ACTM 2012 Regional Statistics Exam

Place the letter corresponding to your answer choice in the proper place on the provided answer sheet. Record your answer by darkening the space with a pencil. Should you need to change an answer choice, erase completely. You have 60 minutes to complete the 25 multiple choice question and work on the tie-breaker items. Tie breaker items will only be graded in the case of a tie for first, second or third position. On the tie breaker items, be sure to put your name on each sheet. You may keep the part of the test that has the multiple choice questions. Round to three decimals when applicable.

1. The table below shows the probabilities generated by rolling one die 50 times and noting the up face. What is the probability of getting an odd up face or a two or less?

<table>
<thead>
<tr>
<th>Roll</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.22</td>
<td>0.10</td>
<td>0.18</td>
<td>0.12</td>
<td>0.18</td>
<td>0.20</td>
</tr>
</tbody>
</table>

- a. 0.66
- b. 0.32
- c. 0.90
- d. 0.68
- e. None of the above

2. A controversial bill is being debated in the state legislature. Representative Williams wants to estimate within 5% points and with 98% confidence the difference in the proportion of her male and female constituents who favor the bill. What sample size should she obtain for male and female constituents?

- a. 769 each
- b. 543 each
- c. 1082 each
- d. 55 each
- e. None of the above

3. A severe drought affected several western states for 3 years. A Christmas tree farmer is worried about the drought's effect on the size of his trees. To decide whether the growth of the trees has been retarded, the farmer decides to take a sample of the heights of 25 trees and obtains the following results (recorded in inches):

60, 57, 62, 69, 46, 54, 64, 60, 59, 58, 75, 51, 49, 67, 65, 44, 58, 55, 48, 62, 63, 73, 52, 55, and 50.

The tree farmer feels the normal height of a tree that was unaffected by the drought would be 65 inches. The z-score for a tree that is 65 inches tall is

- a. 0.979
- b. 0.961
- c. 0.772
- d. 0.836
- e. None of the above
4. Administrators at a large University wanted to know the average number of students enrolled for a class in the Spring of 2012. An administrator randomly selected 100 classes and recorded the number of enrolled students in each of these classes. The population of interest in this study is
   a. All students of the University
   b. All students of the University who are enrolled in Spring 2012
   c. All classes that have been offered by the University until Spring 2012
   d. All classes that are offered by the University in Spring 2012
   e. None of the above.

5. A senator wishes to estimate the proportion of United States voters who favor abolishing the Electoral College. How large a sample is needed in order to be 95\% confident that the sample proportion will not differ from the true proportion by more than 2\%?
   a. 1692
   b. 2401
   c. 25
   d. 4802
   e. None of the above

6. Each year a nationally recognized publication conducts its “Survey of America’s Best Graduate and Professional Schools.” An academic advisor wants to predict the typical starting salary of a graduate at a top business school using GMAT score of the school as a predictor variable. A simple linear regression of SALARY versus GMAT using 25 data points gives the coefficient of determination as \( R^2 = 0.66 \). The practical interpretation of \( R^2 \) is
   a. 66\% of the differences in the SALARY are caused by differences in GMAT scores
   b. We estimate SALARY to increase \$0.66 for every 1-point increase in GMAT
   c. 66\% of the sample variation in SALARY can be explained by using GMAT in a straight-line model
   d. We can predict SALARY correctly 66\% of the time using GMAT in a straight-line model
   e. None of the above.

7. Nine students took the SAT. Their scores are listed below. Later on, they read a book on test preparation and retook the SAT. Their new scores are listed below.

<table>
<thead>
<tr>
<th>Student</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scores before reading book</td>
<td>720</td>
<td>860</td>
<td>850</td>
<td>880</td>
<td>860</td>
<td>710</td>
<td>850</td>
<td>1200</td>
<td>950</td>
</tr>
<tr>
<td>Scores, after reading book</td>
<td>740</td>
<td>860</td>
<td>840</td>
<td>920</td>
<td>890</td>
<td>720</td>
<td>840</td>
<td>1240</td>
<td>970</td>
</tr>
</tbody>
</table>

Construct a 95\% confidence interval for the mean difference \( \mu_d \) of scores (before – after). Assume that the distribution is normally distributed.
   a. (-20.341, 4.852)
   b. (-10.321, 15.436)
   c. (1.651, 30.590)
   d. (-30.5, -0.615)
   e. None of the above
8. Find the standardized test statistic $t$ for a sample with $n = 12$, $\bar{x} = 18.7$, $s = 2.1$, and $\alpha = 0.01$ if $H_I: \mu \neq 19.2$
   a. -0.381
   b. -0.825
   c. -0.008
   d. 0.037
   e. None of the above

9. If we do not reject the null hypothesis when the null hypothesis is in error, then we have made a
   a. Correct decision
   b. Type $\beta$ error
   c. Type II error
   d. Type I error
   e. None of the above

10. A study was conducted to determine if the salaries of librarians from two neighboring cities were equal. Samples of 18 librarians from the first city and 13 from the second city were randomly selected. The mean weekly salary from the first city was $630 with a standard deviation of $40. The mean weekly salary from the second city was $615 with a standard deviation of $25. Find the standardized test statistic, $t$, to test the hypothesis that the mean salary from the first city is more than the second city.
    a. 1.865
    b. 1.282
    c. 2.819
    d. 3.271
    e. None of the above

11. How many ways can five people, A, B, C, D, and E, sit in a row at a concert hall if D and E will not sit next to each other?
    a. 48
    b. 72
    c. 24
    d. 60
    e. None of the above

12. An article in a Florida newspaper reported on the topics that teenagers most want to discuss with their parents. The results of a poll showed that 46% would like more discussion about the family's financial situation, 37% would like to talk about school, and 30% would like to talk about religion. These and other percentages were based on a national sampling of 531 teenagers. Estimate the proportion of all teenagers who want more family discussions about school. Use a 99% confidence level.
    a. 0.37 $\pm$ 0.002
    b. 0.63 $\pm$ 0.002
    c. 0.63 $\pm$ 0.054
    d. 0.37 $\pm$ 0.054
    e. None of the above
13. The National Association of Realtors estimates that 23% of all homes purchased in 2004 were considered investment properties. If a sample of 800 homes sold in 2004 is obtained, what is the probability that at most 200 homes are going to be used as investment property?
   a. 0.093
   b. 0.407
   c. 0.911
   d. 0.594
   e. None of the above

14. The data below are the final exam scores of 10 randomly selected history students and the number of hours they slept the night before the exam. Find the equation of the regression line for the given data. What would be the predicted score for a history student who slept 7 hours the previous night? Is this a reasonable question? Round your answer to the nearest whole number.

<table>
<thead>
<tr>
<th>Hours, x</th>
<th>3</th>
<th>5</th>
<th>2</th>
<th>8</th>
<th>2</th>
<th>4</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scores, y</td>
<td>65</td>
<td>80</td>
<td>60</td>
<td>88</td>
<td>66</td>
<td>78</td>
<td>85</td>
<td>90</td>
<td>90</td>
<td>71</td>
</tr>
</tbody>
</table>

   a. \( \hat{y} = 5.044x + 56.114 \); 91; No, it is not reasonable. 7 hours is well outside the scope of the model.
   b. \( \hat{y} = 5.044x + 56.114 \); 91; Yes, it is reasonable.
   c. \( \hat{y} = -5.044x + 56.114 \); 21; No, it is not reasonable. 7 hours is well outside the scope of the model.
   d. \( \hat{y} = -5.044x + 56.114 \); 21; Yes, it is reasonable.
   e. None of the above

15. A random sample of 10 parking meters in a resort community showed the following incomes for a day:

   $3.60, $4.50, $2.80, $6.30, $2.60, $5.20, $6.75, $4.25, $8.00, and $3.00

   Assume that the incomes are normally distributed. The 95% confidence interval for the true mean is

   a. ($4.81, $6.31)
   b. ($1.35, $2.85)
   c. ($2.11, $5.34)
   d. ($3.39, $6.01)
   e. None of the above

16. In 1997, 4% of mothers smoked more than 21 cigarettes during their pregnancy. An obstetrician believes that the percentage of mothers who smoke 21 cigarettes or more is less than 4% today. She randomly selected 120 pregnant mothers and finds that 3 of them smoked 21 or more cigarettes during pregnancy. The P-value of the hypothesis test to check whether the sample data support the obstetrician’s belief is

   a. 0.201
   b. 0.289
   c. approximately 0
   d. approximately 1
   e. None of the above
17. Can money buy happiness? A researcher wanted to determine whether there was any association between economic status and happiness. She selected a sample of 1000 adults and interviewed them. Each person was asked about their financial situation and their level of happiness was evaluated. The researcher analyzed the results to determine whether there was an association between economic status and happiness. This is an example of
a. a retrospective study; Individuals are asked to look back in time
b. a prospective study; Individuals are observed over long period of time
c. a cross-sectional study; Information is collected at a specific point in time
d. a randomized-block study; Individuals are selected randomly at a specific point in time
e. None of the above.

18. According to government data, the probability that an adult was never in a museum is 15%. In a random survey of 10 adults, what is the probability that at least 8 were in a museum?
   a. 0.002  
   b. 0.820  
   c. 0.800  
   d. 0.200  
   e. None of the above

19. Suppose you are using α = 0.05 to test the claim that μ ≠ 27 using a P-value. You are given the sample statistics n = 35, \( \bar{x} = 26.1 \) and \( \sigma = 2.7 \) Find the P-value.
   a. 0.059  
   b. 0.049  
   c. 0.1  
   d. 0.024  
   e. None of the above

20. The mean age of judges in Dallas is greater than 53.2 years. If a hypothesis test is performed, how should you interpret a decision that fails to reject the null hypothesis
   a. There is sufficient evidence to reject the claim \( \mu > 53.2 \)
   b. There is not sufficient evidence to support the claim \( \mu > 53.2 \)
   c. There is not sufficient evidence to reject the claim \( \mu > 53.2 \)
   d. There is sufficient evidence to support the claim \( \mu > 53.2 \)
   e. None of the above

21. Suppose a 90% confidence interval for \( \mu \) turns out to be (150, 250). Based on the interval, do you believe the average is equal to 260?
   a. Yes, and I am 90% sure of it  
   b. No, and I am 100% sure of it  
   c. Yes, and I am 100% sure of it  
   d. No, and I am 90% sure of it  
   e. None of the above
22. A study was designed to investigate the effects of two variables - (1) a student's level of mathematical anxiety and (2) teaching method - on a student's achievement in a mathematics course. Students who had a low level of mathematical anxiety were taught using the traditional expository method. These students obtained a mean score of 490 with a standard deviation of 30 on a standardized test. Assuming no information concerning the shape of the distribution is known, what percentage of the students scored between 430 and 550?
   a. At least 88.9%
   b. At least 75%
   c. Approximately 88.9%
   d. Approximately 75%
   e. None of the above

23. A human gene carries a certain disease from the mother to the child with a probability rate of 42%. That is, there is a 42% chance that the child becomes infected with the disease. Suppose a female carrier of the gene has 3 children. Assume that the infections of the 3 children are independent of one another. The probability that at least one of the children gets the disease from their mother is
   a. 0.141
   b. 0.195
   c. 0.805
   d. 0.424
   e. None of the above

24. A 1-pound bag of peanuts contains 430 peanuts. The distribution of weights in grams of the peanuts is given below. What is the mean weight of a peanut?
   a. 1.088 g
   b. 0.965 g
   c. 1.029 g
   d. 1.079 g
   e. None of the above

<table>
<thead>
<tr>
<th>Weights (grams)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.755 - 0.814</td>
<td>3</td>
</tr>
<tr>
<td>0.815 - 0.874</td>
<td>2</td>
</tr>
<tr>
<td>0.875 - 0.934</td>
<td>2</td>
</tr>
<tr>
<td>0.935 - 0.994</td>
<td>2</td>
</tr>
<tr>
<td>0.995 - 1.054</td>
<td>2</td>
</tr>
<tr>
<td>1.055 - 1.114</td>
<td>241</td>
</tr>
<tr>
<td>1.115 - 1.174</td>
<td>12</td>
</tr>
</tbody>
</table>

25. A test contains 10 multiple choice questions with each question having 4 choices and exactly one of them is correct. At least 6 questions must be answered correctly to pass the test. If a student takes this test and chooses randomly, what is the probability that the student will pass the test?
   a. 0.020
   b. 0.984
   c. 0.016
   d. 0.996
   e. None of the above
Tie Breaker

Student Name: _______________________

Answer the tie breakers in the order provided. This is the order in which they will be graded and used to break a tied score if needed. Remove the tie breaker pages from the test booklet and hand in with your multiple choice answer sheet.

1. A calculus instructor is interested in the performance of his students from Calculus I that go on to Calculus II. Their final grades in each course (in percent) are given below.

| Calculus I (x) | 88 | 78 | 62 | 75 | 95 | 91 | 83 | 86 | 98 |
| Calculus II (y) | 81 | 80 | 55 | 78 | 90 | 90 | 81 | 80 | 100 |

a. Compute the linear correlation coefficient between Calculus I and II.

b. Find the least-squares regression line.

c. Suppose a student scored 90 in Calculus I. Use the least-square regression line to predict this student’s score in Calculus II.

d. Compute the sum of the squared residuals of the least squared line for the given data.
2. From a group of 8 women and 6 men, a committee consisting of 3 men and 3 women is to be formed. How many different committees are possible if
   a. 2 of the men refuse to serve together?
   b. 2 of the women refuse to serve together?
   c. 1 man and 1 woman refuse to serve together?
Provide necessary details to support your answer.
3. The following data represent the level of health and the highest degree earned for a random sample of individuals from the General Social Survey. Does amount of education play a role in the healthiness of an individual? Conduct a hypothesis test to answer this question using 0.05 level of significance. Write down all necessary steps of the test such as hypotheses, test statistic, P-value, decision, and the conclusion.

<table>
<thead>
<tr>
<th>Education</th>
<th>Health</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Excellent</td>
</tr>
<tr>
<td>Less than High School</td>
<td>72</td>
</tr>
<tr>
<td>High School</td>
<td>465</td>
</tr>
<tr>
<td>Junior College</td>
<td>80</td>
</tr>
<tr>
<td>Bachelor</td>
<td>229</td>
</tr>
<tr>
<td>Graduate</td>
<td>130</td>
</tr>
</tbody>
</table>
2012 ACTM Regional Statistics Exam Key

1. d.  6. c.  11. b.  16. b.  21. d.
2. c.  7. d.  12. d.  17. c.  22. b.
3. d.  8. b.  13. c.  18. b.  23. c.
5. b.  10. b.  15. d.  20. b.  25. a.

Tie Breaker 1.  

a. \( r = .945 \)

b. \( f(x) = 1.044x - 5.989 \)

c. \( f(90) = 88 \)

d. 130.143

Tie Breaker 2.  

a. 896

b. 1000

c. 910

Tie Breaker 3.

H(0) = Health is not dependent on education.

H(a) = Health is dependent on education.

\( \alpha = .05 \)

p value = 6.39 E-54

p value < .05; reject H(0)

There is sufficient evidence to support that one's healthiness depends on one's education.