Regional ACTM Calculus Exam
Spring 2006

Select the best answer for each of the following. Unless otherwise stated, all variables are real numbers.

1. Find an equation for the tangent line to the parabola y= x^2 at the point P(1,1)

A. y = 2x -1 B. y = -2x +1 C. y = -1/2x + 1
D. y =1/2x – 1 E. None of these

2. Find $\lim_{x\to 1} \dfrac{x-1}{\sqrt{x}-1}$.

A. $\sqrt{x}-1$ B. $\sqrt{x}+1$ C. -1
D. 2 E. None of these

3. Find $\lim_{x\to 2} \dfrac{x^2-4x+4}{x^2+2x-6}$.

A. 0 B. 4 C. ∞
D. there is no limit E. None of these

4. $\lim_{x\to +\infty} \sqrt{\dfrac{x^7-4x^5}{2x^7+1}}$

A. 1 B. 5 C. $\sqrt{3/2}$ D. Doesn't exist E. None of these

5. According to Ohm's Law, when a voltage of V volts is applied across a resistor with a resistance of R ohms, a current of $I = \dfrac{V}{R}$ amperes flows through the resistor. Certain alloys become SUPERCONDUCTORS as their temperature approaches absolute zero, meaning that their resistance approaches zero. If the voltage remains constant, what happens to the current in a superconductor as $R\to 0^+$?

A. I approaches ∞ B. It goes to zero C. $I \to V$
D. $I \to 1$ E. None of these

6. What can you say about the continuity of the function $f(x) = \sqrt{9-x^2}$

A. f is only continuous on the open interval (-3,3)
B. f is discontinuous everywhere.
C. f is discontinuous only if x ≤ -3 and x ≥ 3.
D. You can't say anything.
E. f is continuous on the closed interval [-3,3]

7. Is $f(x) = \begin{cases} 2x-3, x \le 2 \\ x^2, x > 2 \end{cases}$ continuous everywhere?

A. yes
B. Not at x = 2
C. Not at x = -2
D. Not at x = 0
E. None of these

8. Suppose that *f* and *g* are continuous functions such that *f*(2) = 1 and $\lim_{x\to 2}[f(x) + 4g(x)] = 13$. Find g(2).

A. 2
B. 4
C. 1
D. 3
E. None of these

9. Suppose that the cost of drilling *x* feet for an oil well is $C = f(x)$ dollars. Estimate the cost of drilling an additional foot, starting at a depth of 300 ft, given that $f'(300)$ = $ 10,000

A. $10,000
B. We need f(x)
C. $300
D. $300,000
E. None of these

10. Find $f'''(2)$ if $f(x) = 3x^2 - 2$.

A. 6
B. 12
C. 0
D. -1
E. None of these

11. Find *k* if the curve $y = x^2 + k$ is tangent to the line $y = 2x$.

A. 0
B. 1
C. 2
D. 3
E. None of these

12. Find the derivative of $f(x) = \dfrac{3}{(2x+1)^2}$.

A. $-6(2x+1)^{-3}$
B. $-12(2x+1)^{-3}$
C. 12
D. $2(2x+1)^2$
E. None of these

13. If $x^2 - xy + y^2 = 1$, find y'.

A. $y' = \dfrac{y-2x}{2y-x}$
B. $y' = \dfrac{y+2x}{x-2y}$
C. $y' = \dfrac{x^2}{x-2y}$
D $y' = \dfrac{2x}{x-1}$
E. None of these

14. Find the equation of the line tangent to the graph of $\tan x$ at $x = \frac{\pi}{4}$.

A. $y = 2(x - \frac{\pi}{4})$ B. $y = 2x$ C. $y = 2x + 1 - \pi/2$

D. $y = \frac{\sin x}{\cos x}$ E. None of these

15. If $f(x) = x^3 - 6x^2 + 9x$, then $f(x)$ is increasing on the interval(s)

A. for all x B. $x < 1$ only C. $x < 1$ and $x > 3$
D. $x > 3$ only E. none of these

16. Find y' if $y = \sin(\cos 2x)$.

A. $y' = \cos^2(2x)\sin(2x)$ B. $y' = 2\cos(2x)$ C. $y' = \cos(\cos(2x))(-\sin(2x))\cdot 2$
D. $y' = \tan(2x)$ E. none of these

17. Find the value of the constant A so that $y = A\sin 3t$ satisfies the equation $\frac{d^2y}{dt^2} + 2y = 4\sin 3t$.

A. $\frac{-4}{7}$ B. $\frac{4}{7}$ C. 4 D. $\frac{3}{4}$ E. None of these

18. The area bound between $y = x^3 - x$ and the x-axis is given by

A. $\int_0^1 x^3 - x\ dx$ B. $\int_{-1}^1 x^3 - x\ dx$ C. $\int_{-1}^0 x^3 - x\, dx - \int_0^1 x^3 - x\, dx$

D. $\int_{-2}^2 x^3 - x\, dx$ E. None of these

19. Find all the values of a such that the curves $y = \frac{a}{x-1}$ and $y = x^2 - 2x + 1$ intersect at right angles.

A. -1 B. 1 C. $\pm\frac{\sqrt{2}}{4}$ D. $\pm\frac{1}{2}$ E. None of these

Name____

PUT YOUR NAME AT THE TOP OF THESE LAST PAGES. HAND THEM IN WITH YOUR ANSWER SHEET. THE NEXT THREE PROBLEMS ARE TIE-BREAKERS. SHOW ALL NECESSARY WORK! NEATNESS MAY COUNT. DO NOT SKIP THESE PROBLEMS!! IF ALL YOU CAN DO IS DRAW A PICTURE, THEN DO IT! PARTIAL CREDIT WILL BE GIVEN TO BREAK TIES.

1.The population P of the United States (in millions) in year t may be modeled by the function

$$P = \frac{50371.7}{151.3 + 181{,}626e^{-0.031636e(t-1950)}}$$

a. Based on this model, what was the population in 1950?

b. By evaluating the appropriate limit, show that the graph of P versus t has a horizontal asymptote $P = C$ and find C.

c. What is the significance of the constant C in the last part for the population predicted by this model?

Name ____

2. The graph of $f(x) = |x|$ has a corner at $x = 0$, which implies that $f(x) = |x|$ is not differentiable at $x = 0$. Find a formula for $f'(x)$.

Name ____

3. A 10 ft. ladder leans against a wall at an angle of θ with the horizontal. The top of the ladder is x feet above the ground. If the bottom of the ladder is pushed toward the wall, find the rate at which x changes with respect to θ when $\theta = 60°$. Express the answer in units of feet/degree.

Name ____

4. Find a number δ such that $|x^2 - 9| < .5$ if $0 < |x - 3| < \delta$.

2006 Regional Calculus Exam Answers

1. A
2. D
3. A
4. C
5. A
6. E
7. B
8. D
9. A
10. C
11. B
12. B
13. A
14. C
15. C
16. C
17. A
18. C
19. C

Tie Breaker 1.

a. If x = 1950, then P = 331.4 million
b. $\lim_{x \to \infty} P = .277$
c. Population will tend upward .277 million as time goes by.

Tie Breaker 2

$$f'(x) = \begin{cases} 1, x > 0 \\ -1, x < 0 \\ undefined, x = 0 \end{cases}$$

Tie Breaker 3

Consider the 10 ft ladder leaning against a vertical wall hitting the wall at a distance of x up and a distance z out from the bottom. Let θ be the bottom angle at the ladder, ground contact point. Then we have x = 10sinθ. Differentiating with respect to t we have $\frac{x}{t} = 10\cos\theta \frac{d\theta}{dt.}$. Since cos 60° = ½ we have $\frac{dx}{dt} = .5\frac{d\theta}{dt}$

Tie Breaker 4

a. A geometric constructive procedure is okay if they check their answer and show it works.
b. Since $|x| = \sqrt{x^2}$, we have $\sqrt{(\sqrt{5x+1}-4)^2} \le .5^2 or (\sqrt{5x+1}-4)^2 - .25 \le 0.$ Factoring we have $(\sqrt{5x+1}-4)-.5)(\sqrt{5x+1}-4)+.5) \le 0.$ Rewriting we have $(\sqrt{5x+1}-4.5)(\sqrt{5x+1}-3.5) \le 0$.

Solving we obtain $3.5 \le \sqrt{5x+1} \le 4.5$. Since all terms are positive we have $3.5^2 \le 5x + 1 \le 4.5^2$. Now solving for x we get $2.25 \le x \le 3.85$ or $-.75 \le x-3 \le .85$. Taking the minimum value we let $\delta < .75$

c. Actually there are many ways to work this problem, but checking backwards it appears .75 is an upper bound so any number less than .75 is a good answer.