State Precalculus/Trigonometry Contest 2006

Select the best answer for each of the following questions and mark it on the answer sheet provided. Be sure to read all the answer choices before making your selection. When you are finished with the multiple choice, attempt the tiebreaker questions.

1. Find the inverse function $f^{-1}(x)$ of $f(x) = \dfrac{3x+1}{x-1}$.

a) $f^{-1}(x) = \dfrac{x-1}{3x+1}$ b) $f^{-1}(x) = \dfrac{\frac{1}{3}x-1}{x+1}$ c) $f^{-1}(x) = \dfrac{x+1}{x-3}$ d) $f^{-1}(x) = 3 + \dfrac{1}{x} - 1$

e) 0

2. How long does it take to triple your money if you invest in an account that draws 5% annual interest compounded continuously.

a) 4.09 years b) 13.86 years c) 20.10 years d) 21.97 years e) 22.52 years

3. Which of the following is the vertex of the parabola $y = 2x^2 + 3x + 1$.

a) $(-\dfrac{3}{4}, -\dfrac{1}{8})$ b) $(\dfrac{3}{4}, -\dfrac{1}{8})$ c) (0,4) d) (2,3) e) (2,-3)

4. A parabola has vertex (2,0) and passes through the point (3,5). Which of the following points is on the parabola?

a) (4,20) b) (0,18) c) (1,6) d) (-1,32) e) (-2,32)

5. The solution set of $|3x - 5| > 1$ is:

a) $\{x|\ x < 2\}$ b) $\{x|\ x > 2\}$ c) $\{x|\ x > \frac{4}{3}\}$ d) $\{x|\ x < \frac{4}{3}$ or $x > 2\}$ e) $\{x|\ \frac{4}{3} < x < 2\}$

6. The center of the circle $x^2 + y^2 + 4x - 2y - 11 = 0$ is:

a) (-2,1) b) (1,-2) c) (1,1) d) (4,-2) e) (4,1)

7. The complex number $\dfrac{2}{3+i}$ is equal to:

a) $\dfrac{2}{3} + \dfrac{2}{i}$ b) $3 - i$ c) $\dfrac{3}{5} - \dfrac{1}{5}i$ d) $\dfrac{3}{4} - \dfrac{1}{4}i$ e) $\dfrac{3}{4} + \dfrac{1}{4}i$

8. If θ is an angle in standard position whose terminal side passes through the point (-3,4), what is the value of $\cos\theta + \tan\theta$?

a) $-\dfrac{8}{15}$ b) $-\dfrac{29}{15}$ c) $\dfrac{1}{5}$ d) 1 e) 0

9. If α and β are first quadrant angles for which $\sin\alpha = \dfrac{1}{2}$ and $\cos\beta = \dfrac{1}{3}$, what is the value of $\cos(\alpha+\beta)$?

a) $\dfrac{5}{6}$ b) $\dfrac{3\sqrt{3}+2}{6}$ c) $\dfrac{\sqrt{3}-2\sqrt{2}}{6}$ d) $\dfrac{3\sqrt{3}-2}{6}$ e) $\dfrac{1}{2}$

10. In a triangle with sides of lengths 3 ft, 4 ft, and 6 ft, what is the measure of the angle between the sides of lengths 4 ft and 6 ft?

a) 26.4^o b) 48.2^o c) 114.2^o d) 117.3^o e) 122.8^o

11. What is the exact value of $\cot \dfrac{11\pi}{6}$?

a) $\dfrac{2}{\sqrt{3}}$ b) $\dfrac{3}{\sqrt{2}}$ c) $-\dfrac{\sqrt{3}}{3}$ d) -2 e) $-\sqrt{3}$

12. Which of the following are true for all real numbers x: (I) $\sin x = \sin(-x)$, (II) $\cos x = \cos(-x)$, (III) $\tan x = \tan(x + \pi)$?

a) Only I. b) Only II. c) Only III. d) Only II and III. e) Only I and II.

13. If $0 < x < \dfrac{\pi}{2}$, what is the value of $\tan(\sin^{-1} x)$?

a) $\dfrac{x}{\sqrt{x^2+1}}$ b) $\dfrac{\sqrt{x^2+1}}{x}$ c) $\dfrac{1}{x}$ d) $\dfrac{x}{\sqrt{1-x^2}}$ e) $\dfrac{\sqrt{1-x^2}}{x}$

14. In a right triangle with a 30^o angle, how is the hypotenuse related to the shorter leg?

a) The length of the hypotenuse is $\sqrt{2}$ times the length of the shorter leg.

b) The length of the hypotenuse is $\sqrt{3}$ times the length of the shorter leg.

c) The length of the hypotenuse is 2 times the length of the shorter leg.

d) The length of the hypotenuse is 3 times the length of the shorter leg.

e) The length of the hypotenuse is 5 times the length of the shorter leg.

15. The expression $(\dfrac{\sqrt{3}}{2} + \dfrac{i}{2})^{18}$ is equal to

a) $9\sqrt{3} + 9i$ b) i c) $-i$ d) 1 e) -1

16. Which of the following is an equation of the line passing through the point (5,1) and perpendicular to the line $y = \dfrac{1}{2}x - 3$?

a) $y - 1 = -2(x - 5)$ b) $y - 5 = -2(x - 1)$ c) $y - 1 = 2(x - 5)$ d) $y - 5 = 2(x - 1)$

e) $y - 1 = -\dfrac{1}{2}(x - 5)$

17. Which of the following statements is true about the graph of a polynomial function of degree 3.

a) It has a maximum value and a minimum value.

b) It is monotonic.

c) It intersects the x-axis at three distinct points.

d) It intersects the y-axis at exactly one point.

e) It has a vertical asymptote.

18. What is the area of a triangle with sides of lengths 2 m, 3 m, and 4 m?

a) 4.000 sq m b) 3.000 sq m c) 2.905 sq m d) 2.675 sq m

e) 2.625 sq m

19. The foci of the ellispe $x^2 + 4y^2 = 16$ have coordinates

a) (-4,0) and (4,0) b) $(-2\sqrt{3}, 0)$ and $(2\sqrt{3}, 0)$ c) (-2,0) and (2,0) d) $(-2\sqrt{5}, 0)$ and $(2\sqrt{5}, 0)$

e) (0,2) and (0,-2)

20. When $(x + y)^{20}$ is expanded, you get the polynomial $x^{20} + 20x^{19}y + \cdots + y^{20}$. What is the coefficient on $x^{12}y^8$?

a) 125,970 b) 38,760 c) 9,216 d) 96 e) 12

21. The determinant of the matrix $\begin{pmatrix} 2 & 1 & 3 \\ -1 & 0 & 4 \\ 5 & 2 & 1 \end{pmatrix}$ is

a) 29 b) 12 c) 3 d) 0 e) -1

22. Which of the following numbers does $(1 + \frac{1}{n})^n$ approach as n becomes larger and larger?

a) π b) e c) 1 d) $\sqrt{7}$ e) 255.6067

23. If θ is a fourth quadrant angle, which of the following gives $\sin\frac{\theta}{2}$?

a) $\sqrt{\frac{1}{2} + \frac{1}{2}\sin\theta}$ b) $-\sqrt{\frac{1}{2} + \frac{1}{2}\sin\theta}$ c) $\sqrt{\frac{1}{2} - \frac{1}{2}\cos\theta}$ d) $-\sqrt{\frac{1}{2} - \frac{1}{2}\cos\theta}$ e) $\frac{1}{2}\sin\theta$

24. What value of x solves $\log_6 x + \log_6(x + 2) = 1$.

a) 1 b) 1.65 c) $\sqrt{8}$ d) $-1 + \sqrt{7}$ e) 0

25. What's the area of the region $\{(r, \theta) |\ 2 \le r \le 3, \frac{\pi}{2} \le \theta \le \pi\}$, where r and θ are polar coordinates?.

a) $\frac{5\pi}{4}$ b) 5π c) $\frac{9\pi}{4}$ d) π e) $\frac{5}{4}$

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Tiebreakers

1. Find the exact values of all x that solve the equation $\cos 2x = \sin x$.

2. Find an equation in the form $(x-h)^2+(y-k)^2=r^2$ of the circle passing through the points (0,2), (7,1), and (3,3).

3. If $1 + i$ is a zero of the polynomial $f(x) = x^4 - 2x^3 - 3x^2 + 10x - 10$, find the exact values of all the real zeros of $f(x)$.

Key for the Regional Precalculus/Trigonometry Contest 2006

1. C
2. D
3. A
4. A
5. D
6. A
7. C
8. B
9. C
10. A
11. E
12. D
13. D
14. C
15. E
16. A
17. D
18. C
19. B
20. A
21. E
22. B
23. C
24. D
25. A

Tiebreaker 1.

$\cos^2 x - \sin^2 x = \sin x \ \Rightarrow \ 1 - 2\sin^2 x = \sin x \ \Rightarrow \ 2\sin^2 x + \sin x - 1 = 0 \Rightarrow \ (2\sin x - 1)(\sin x + 1) = 0 \ \Rightarrow$ $\sin x = \dfrac{1}{2}$ or $-1 \ \Rightarrow \ x = \dfrac{\pi}{6} + 2n\pi, \dfrac{5\pi}{6} + 2n\pi,$ or $\dfrac{3\pi}{2} + 2n\pi$, where n is an integer

Tiebreaker 2.

Let (h, k) be the center. Then $(h-0)^2 + (k-2)^2 = (h-7)^2 + (k-1)^2 = (h-3)^2 + (k-3)^2 \ \Rightarrow$ $-4k + 4 = -14h + 49 - 2k + 1 = -6h + 9 - 6k + 9 \ \Rightarrow \ 14h - 2k = 46$ and $6h + 2k = 14$ (setting the first two expressions equal and then the first and last expressions equal in the string of equalities) $\Rightarrow \ 20h = 60$ $\Rightarrow \ h = 3$ and $k = -2 \ \Rightarrow \ r^2 = (0-3)^2 + (2+2)^2 = 25 \ \Rightarrow \ (x-3)^2 + (y+2)^2 = 25$

Tiebreaker 3.

Since $1 + i$ is a zero, so is $1 - i$. Hence, $(x - (1+i))(x - (1-i)) = x^2 - 2x + 2$ is a factor of $f(x)$. Dividing $x^2 - 2x + 2$ into $f(x)$ you get $x^2 - 5$. Therefore, $f(x) = (x - (1+i))(x - (1-i))(x^2 - 5)$, so you can see that the only real zeros are $\sqrt{5}$ and $-\sqrt{5}$.