

ACTM State Algebra II Exam  
April 26, 2008

Place the letter of the correct answer in the space provided as well as on the answer sheet.

\_\_\_\_\_ 1. Perform the indicated operations and simplify:  $7x - 3x[5 - (x - 2)]$

- A)  $28x - 4x^2$                       B)  $12x - 4x^2$                       C)  $-2x + 3x^2$                       D)  $-14x + 3x^2$

\_\_\_\_\_ 2. Factor completely, relative to the integers:  $64x^3 - 27$

- A)  $(4x - 3)(16x^2 + 12x + 9)$     B)  $(4x - 3)(4x + 3)$     C)  $(4x - 3)(16x^2 - 12x + 9)$     D)  $(4x - 3)(16x^2 - 12x - 9)$

\_\_\_\_\_ 3. Perform the indicated operations and simplify:  $\frac{\frac{x}{y} + 4}{\frac{x^2}{y^2} - 16}$

- A)  $\frac{1}{x - 4}$                       B)  $\frac{x + 4y}{x^2 - 16y^2}$                       C)  $\frac{y}{x - 4y}$                       D)  $\frac{y(x + 4y)}{(x - 16y)(x + 16y)}$

\_\_\_\_\_ 4. Perform the indicated operations and simplify:  $\frac{4x^2}{x^2 - 25} \div \frac{2x}{3x + 15}$

- A)  $\frac{8x^3}{3(x + 5)^2(x - 5)}$                       B)  $\frac{6x}{x(x - 5)}$                       C)  $\frac{3x}{x - 5}$                       D)  $\frac{6x}{(x - 5)}$

\_\_\_\_\_ 5. Simplify and write your answer using positive exponents only:  $\left(\frac{5x^{-2}y^5}{10x^3y^2}\right)^{-2}$

- A)  $\frac{4x^{25}}{y^9}$                       B)  $\frac{4x^{10}}{y^6}$                       C)  $\frac{4x^2}{y^6}$                       D)  $\frac{y^6}{4x^{10}}$

\_\_\_\_\_ 6. Simplify:  $\left(4x^{\frac{1}{2}}\right)\left(6x^{\frac{3}{4}}\right)$

- A)  $24x^{\frac{4}{6}}$                       B)  $24x^{\frac{3}{8}}$                       C)  $10x^{\frac{2}{3}}$                       D)  $24x^{\frac{5}{4}}$

\_\_\_\_\_ 7. Solve the equation for x:  $\frac{4}{x+5} + \frac{3}{x-3} = \frac{31}{(x+5)(x-3)}$

- A)  $x = -5, x = 3$       B)  $x = 4$       C)  $x = \frac{31}{7}$       D)  $x = \frac{29}{7}$

\_\_\_\_\_ 8. Solve the inequality and write the final answer using interval notation:  $|3x - 6| \geq 9$

- A)  $(-\infty, -1) \cup (5, \infty)$       B)  $(-1, 5)$       C)  $(-\infty, -1] \cup [5, \infty)$       D)  $[-1, 5]$

\_\_\_\_\_ 9. Solve the inequality and write the final answer using interval notation:  $-4 < -3x + 5 < 11$

- A)  $(-3, 2)$       B)  $(-2, 3)$       C)  $(3, -2)$       D)  $\left(\frac{-16}{3}, \frac{1}{3}\right)$

\_\_\_\_\_ 10. Solve the radical equation for x and check your answer:  $\sqrt{x+11} = x-1$

- A)  $\{5\}$       B)  $\{-2, 5\}$       C)  $\{-5, 2\}$       D)  $\{2\}$

\_\_\_\_\_ 11. Solve for x:  $x^2 + 7x - 5 = 0$

- A)  $x = -7 \pm \frac{\sqrt{69}}{2}$       B)  $x = \frac{-7 \pm \sqrt{29}}{2}$       C)  $x = \frac{-7 \pm \sqrt{69}}{2}$       D)  $x = -7 \pm \frac{\sqrt{29}}{2}$

\_\_\_\_\_ 12. Solve for x using the square root property:  $25x^2 + 9 = 0$

- A)  $x = -\frac{3}{5}$       B)  $x = \pm \frac{3}{5}i$       C)  $x = \pm \frac{3}{5}$       D)  $x = \pm \frac{5}{3}i$

\_\_\_\_\_ 13. Find an equation of the line with slope  $m = -\frac{1}{3}$ , that passes through the point  $(-2, 5)$

- A)  $y = \frac{1}{3}x + \frac{13}{3}$       B)  $y = -\frac{1}{3}x - \frac{13}{3}$       C)  $y = -\frac{1}{3}x - \frac{17}{3}$       D)  $y = -\frac{1}{3}x + \frac{13}{3}$

\_\_\_\_\_ 14. Find the domain of the function and write your answer in interval notation:  $f(x) = \sqrt{6x - 24}$

- A)  $[4, \infty)$       B)  $(-\infty, 4]$       C)  $(4, \infty)$       D)  $(-\infty, 4) \cup (4, \infty)$

\_\_\_\_\_ 15. Solve the inequality and write your final answer in interval notation:  $\frac{x+4}{x-1} \leq 0$

- A)  $(-\infty, -4] \cup (1, \infty)$       B)  $(-\infty, -4] \cup [1, \infty)$       C)  $[-4, 1)$       D)  $[-4, 1]$

\_\_\_\_\_ 16. If  $f(x) = 3x + 5$  and  $g(x) = x^2 - 8$ , find  $(g \circ f)(x)$  and simplify.

- A)  $3x^3 + 5x^2 - 24x - 40$       B)  $9x^2 + 30x + 17$       C)  $9x^2 + 17$       D)  $3x^2 - 19$

\_\_\_\_\_ 17. Determine the vertex of the parabola given by:  $f(x) = 3x^2 + 36x + 76$

- A) (6, 2)      B) (-6, -2)      C) (6, -2)      D) (-6, 2)

\_\_\_\_\_ 18. Given that  $x = 6$  is a root of  $P(x) = x^3 - 4x^2 - 20x + 48$ , find the remaining roots.

- A)  $x = -4, x = 2$       B)  $x = -2, x = 4$       C)  $x = -4, x = 12$       D)  $x = -12, x = 4$

\_\_\_\_\_ 19. Solve for  $x$  in the system of equations using any method: 
$$\begin{aligned} x + 2y &= 15 \\ 2x - y &= 5 \end{aligned}$$

- A)  $x = 4$       B)  $x = 2$       C)  $x = 5$       D)  $x = 1$

\_\_\_\_\_ 20. A rectangular family room is twice as long as it is wide, and its perimeter is 84 feet. Find the dimensions of the family room.

- A) 18 ft x 36 ft      B) 16 ft x 32 ft      C) 12 ft x 24 ft      D) 14 ft x 28 ft

\_\_\_\_\_ 21. Find the value of  $x$ :  $\frac{1}{2} \log_5 9 + \frac{3}{2} \log_5 4 - \log_5 6 = \log_5 x$

- A) 4      B) 5      C) 6      D) 7

\_\_\_\_\_ 22. Simplify:  $\frac{e^{2-3x}}{e^{x+1}}$

- A)  $e^{1-4x}$       B)  $e^{1-2x}$       C)  $e^{3-4x}$       D)  $e^{3-2x}$

\_\_\_\_\_23. Suppose you drive for 12.5 hours and you travel 786 miles. Which answer best describes your rate of travel?

A) 65 mi/hr

B) 67 mi/hr

C) 63 mi/hr

D) 61 mi/hr

\_\_\_\_\_24. Suppose \$12,000 is invested at 7% interest compounded monthly. How much money will be in the account at the end of 9 years?

A) \$5,475,310

B) \$27,706

C) \$547,531

D) \$22,490

\_\_\_\_\_25. An urn contains 100 marbles. Fifty are purple, ten are green, fifteen are red, twenty-five are orange. Two marbles are drawn at random one after another without replacement. Calculate the following:

***P(green and red)***

A)  $\frac{150}{9900}$

B)  $\frac{150}{10000}$

C)  $\frac{140}{9900}$

D)  $\frac{140}{10000}$

1. How many liters of a 70% alcohol solution must be added to 50 liters of a 40% alcohol solution to produce a 50% alcohol solution?

## Tie Breaker #2

For the function  $f(x) = \sqrt{2x-1}$  determine if an inverse function,  $f^{-1}(x)$  exists. If the inverse exists, find the inverse:

Show that the functions  $f(x)$  and  $f^{-1}(x)$  are inverse functions.

## Tie Breaker #3

Consider the rational function,  $f(x) = \frac{x-1}{x^2-2x-3}$

Find the following:

1. Horizontal asymptote(s)
2. Vertical asymptote(s)
3. X- intercept(s)
4. Y- intercept

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1. D
2. A
3. C
4. D
5. B
6. D
7. B
8. C
9. B
10. A
11. C
12. B
13. D
14. A
15. C
16. B
17. B
18. A
19. C
20. D
21. A
22. A
23. C
24. D
25. A

Tie Breakers

1. 25 liters

2.

$$f^{-1} = \frac{x^2 + 1}{2}$$

$$f \circ f^{-1} = \sqrt{2 \left( \frac{x^2 + 1}{2} \right) - 1}$$

$$f \circ f^{-1} = x$$

3.

Horizontal asymptote  $y = 0$

Vertical asymptotes  $x = 3$  and  $x = -1$

X – intercept  $(1,0)$  or  $x = 1$

Y – intercept  $(0,1/3)$  or  $y = 1/3$