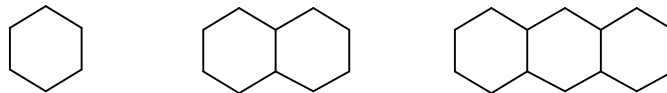


2008 Regional Geometry Exam  
Arkansas Council of Teachers of Mathematics

In each of the following questions choose the best answer and bubble the corresponding letter on the answer sheet. Note: The geometric figures on this exam are not necessarily drawn to scale. When you have completed the first 25 questions please work on the tie-breaker questions. These will be used to break ties for determining first, second and third place, should a tie occur.

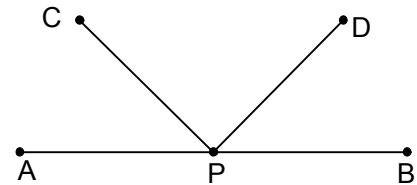
1. A chain of hexagons is made with toothpicks. Chains of length one, two and three are displayed below. How many toothpicks would be needed to make a chain that is 15 hexagons in length?



Problem #1

- A. 90                      B. 62                      C. 75                      D. 76
- E. None of these
2. In the figure  $\overline{CP} \perp \overline{DP}$  and the points A, P and B are collinear. A pair of complementary angles is

- A.  $\angle APC, \angle BPC$                       B.  $\angle APD, \angle BPD$
- C.  $\angle APC, \angle BPD$                       D.  $\angle BPC, \angle BPD$
- E. None of these



Problem #2

3. In a building project Jane needs three boards that are 2 feet 5 inches long. If one inch is allowed for waste, how long a board should she buy?

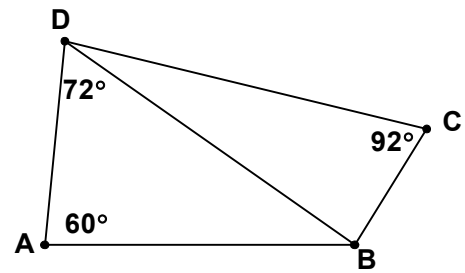
- A.  $7\frac{1}{4}$  ft                      B.  $7\frac{1}{3}$  ft                      C.  $7\frac{1}{2}$  ft
- D.  $6\frac{1}{3}$  ft                      E. None of these

4. In  $\triangle ABC$ ,  $\angle BCD$  is an exterior angle. If  $m\angle BCD = 117^\circ$ , which of the following could be the measures of  $\angle ABC$  and  $\angle BAC$ ?

- A.  $31^\circ, 34^\circ$                       B.  $42^\circ, 19^\circ$                       C.  $35^\circ, 18^\circ$
- D. Not enough information                      E. None of these

5. The following figure is **not** drawn to scale. According to the angle measures indicated, which will be the longest segment?

- A.  $\overline{AB}$                       B.  $\overline{AD}$   
 C.  $\overline{BD}$                       D.  $\overline{BC}$   
 E.  $\overline{CD}$



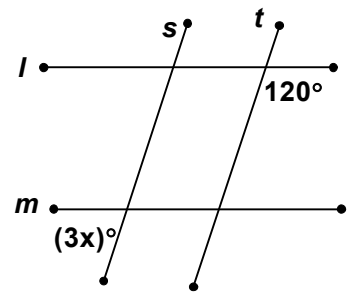
Problem #5

6. A prism has an octagonal base. The number of edges in the prism is

- A. 8                      B. 16                      C. 24                      D. 32  
 E. None of these

7. In the figure,  $l \parallel m$  and  $s \parallel t$ . With the indicated angle measures, what is the value of  $x$ ?

- A. 20  
 B. 40  
 C. 60  
 D. Not enough information  
 E. None of these



Problem #7

8. Consider the following sets of numbers. Which could be the sides of a right triangle?

$P = \{20, 21, 29\}$                        $Q = \{9, 40, 43\}$                        $R = \{12, 35, 37\}$

- A. P                      B. Q                      C. R                      D. P and R  
 E. Q and R

9. The width ( $W$ ) of a rectangle is increased by 2 and the length ( $L$ ) is increased by 3. The amount of change in the area is given by

- A.  $2L + 3W + 6$                       B.  $WL + 3L + 2W + 6$                       C.  $WL + 3W + 2L + 6$   
 D.  $3L + 2W + 6$                       E. None of these

10. The sum of the measures of the interior angles of a polygon is 2160. The number of sides in the polygon is

- A. 13                      B. 15                      C. 12                      D. 16  
 E. None of these

11. Consider the following statements about the diagonals of a rectangle.

- I. The diagonals are perpendicular.
- II. The diagonals are congruent.
- III. The diagonals bisect each other.

Which of the above statement are always true?

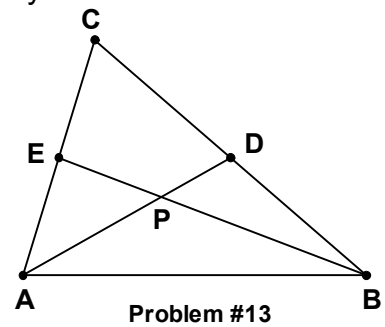
- A. I, II and III
- B. I and II
- C. I and III
- D. II and III
- E. None of these

12. A quadrilateral is formed by joining the midpoints of the sides of a rectangle. The quadrilateral is best described as a

- A. parallelogram
- B. square
- C. rectangle
- D. rhombus
- E. None of these

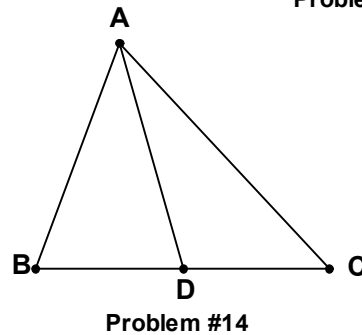
13. In the figure D and E are midpoints of  $\overline{BC}$  and  $\overline{AC}$ , respectively. If  $\overline{AD}$  and  $\overline{BE}$  intersect at P and  $AD = 18$ , then  $PD =$

- A. 12
- B. 6
- C. 9
- D. Not enough information
- E. None of these



14. In  $\triangle ABC$ ,  $\overline{AD}$  is a median and  $\overline{AD} \perp \overline{BC}$ . Which of the following is true?

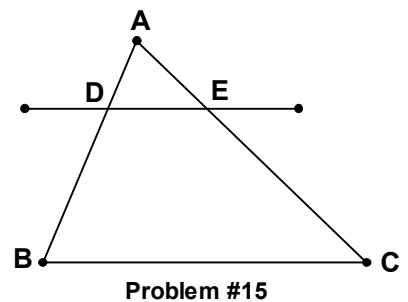
- I.  $\triangle ABC$  is an equilateral triangle.
- II.  $\overline{AD}$  bisects  $\angle BAC$
- III.  $\angle ABD \cong \angle ACD$



- A. I and II
- B. I and III
- C. II and III
- D. I, II and III
- E. None of these

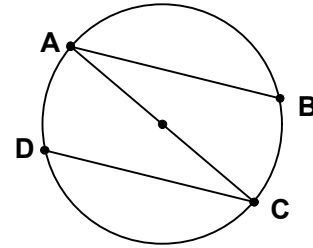
15. In  $\triangle ABC$ ,  $\overline{DE} \parallel \overline{BC}$ . If  $AD = 6$ ,  $BD = 15$ , and  $AC = 49$ , then  $CE =$

- A. 14
- B. 35
- C. 24.6
- D. 29.4
- E. None of these



16. In a circle with diameter  $\overline{AC}$ ,  $\overline{AB} \parallel \overline{CD}$ . If  $m\angle BAC = 42^\circ$ , then the measure of the minor arc CD is

- A.  $84^\circ$       B.  $42^\circ$   
 C.  $96^\circ$       D.  $159^\circ$   
 E. None of these



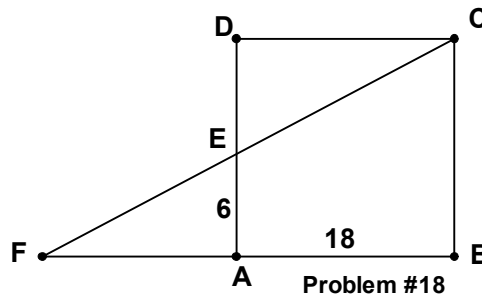
Problem #16

17. A regular polygon is inscribed in a circle of radius R. If the perpendicular distance from the center of the circle to a side of the polygon is P then the side of the polygon is given by

- A.  $\sqrt{R^2 + P^2}$       B.  $\sqrt{R^2 - P^2}$       C.  $2\sqrt{R^2 + P^2}$   
 D.  $2\sqrt{R^2 - P^2}$       E. None of these

18. In the figure at the right ABCD is a square. If  $AB = 18$  and  $AE = 6$ , then  $CF =$

- A.  $3\sqrt{13}$   
 B. 27  
 C.  $6\sqrt{10}$   
 D.  $9\sqrt{13}$



Problem #18

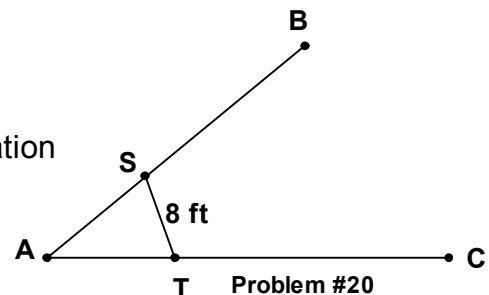
- E. None of these

19. A diagonal of a rectangle makes an angle of  $42^\circ$  with a side of the rectangle. If the length of the diagonal is 12 then the perimeter of the rectangle (to the nearest thousandth) is

- A. 16.947      B. 33.895      C. 34.081  
 D. 68.163      E. None of these

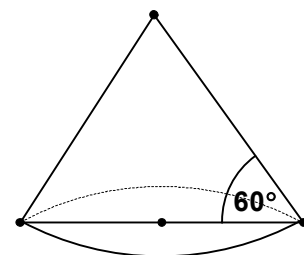
20. Two points S and T are moving at the same rate along rays  $\overrightarrow{AB}$  and  $\overrightarrow{AC}$ , respectively. If after 2 minutes the distance between S and T is 8 feet, how far are they apart after 5 minutes?

- A. 20 feet      B. 40 feet  
 C. 24 feet      D. Not enough information  
 E. None of these



Problem #20

21. At a particular time of day a person who is 6 feet tall casts a shadow along level ground that is 8 feet in length. At the same time of day, what will be the distance from the top of a tree that is 15 feet tall to the tip of the shadow of the tree?
- A. 20 feet,                      B. 25 feet                      C. 24 feet  
 D. 30 feet                      E. None of these
22. Let  $A = (-2, 1)$  and  $B = (3, -11)$  be points in the rectangular coordinate plane. The distance  $AB$  is
- A.  $\sqrt{101}$                       B. 18                      C. 13                      D.  $\sqrt{205}$   
 E. None of these
23. The points  $(-3, 2)$ ,  $(1, 5)$  and  $(-2, 9)$  are three coordinates of vertices of a square. What are the coordinates of the fourth vertex?
- A.  $(-6, 6)$                       B.  $(6, -6)$                       C.  $(-6, 4)$   
 D.  $(-6, 5)$                       E. None of these
24. Which of the following lines is perpendicular to the line  $2x - 3y = 6$ ?
- A.  $2x + 3y = 6$                       B.  $6x + 4y = 15$                       C.  $3x - 2y = 6$   
 D.  $2x - 3y = 12$                       E. None of these
25. In a right circular cone a line segment drawn on the surface of the cone from the vertex to the end of a diameter of the base of the cone makes an angle of  $60^\circ$  with the diameter. If the diameter of the base is 12, then the volume of the cone is
- A.  $72\sqrt{3}\pi$                       B.  $108\sqrt{3}\pi$   
 C.  $288\sqrt{3}\pi$                       D.  $72\pi$   
 E. None of these



Problem #25

**TIE BREAKER PROBLEMS**  
**GEOMETRY - 2008**

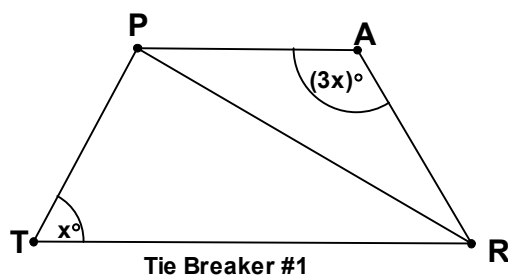
Name \_\_\_\_\_  
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School \_\_\_\_\_

The tie breaker problems should be done when the first 25 questions have been answered. They will be used, in the order they are given, to break any ties for first, second, or third place.

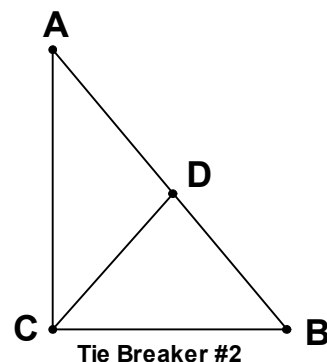
**Tie Breaker #1**

In trapezoid TRAP with  $\overline{PT} \cong \overline{PA} \cong \overline{AR}$  and  $m\angle PTR = x^\circ$  and  $m\angle PAR = (3x)^\circ$ , find the measure of  $\angle ARP$ .



**Tie Breaker #2**

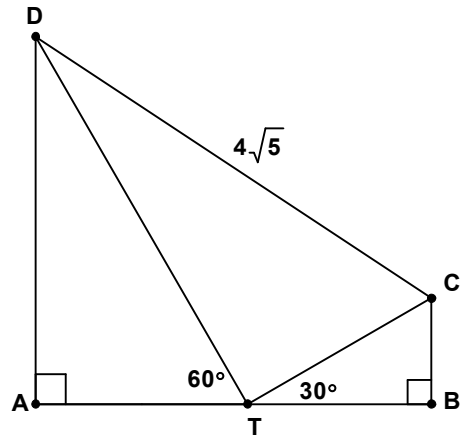
$\triangle ABC$  is a right triangle with right angle at C. A segment is drawn from C to the midpoint D of the hypotenuse. If  $AC = 8$  and  $BC = 6$ , find the measure of  $\angle BCD$  to the nearest tenth of a degree.



Name \_\_\_\_\_ School \_\_\_\_\_

**Tie Breaker #3**

Find the length of  $\overline{AB}$  if the ratio of the perimeter of  $\triangle TBC$  to that of  $\triangle DAT$  is  $1 : 2$ .



**Tie Breaker #3**

**TIE BREAKER PROBLEMS  
GEOMETRY - 2008**

Name KEY  
PLEASE PRINT

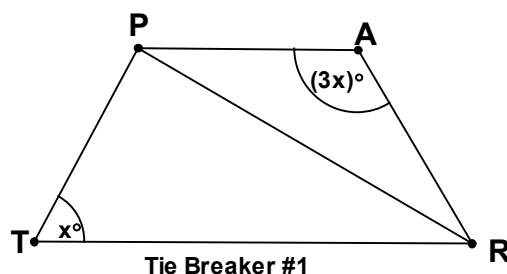
School \_\_\_\_\_

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**Tie Breaker #1**

In trapezoid TRAP with  $\overline{PT} \cong \overline{PA} \cong \overline{AR}$  and  $m\angle PTR = x^\circ$  and  $m\angle PAR = (3x)^\circ$ , find the measure of  $\angle ARP$ .

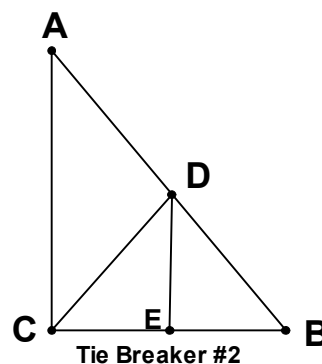
**TRAP is isosceles so  $m\angle APT = (3x)^\circ$ .**  
 **$\angle APT$  and  $\angle PTR$  are supplementary.**  
**So  $3x + x = 180$  and  $x = 45^\circ$ . So  $m\angle PAR = 135^\circ$ .**  
 **$\triangle APR$  is isosceles so  $m\angle ARP = \frac{1}{2}(180 - 135)^\circ$ .**  
**So  $m\angle ARP = 22\frac{1}{2}^\circ$ .**



**Tie Breaker #2**

$\triangle ABC$  is a right triangle with right angle at C. A segment is drawn from C to the midpoint D of the hypotenuse. If  $AC = 8$  and  $BC = 6$ , find the measure of  $\angle BCD$  to the nearest tenth of a degree.

**A perpendicular from D to  $\overline{BC}$  bisects  $\overline{BC}$  at E.**  
**Now,  $DE = \frac{1}{2} AC = 4$ .  $CD = \frac{1}{2} AB$ .  $AB = 10$  so  $CD = 5$ .**  
 **$\sin(\angle BCD) = \frac{4}{5}$  and  $\angle BCD = 53.1^\circ$ .**





Name \_\_\_\_\_

School \_\_\_\_\_

**Tie Breaker #3**

Find the length of  $\overline{AB}$  if the ratio of the perimeter of  $\triangle TBC$  to that of  $\triangle DAT$  is 1 : 2.

Let  $BC = a$ . Therefore  $CT = 2a$ . Thus,

$$DT = \sqrt{80 - 4a^2}.$$

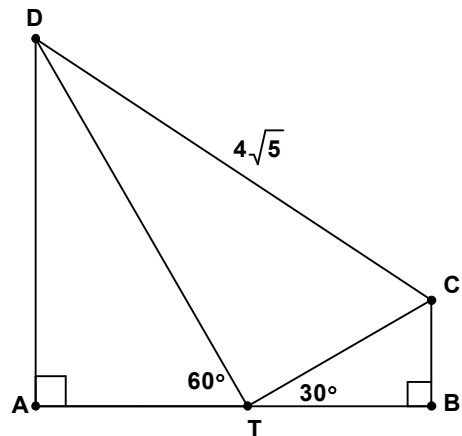
But  $AT = 2a$  since the ratio of the perimeters is 1 : 2 and  $\triangle TBC$  is similar to  $\triangle DAT$ . Therefore,  $DT = 4a$ .

Therefore,  $4a = \sqrt{80 - 4a^2}$ . So  $16a^2 = 80 - 4a^2$ . Then  $20a^2 = 80$  and  $a^2 = 4$ . Thus  $a = 2$ .

$$\text{Now } AB = AT + BT = 2a + a\sqrt{3} = 2(2 + \sqrt{3})$$

**Alternate solution:**

Let  $BC = a$ , then  $TC = 2a$  and  $AT = 2a$ . So  $DT = 4a$ . Therefore,  $16a^2 + 4a^2 = 80$ . Thus,  $a^2 = 4$  and  $a = 2$ . Since  $BT = 2\sqrt{3}$  then  $AT = 4 + 2\sqrt{3}$ .



Tie Breaker #3

## Key – Regional Geometry 2008

- |     |   |     |   |
|-----|---|-----|---|
| 1.  | D | 14. | C |
| 2.  | C | 15. | B |
| 3.  | B | 16. | C |
| 4.  | E | 17. | D |
| 5.  | A | 18. | D |
| 6.  | C | 19. | B |
| 7.  | A | 20. | A |
| 8.  | D | 21. | B |
| 9.  | A | 22. | C |
| 10. | E | 23. | A |
| 11. | D | 24. | B |
| 12. | D | 25. | A |
| 13. | B |     |   |