In each of the following choose the best answer and place the corresponding letter on the Scantron Sheet. If you erase on the answer sheet, be sure to erase completely. Answer questions 1 – 25 before you attempt the tie-breaker questions. The tie-breaker questions will be used to break ties for 1\textsuperscript{st}, 2\textsuperscript{nd}, and/or 3\textsuperscript{rd} place. The figures are not necessarily drawn to scale. On some of the questions if a figure is not drawn the student should draw a figure to assist in answering the question.

1. Which of the following statements is not always true about an exterior angle of a triangle?
   A. The exterior angle and the adjacent interior angle are supplementary.
   B. The measure of the exterior angle is equal to the sum of the measures of the two non-adjacent interior angles.
   C. The measure of the exterior angle is greater than the measure of the adjacent interior angle.
   D. The measure of the exterior angle is greater than the measure of either of the non-adjacent interior angles.
   E. None of these

2. The point of intersection of the medians of a triangle is called the
   A. Incenter
   B. Circumcenter
   C. Orthocenter
   D. Centroid
   E. None of these

3. A central angle in a unit circle has a measure of 54°. The related arc has a measure of
   A. \( \frac{3\pi}{10} \)
   B. \( \frac{3\pi}{5} \)
   C. \( \frac{3\pi}{20} \)
   D. \( \frac{\pi}{10} \)
   E. None of these

4. If each diagonal of a quadrilateral ABCD divides it into congruent triangles, then which of the following could ABCD be?
   A. square
   B. rhombus
   C. parallelogram
   D. any of the above
   E. None of these

5. ABCD is a trapezoid. Which of the following is true?
   A. \( \angle BAD \) and \( \angle ABC \) are supplementary
   B. \( \angle BAD \) and \( \angle BCD \) are supplementary
   C. \( \angle BAD \) and \( \angle ABC \) are complementary
   D. \( \angle BAD \cong \angle CDA \)
   E. None of these
6. The diagonals of a quadrilateral ABCD are perpendicular. Then ABCD must be a
   A. square   B. rhombus   C. kite   D. rectangle
   E. A, B or C

7. In the figure area(\(\Delta ABD\)) = area(\(\Delta ACD\)). Which of the following is true?
   A. \(\overline{AD}\) bisects \(\angle BAC\)    B. \(\overline{AD}\) is an altitude of \(\Delta ABC\)
   C. \(\overline{AD}\) is a median of \(\Delta ABC\)    D. Not enough information
   E. None of these

8. The altitude of an equilateral triangle is 10. The perimeter of the triangle, to the nearest hundredth, is
   A. Not enough information    B. 34.64    C. 34.62
   D. 51.96    E. None of these

9. \(\overline{AD}\) is an altitude of \(\Delta ABC\). What is the value of AD to the nearest hundredth?
   A. 5.74    B. 8.19    C. 7.00
   D. 7.34    E. None of these

10. The measure of an interior angle of a regular octagon is
    A. 135°    B. 144°    C. 45°    D. 36°
    E. None of these

11. In the figure \(\overline{AD}\) bisects \(\angle BAC\) and \(\overline{AE} \cong \overline{AB}\). Which of the following must be true?
    P: \(m\angle ABD = m\angle AED\)    Q: \(m\angle AED > m\angle ACD\)    R: \(m\angle ABD > m\angle ACD\)
    A. P    B. P, Q    C. P, R
    D. P, Q, R    E. None of these

12. ABCD is a square with \(\overline{AD} = 6\) and BEFG is a rectangle with \(\overline{BE} = 2\). If the square
    and the rectangle have the same area then \(\overline{AF} = \)
    A. \(6\sqrt{10}\)    B. \(2\sqrt{82}\)    C. \(2\sqrt{85}\)
    D. \(4\sqrt{10}\)    E. None of these
13. The line \( t \) intersects the lines \( l \) and \( m \) in points P and Q. A pair of alternate interior angles is

A. \( \angle CPQ, \angle AQP \)  
B. \( \angle BPD, \angle CPE \)  
C. \( \angle BPQ, \angle AQP \)  
D. \( \angle PBQ, \angle PBQ \)  
E. None of these

14. Parallel lines \( l \), \( m \) and \( n \) are intersected by transversals \( s \) and \( t \). If \( AB = 2x \), \( BC = 8 \), \( DE = 3x - 2 \) and \( DF = 3x + 4 \) then \( x = \)

A. \( -\frac{3}{4} \)  
B. \( \frac{3}{4} \)  
C. \( \frac{4}{3} \)  
D. \( -\frac{4}{3} \)  
E. None of these

15. The perimeter of the triangle at the right is

A. \( 20 + 5\sqrt{3} + 5\sqrt{2} \)  
B. \( 15 + 5\sqrt{3} + 5\sqrt{6} \)  
C. \( 20 + 10\sqrt{5} \)  
D. 45  
E. None of these

16. In the figure \( \angle ADE \cong \angle ABC \), \( AD = 2 \) and \( BD = 3 \). If \( AC = 8 \), then \( EC = \)

A. 3.8  
B. 4.2  
C. 3.2  
D. 4.8  
E. None of these

17. In the figure \( DE \parallel BC \). If \( AD = 6 \), \( BD = 2 \) and \( \text{Area}(\triangle ABC) = 32 \) then \( \text{Area}(\triangle ADE) = \)

A. 18  
B. 24  
C. 16  
D. 20  
E. None of these

18. In the right triangle \( \triangle ABC \), \( DE \perp AC \). If \( AE = 5 \), \( AD = 4 \) and \( AB = 20 \), then \( CE = \)

A. 12  
B. \( 3\sqrt{17} \)  
C. \( 3\sqrt{15} \)  
D. Not enough information  
E. None of these
19. The diagonal of a rectangle is 10 feet. If the length is 3 feet longer than the width, what quadratic equation would be solved to determine the width of the rectangle?
   A. \(2w^2 + 6w - 91 = 0\)  
   B. \(2w^2 + 6w - 109 = 0\)  
   C. \(2w^2 + 9w - 91 = 0\)  
   D. \(2w^2 + 9w - 109 = 0\)  
   E. None of these

20. Jack views a radio tower that is 80 feet tall. If his line of sight makes an angle of \(30^\circ\) with the ground, to nearest foot, how far is he standing from the base of the tower? [Assume his eyes are 5 feet above the ground.]
   A. 150  
   B. 139  
   C. 92  
   D. 130  
   E. None of these

Questions 21 and 22 use the following information.
Let \(A = (-1, 5)\), \(B = (7, -1)\) be points in the coordinate plane.

21. The midpoint of \(AB\) is
   A. (2,3)  
   B. (3,3)  
   C. (3,2)  
   D. (2,4)  
   E. None of these

22. The equation of the circle that has \(AB\) for a diameter is
   A. \((x - 2)^2 + (y - 3)^2 = 100\)  
   B. \((x - 3)^2 + (y - 2)^2 = 25\)  
   C. \((x - 2)^2 + (y - 3)^2 = 25\)  
   D. \((x - 3)^2 + (y - 2)^2 = 100\)  
   E. None of these

23. \(AB\) and \(CD\) are chords in the circle intersecting at \(E\). Which of the following is true?
   A. \(AB\) bisects \(CD\)  
   B. \((AE)(BE) = (CE)(DE)\)  
   C. \(AC \cong BD\)  
   D. \(\angle ABC \cong \angle BCD\)  
   E. None of these

24. \(\triangle ABC\), \(\triangle DEF\) and \(\triangle GEC\) are equilateral triangles with sides 12, 6, and 4, respectively. The area of the shaded portion of the figure is
   A. 37  
   B. 40  
   C. 37\(\sqrt{3}\)  
   D. 41\(\sqrt{3}\)  
   E. None of these

25. The diagonals of a kite are 10 cm and 24 cm. The shorter diagonal divides the longer diagonal into a 3 to 5 ratio. What is the perimeter of the kite, to the nearest hundredth of a centimeter?
   A. 52.21  
   B. 52.98  
   C. 53.00  
   D. 51.50  
   E. None of these
Tie-Breaker Questions

Name__________________________________ School____________________________

Please Print Please Print

Please answer the 25 questions on the exam before attempting the tie-breaker questions. They will be used, in the order given, to resolve ties for 1st, 2nd, and/or 3rd. Be sure to give clear reasons for any of your conclusions.

1. ABCD is a square with a side of length a. If $AB$ is extended to F so that BF = AB, determine CE in terms of a.

2. The perimeter of the smaller square is 8 cm and the area of the larger square is 25 square centimeters. Determine lengths of $AD$ and $DE$.

3. A roll of plastic is 4 yards long. If the diameter of the roll is 6 inches and the thickness of the plastic is 0.02 inches, what is the volume of plastic, in cubic feet, in a sheet of plastic created when two revolutions of the roll is removed from the roll? [Find your answer correct to 5 decimal places.]
1. ABCD is a square with a side of length a. If $\overline{AB}$ is extended to F so that $BF = AB$, determine $CE$ in terms of $a$.

$\triangle DGC \sim \triangle FGB$ and $\triangle GEC \sim \triangle DEA$. ∴ $GC = \frac{a}{3}$.

Let $CE = x$, then $AE = a\sqrt{2} - x$.

$$\frac{x}{a/2} = \frac{a\sqrt{2} - x}{a}.$$ So $2x = a\sqrt{2} - x$.

$$\therefore 3x = a\sqrt{2}.$$ So $CE = \frac{a\sqrt{2}}{3}$.

2. The perimeter of the smaller square is 8 cm and the area of the larger square is 25 square centimeters. Determine lengths of $\overline{AD}$ and $\overline{DE}$.

$AC = 2$ and $BE = 5$. $\triangle ACD \sim \triangle BED$. Let $DE = x$. Then $CD = 3 - x$.

$$\frac{CD}{AC} = \frac{DE}{BE}.$$ Therefore, $\frac{x}{5} = \frac{3-x}{2}$.

Thus, $2x = 15 - 5x$ and $x = \frac{15}{7}$. So $CD = \frac{6}{7}$.

$$\therefore AD = \sqrt{4 + \frac{36}{49}} = \frac{\sqrt{232}}{7}.$$ So $AD = \frac{2\sqrt{58}}{7}$.

3. A roll of plastic is 4 yards long. If the diameter of the roll is 6 inches and the thickness of the plastic is 0.02 inches, what is the volume of plastic, in cubic feet, in a sheet of plastic created when two revolutions of the roll is removed from the roll? [Find your answer correct to 5 decimal places.]

$$V = 144\pi(.02)(6 + 5.96)/12^3$$

$$V \approx .06262 \text{ cubic feet}$$