2 x$
C. $\sec^2 x \csc^2 x$
D. $-\sec^2 x \csc^2 x$
E. None of these

19. Determine the inflection point for the function $g(x) = \dfrac{x-1}{x^3}$.

A. x = 0
B. x = 1
C. x = 2
D. x = 3
E. None of these

20. Determine the value of *c* so that $f(x) = x^4 - cx^2$ has a local minimum at $x = -4$.

A. 32
B. -32
C. 0
D. -64
E. None of these

21. Calculate $\lim\limits_{h\to 0} \dfrac{(h+a)^2 - a^2}{h}$, where *a* is a constant.

A. 0
B. *a*
C. 2*a*
D. $+\infty$
E. None of these

22. Determine the constants *a* and *b* such that the function is continuous on the entire real number line:

$$f(x) = \begin{cases} \dfrac{x^2-1}{x-1}, & x < 1 \\ ax + b, & 1 \le x \le 2 \\ \dfrac{x^2-4}{x-2}, & 2 < x \end{cases}$$

A. $a = 2, b = 0$
B. $a = 1, b = 0$
C. $a = 0, b = 2$
D. $a = 2, b = 1$

E. None of these

23. Calculate the derivative of $y = \left(\cos x^3\right)\left[\ln(2x+1)\right]$.

A. $\frac{2}{2x+1}\cos(x^3) - \ln(2x+1)\sin(x^3)$
B. $\frac{2}{2x+1}\cos(x^3) + \ln(2x+1)\sin(x^3)$
C. $\frac{\cos(x^3)}{2x+1} - \ln(2x+1)\sin(x^3)$
D. $\frac{2}{2x+1}\cos(x^3) + 3x^2\ln(2x+1)\sin(x^3)$
E. None of these

24. Write the equation of the line that represents the linear approximation of $f(x) = x^3 - 2x + 8\sqrt{x}$ at the point a = 4.

A. *L(x)* = 44x – 104
B. *L(x)* = 48x – 120
C. *L(x)* = 48x – 104
D. *L(x)* = 44x – 120
E. None of these

25. Find the value for c such that $f(x) = \left(2x^2 + c\right)(1 - x)$ has a horizontal tangent line at $x = 2$.

A. 16
B. 0
C. -8
D. -16
E. None of these

ACTM State Calculus Competition **Name:_____**
Tie Breaker Questions
April 26, 2014 **School/Teacher:_____**

Reminder: Attempt the tie-breaker questions in sequential order (Do #1 first, followed by #2, and then #3 last).

1. An open box with a square base is to have a volume of 2,048 cm^3. What should the dimensions of the box be if the amount of material used is to be a minimum?

2. The position of a particle moving in a straight line during a 5 second trip is $s(t) = t^2 - t + 10$ cm. Find a time t at which the instantaneous velocity is equal to the average velocity for the entire trip.

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3. Water pours into a conical tank of height 10 m and radius of 4 m at a rate of 6 m^3/min. At what rate is the water level rising when the level is 5 m high?

Calculus Key

1. D
2. C
3. B
4. C
5. B
6. D
7. A
8. E
9. B
10. C
11. B
12. B
13. C
14. A
15. E
16. A
17. B
18. D
19. C
20. A
21. C
22. A
23. E
24. B
25. D

1. 16 cm x 16 cm x 8 cm

2. 2.5 seconds

3. 0.48 m/min