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Abstract Wireless Sensor Networks (WSNs) are autonomous wireless systems consists of a variety of collaborative sensor nodes forming a self-configuring network with or without any pre-defined infrastructure. The common challenges of a WSN are network connectivity, node mobility, energy consumption, data computation and aggregation at sensor nodes. In this paper we focus on intermittency in network connectivity due to mobility of sensor nodes. We propose a new mathematical model to capture a given entire WSN as is with intermittency introduced between the communication links due to mobility. The model involves open $GI/G/1/N$ queuing networks whereby intermittency durations in communication links are captured in terms of mobility models. The analytical formulas for the performance measures such as average end-to-end delay, packet loss probability, throughput, and average number of hops are derived using the queuing network analyzer and expansion method for models with infinite- and finite-buffer nodes, respectively. For models with 2-state intermittency, we analyze the performance measures by classifying these models into three types: namely, model with intermittent reception, model with intermittent transmission and/or reception, and model with intermittent transmission. We extend the analysis to multi-state intermittency models. We demonstrate the gained insight of WSNs through extensive numerical results.

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1 Introduction

Wireless Sensor Networks (WSNs) are autonomous wireless systems consists of a variety of collaborative sensor nodes forming a self-configuring network with or without any pre-defined infrastructure [1]. None, some or all of sensor nodes of a WSN can be mobile in nature depending on the requirement of WSN. WSNs are special type of Mobile Ad Hoc Networks [2]. Since WSNs are decentralized ad hoc networks; they confer advantageous such as ready deployability.

For the past decade WSNs have been the topic of great interest in both academia and industry for their potential applications across many fields [1]. Some of these applications include ecological habitat monitoring [3], military surveillance and target targeting [4], intrusion detection systems [5], structural and seismic monitoring [6], and in industrial and commercial networked sensing to replace wired sensor networks to wireless ones ([7], [8]), component failure for aircraft [9], sensor-based information home appliances [10], container tracking [11], environmental monitoring [12], health care monitoring systems [13], and disaster management systems [14].

The common challenges of a WSN are network connectivity, node mobility, energy consumption, data computation and aggregation at sensor nodes. Researchers have developed algorithms and protocols to address these challenges [15]. Intermittency in network connectivity could be due to mobility of nodes, environment where the nodes are deployed, and/or due to low battery power of sensor nodes. In this paper, we focus on performance analysis of WSNs such as end-to-end delay and packet loss probability due to intermittency in the network connectivity. In particular, we analyze the intermittency due to mobility of sensor nodes by making use some of the mobility models available in WSNs' literature.

Basagni et al. [16] have motivated and illustrated the use of mobility in WSNs. They compared different approaches such as single-hop routing, multi-hop routing with and without controlled sink mobility, and reviewed pros and cons of using mobility in WSNs. Camp et al. [17] surveyed a variety of mobility models which can be used to simulate WSNs. In [18], [19], [20], the authors have carried out connectivity analysis of mobile wireless ad hoc networks. None of these works have focused on a model to capture a given WSN as a whole and study its performance measures.

In this paper we propose a new model to capture a given entire WSN as is with intermittency introduced between the communication links due to mobility. The model facilitates the use of intermittency distributions of different mobility models ([17], [21]). The model is different from the one proposed by Bisnik and Abouzeid [22] whereby the routing of packets are governed by functions of communication area of nodes and hence not amenable to applying intermittency distributions of well-known mobility models.

Our model is derived from open queuing networks - a special field of stochastic processes [23]. In this model, each sensor node of the WSN is viewed as a queuing node, the wireless communication link between sensor nodes are viewed as intermittent links between queuing nodes. The derivation of the model involves the modification of existing open queuing networks by introducing intermittency among the links in order to capture the intermittency in network connectivity in WSN due to mobile nodes. We present different variations of our model to capture different intermittency scenarios and nodes' buffer sizes of WSNs. We provide a comprehensive set of parameters to analyze WSNs with sparse mobile nodes, medium mobile nodes and high mobile nodes. To the best of our knowledge, there is no model in the literature devoted to the usage of mobility models to analytically study performance of mobile WSNs.

The paper is structured as follows: In section 2, we describe the generic model and different variations of the generic model based on the nature of intermittency. We derive analytical formulas for performance measures of 2-state and multi-state intermittent models in section 3. In section 4, we provide numerical results for the performance measures of different networks. We conclude the paper in section 5 with some remarks and possible extension of the proposed model as future work.

2 Model Description

2.1 Model

Consider a two-dimensional WSN with mobile nodes. A typical sensor node consists of a sensor, microcontroller, battery, and antenna with fixed transmitting and receiving range (radio range). A sensor node could generate its own packets (sensed data) to be sent to a sink (destination) or it could forward packets from other nodes to be sent to a sink. If a sink is not in the radio range of the node, the node sends the packets to intermediate nodes which in turn forward the packets to the destined sink. When the battery level of a node goes down, all its activities start going down including its radio range (weak signal strength). The signal strength between two nodes can also go down when the two nodes move away from each other. Also due to mobility of nodes, it may so happen that a node would forward its own packets if the mobility is too high. This is due to the routing delay which occurs at each sensor node.

We consider a model which will have a buffer (finite or infinite) to collect arriving packets and a processing unit to process packets one or many at a time (Figure 1). If the processing unit is busy in processing a packet, an arriving packet joins the buffer, or else goes straight to the processing area. We refer such a model as a queuing node. We model each sensor node as a queuing node wherein packets are customers, packets to be forwarded are arrival of customers, the packet processing place is the server and the routing delay is the service time.

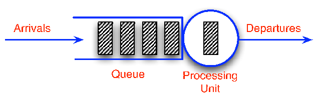
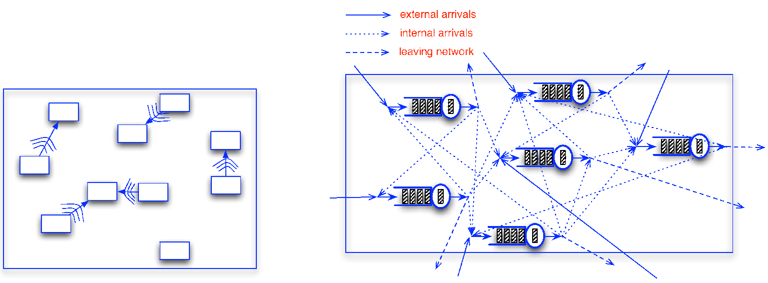


Fig. 1 Queuing model to model a sensor node

We now consider a collection of queuing nodes which have random connections among them according to an intermittent duration probability law (see Figure 2.1). We refer this collection as a queuing network whereby at a queuing node, customers can arrive from outside the network (external arrivals) and from other queuing nodes (internal arrivals) according to a routing probability distribution law. Customers who finish service in a node may leave the network according to the routing probability distribution law (departing the network). That is, a customer who finishes service in a queuing node can go to any of the nodes (provided the random connection between the leaving node and the next node is available) or leave the network. We refer such a model as an intermittent open queuing network. It is open because external arrivals are allowed in the network.

We model a WSN as a queuing network wherein each queuing node is a sensor node, external arrivals at a queuing node are the sensed data (packets) of a sensor node, internal arrival at a queuing node are the packets to be forwarded by a sensor node, customers departing a queuing node are the packets which reached their destination sensor node - sink node, buffer of a queuing node is the buffer of a sensor node, the random connections among the queuing nodes are the intermittent wireless links among the sensor nodes. The intermittent duration probability law which governs the connections among queuing nodes is mapped to a mobility model of sensor nodes which triggers intermittency in the wireless network connectivity among sensor nodes. The routing distribution law of the open queuing network is mapped to the routing algorithm (protocol) of the WSN (see Figure 2).



(a) WSN (b) Open queuing network to model a WSN

Fig. 2 WSN and Open queuing network

We note that in the above mapping between an open queuing network and a WSN, we do not consider details like nature of microcontroller and antenna, battery life, collision of packets, and retransmission of packets. Our focus is mainly on the intermittency in network communication among sensor nodes due to mobility. We assume the rest are assured to function well.

Having explained how a WSN can be viewed as an intermittent open queuing network, from now on we focus on our model derived from open queuing networks. We characterize our model in the next section.

2.2 Model Characterization

We use the following notations to characterize and analyze WSN models for the rest of the paper.

- M - Number of nodes
- N - Buffer size (finite or infinite)
- $\frac{1}{\mu_i}$ - Average processing time of a packet at node i
- $\frac{1}{\lambda_i}$ - Average time between two consecutive arrival (inter-arrival time) of packets which are to be forwarded and generated by node i
- $\frac{1}{\lambda_{0i}}$ - Average inter-generation time of packets sensed by node i
- $\frac{1}{\lambda_{i0}}$ - Average inter-sinking time of packets at node i
- $1/\lambda_{ij}$ - Average inter-arrival time of packets which are routed from node i to node j
- c_{0i}^2 - Squared coefficient of variation (SCV) of generation time of sensed packets generated at node i
- c_{ai}^2 - SCV of inter-arrival times of routed packets from other nodes to node i generation time of sensed packets generated at node i
- c_{ij}^2 - SCV of inter-arrival times of routed packets from node i to node j
- c_{si}^2 - SCV of processing time of packets at node i
- r_{ij} - Routing probability that a packet from node i is routed to node j
- $\mathcal{S} = \{0, 1, 2, \ldots, K\}$ - Set of possible states of intermittency among nodes
- t_{km} - Transition probability that an intermittency state is changing from k to m $(t, k \in \mathcal{S})$
- p_k - Probability of successful transmission of packets while the intermittency state is $k \in \mathcal{S}$
- s_k - Amount of time that the network connectivity between two nodes stays in state $k \in \mathcal{S}$
- ρ_i - Traffic load of sensor node i
- $\mathcal{T}$ - Throughput of the network
- $\mathcal{G}$ - Average generation rate of packets in the network
- W_{sj} - Average delay of a packet at node i
- W_{qj} - Average delay of a packet at the buffer of node i
- W_s - Average end-to-end delay a packet (from time it is created until it reaches the sink)
- $p_N^{(i)}$ - Probability that a routed packet to node i is lost due to full buffer of node i
- P_L - Probability that a packet is lost due to intermittency in the communication and/or full buffer of the node where the packet is routed.

We consider a WSN model *without intermittency in connectivity* and with M nodes whereby server in each node serves one packet at a time and packets join the node on the FIFO basis. A non-intermittent network is the one where all the nodes can communicate to other nodes without any communication failure due to mobility. For this non-intermittent network the following performance measures are available in the literature [23].

$$\lambda_j = \lambda_{0j} + \sum_{i=1}^{M} r_{ij}\lambda_i(1 - p_N^{(i)}). \tag{1}$$

$$\lambda_{j0} = \left(1 - \sum_{i=1}^{M} r_{ji}\right)\lambda_j. \tag{2}$$

$$\rho_i = \frac{\lambda_i}{\mu_i}. \tag{3}$$

$$\lambda_{ij} = r_{ij}\lambda_i. \tag{4}$$

$$\mathcal{T} = \sum_{j=1}^{M} \lambda_{j0}. \tag{5}$$

$$P_L = \frac{\mathcal{G} - \mathcal{T}}{\mathcal{G}}, \text{ with } \mathcal{G} = \sum_{j=1}^{M} \lambda_{0j}. \tag{6}$$

$$W_s = \frac{1}{\mathcal{T}} \sum_{j=1}^{M} \lambda_j W_{sj}, \text{ with } W_{sj} = W_{qj} + \frac{1}{\mu_j}. \tag{7}$$

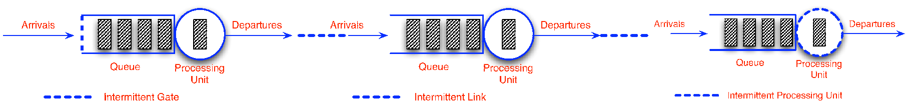
If N is infinite, then $p_N^{(i)} = 0$.

When the generation of sensed data follow Poisson process and if the process times at the nodes follow exponential distribution, then the above performance measures can be obtained analytically in closed form [24]. For this non-intermittent network, when the generation of sensed data and/or their processing times are non-Poisson and non-exponential, approximate methods known as queuing network analyzer (QNA) [25] and expansion method [26] have been developed, respectively, for non-intermittent networks with infinite buffer nodes and finite buffer nodes.

An open queuing network without intermittency and with restricted packet generation process and service distribution is not a practical model to model a WSN. In the following sections we modify this model by introducing intermittency in network connectivity and study its performance for different packet generation processes and processing times distributions. We suitably modify the QNA and expansion method for the model with intermittent communications. We provide these two methods precisely to our requirements in appendix A for the sake of completeness.

We propose three variations in our model to capture the following three scenarios:

1. **Intermittent reception:** Consider a WSN with mobile nodes that have the ability to stop listening for a random amount of time but continue to generate its own data (sensing the environment) and transmit them to reachable nodes. Such capabilities help the WSN to save energy by making nodes to behave selfish for a while, but the data, that are being transmitted by other nodes to the nodes that stopped receiving data, are lost.



(a) With intermittent reception (b) With intermittent transmission and/or reception (c) With intermittent transmission

Fig. 3 Sensor node models with three intermittent scenarios

2. **Intermittent transmission and/or reception:** Consider a WSN with mobile nodes in which (some or most) sensor nodes can be out of radio range of other nodes due to their (less or more) mobility. Whenever nodes go out of communication range, the data which are being transmitted among them are lost. In this scenario, it may so happen that a node, say n, may not be able to receive any data from a node, say m, but n can transmit packets to m as n may have more transmission power than m.
3. **Intermittent transmission** As opposed to scenario 1, consider a WSN with mobile nodes that have the ability to stop processing data whenever they go out of transmission range from other nodes but continue to receive data from reachable nodes. In this scenario, there is no loss of packets as the nodes stop transmitting packets whenever they go out of transmission range.

The corresponding sensor node models are shown in Figure 3. We consider these three variations for the reason that the performance measures of the corresponding models are amenable to analytical derivation.

2.3 Intermittent Model

We discuss now how the three types of intermittencies discussed in previous section can be modeled. Before discussing them we first look in to the ways in which intermittencies in network connectivity can happen. Consider a pair of mobile sensor nodes and assume that they are well within their communication radio range and hence transmission of data among these nodes can happen without any data loss. When the nodes start moving randomly, it is very well possible that the signal (communication) strength between them can keep varying (strong to weak and vice versa) and would also lose their connectivity completely sometimes. It is also possible when one of these nodes go down due to physical damage, sleep or low battery power, the signal strength can go from strong to none and vice versa when one of the sleeping nodes wakes up. The loss in data transmission among these nodes is directly proportional to the strength of signal between them. Also, when the nodes move fast, the signal strength goes from none to strong much quicker than the case where they move slowly.

We model the intermittency using a continuous time semi-Markov chain [24] whereby the chain could be in any one of the states $k \in \mathcal{S} = \{0, 1, 2, \ldots, K\}$ for random amount of time s_k (sojourn time) and changes to another state $m \in \mathcal{S}$ with transition probability t_{km}. The state diagram of this chain is shown in Figure 4.

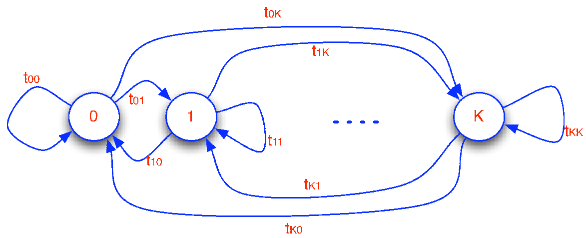


Fig. 4 State diagram of the model for intermittency

In order to capture the intermittencies in a mobile WSN as explained above, in our analysis we assume that

$$p_k = \frac{k}{K}, \quad k \in \mathcal{S}, \tag{8}$$
$$t_{km} = \frac{1}{K}, \quad \forall k, m \in \mathcal{S}, \tag{9}$$
$$s_k = s, \quad k \in \mathcal{S} \tag{10}$$

where p_k is the probability of successful transmission of packets while the intermittency state is k. That is, if the intermittent state of the link from node, say i, to node, say j, is 0, then all transmitted packets from node i and node j are dropped (lost) and hence $p_0 = 0/K = 0$.

These assumptions ensure that by changing the value of K, we can capture the intermittency in connectivity among mobile sensor nodes in terms of their speed – small values of K corresponds to faster movements of sensor nodes and large values of K correspond to slow movements of sensor nodes. This is because, when K is small, say, 1, then the intermittent states are 0 (off state - all data are dropped) and 1 (on state - all data are transmitted successfully). When the nodes move fast, the signal strength between any two nodes change from strong to none and vice versa. When K is a large value, say 10, then the intermittent states are $\{0, 1, \ldots, 10\}$ with corresponding probability of successful transmission of packets $\{0, 0.1, 0.2, \ldots, 1\}$. When two nodes move slowly, the signal strength between them vary gradually from strong to weak and vice versa and in this case, variations are being captured in discrete values instead of continuous values.

Since finding analytical formulas in closed form for the performance measures of the model with multi-state intermittent communication links is difficult in general, we derive analytical formulas for special cases of multi-state intermittent models. But for the case where $K = 1$ (on and off intermittent states), we derive analytical formulas in detail for all possible variation of parameters. We carry out a detailed analysis in Section 3 by suitably modifying both the QNA and expansion methods for models with infinite and finite buffer nodes, respectively.

3 Analysis

In this section, we first provide closed form analytical formulas, in terms of theorems, to compute the performance measures of WSN models with 2-state and intermittency and with infinite (section 3.1) and finite buffer (section 3.2) nodes. We then provide closed form analytical formulas for models with multi-state intermittency for a specific set of parameters.

In all these models we assume the following:

$$\begin{aligned}
K &= 1 \\
p_k &= k, \quad k = 0, 1, \\
t_{00} &= 0, t_{01} = 1, t_{10} = 1, \text{ and } t_{11} = 0 \\
s_0 &= \frac{1}{\alpha}, \text{ and } s_1 = \frac{1}{\beta}.
\end{aligned}$$

3.1 WSN model with infinite buffer nodes

3.1.1 Model with intermittent reception

In this model, each node's reception goes on and off with average rates α and β and variances v_{on} and v_{off}. When the reception is on, the nodes receives data from other nodes, otherwise, the data are lost. Let λ'_j denote the average arrival rate of data at node j due to its intermittent reception. We note that if there is no intermittency in the reception, the average arrival of data at node j is given by λ_j (as in (1) with $p_j^{(N)} = 0$). Let c'^2_{aj} be the SCV of the intermittent arrival process of data at node j.

Theorem 1 *For this model*

$$\lambda'_j = p_{on}\lambda_j \tag{11}$$

$$c'^2_{aj} = c^2_{aj} + k\lambda_j, \tag{12}$$

where p_{on} is the probability that node j is receiving packets from other nodes and is given by

$$p_{on} = \frac{\beta}{\alpha+\beta}, \tag{13}$$

$$k = \frac{\alpha(v_{on}\alpha^2 + v_{off}\beta^2)}{(\alpha+\beta)^2}, \tag{14}$$

and c^2_{aj} is given in the QNA algorithm (Appendix A.1).

Proof Due to the presence of the on-off reception at node j, we can say that the arrival process, say X, at node j switches between two general renewal processes X_1 and X_2 with rates λ_j and 0 according to a general renewal switching (on-off) periods Y_1 and Y_2 with average switching rates α and β, and variances v_{on} and v_{off}, respectively. Clearly the average arrival rate of packets at the node is given by

$$\lambda'_j = \frac{1/\alpha}{1/\alpha + 1/\beta}\lambda_j = \frac{\beta}{\alpha+\beta}\lambda_j. \tag{15}$$

The SCV of X is given by (due to [32]):

$$c_X^2 = c_{X_1}^2 + \lambda_j \frac{\alpha(v_{off}\alpha^2 + v_{on}\beta^2)}{(\alpha+\beta)^2}. \tag{16}$$

Replacing c_{X_1} by c'^2_{aj}, and c_X by c^2_{aj} in (16) proves the theorem. □

By replacing λ_j by λ'_j and c^2_{aj} by c'^2_{aj} in the QNA algorithm in Appendix A.1 , the performance measures of this intermittent model can be computed.

3.1.2 Model with intermittent transmission and/or reception

In this model, each node's transmission and/or reception go on and off with average rates α and β and variances v_{on} and v_{off}. When a node, say j is in the receiving distance of a node, say i, we say that connectivity link is from i to j is on so that i can route packets to j with routing probability r_{ij}. Otherwise, we say that the link from i to j is off. When the link is off, all the routed packets are lost.

Let λ'_j denote the average arrival rate of data at node j due to its intermittent link. We note that if the links are non-intermittent, the average arrival of data at node j is given by λ_j (as in (1) with $p_j^{(N)} = 1$). Let c'^2_{aj} be the SCV of the intermittent arrival process of data at node j.

Theorem 2 *For this model*

$$\lambda_j = \lambda_{0j} + p_{on}\sum_{i=1}^{M}\lambda_{ij}, \tag{17}$$

$$c'^2_{aj} = 1 - w_j(1 - \sum_{i=1}^{M} p_{ij}c^2_{ij} - k\sum_{i=1}^{M} p_{ij}\lambda_{ij}), \tag{18}$$

where p_{on} is the probability that the link between any two nodes is on and is given by (13), k is given by (14), and c^2_{ij} is given in the QNA algorithm (Appendix A.1).

Proof Due to the presence of on-off links between nodes i and j, the average arrival rate λ'_{ij} between these nodes is given by

$$\lambda'_{ij} = p_{on}\lambda_{ij}. \tag{19}$$

The total average arrival rate at node j is then given by $\lambda_j = \lambda_{0j} + \sum_{i=1}^{M}\lambda'_{ij}$. Substituting (19) in this equation, we get (17).

The SCV c_{ij}^2 of the traffic flow between node i and node j is obtained by replacing c_X^2 by $c_{ij}'^2$, $c_{X_1}^2$ by c_{ij}^2, and λ_j by λ_{ij} in (16). That is,

$$c_{ij}'^2 = c_{ij}^2 + k\lambda_{ij}, \tag{20}$$

where k is given by (14). Replacing c_{ij}^2 in the QNA algorithm by $c_{ij}'^2$ from (20), yields (18). $\square$

By replacing λ_j by λ_j' and c_{aj}^2 by $c_{aj}'^2$ in the QNA algorithm given in Appendix A.1, the performance measures of this intermittent model can be computed.

3.1.3 Model with intermittent transmission

In this model, the processing unit in each node goes on and off with average rates α and β, respectively, according to a general renewal processes. Independent of the intermittent state of the processing unit, the node continues to receive data from other reachable nodes. We recall that in this model there is no loss of packets as the nodes stop transmitting packets whenever they go out of transmission range.

Let μ_j' denote the average processing rate of data at node j due to its intermittency. We note that if the links are non-intermittent, the average processing rate of data at node j is given by μ_j. Let $c_{sj}'^2$ be the SCV of the intermittent processing time of data at node j.

Theorem 3 *For this model*

$$\mu_j' = p_{on}\mu_j, \tag{21}$$
$$c_{sj}'^2 = c_{sj}^2 + k\mu_j, \tag{22}$$

where p_{on} is the probability the processing unit at node j goes on and is given by (13), and

$$k = \alpha p_{on}^2 \frac{(1 + c_{off}^2)}{\beta^2}$$

with c_{off}^2 being the SCV of the processing unit off distribution.

Proof Let S, Y_1, and Y_2 denote the processing time, on time, and off time distributions of the processing unit at node j, respectively. Let S' denote the effective processing time distribution of the processing unit due to its intermittency. Then the average of S' is obtained by conditioning on S as follows:

$$E[S'] = \int_{t=0}^{\infty} E\left[S' \mid t \le S \le t + dt, N(S) = n\right] \times Pr\{N(S) = n\} f_S(t)\mathrm{d}t,$$

where $N(S)$ denotes the number of times the processing unit was down (renewals) during $[0, S]$. Then

$$\begin{aligned} E[S'] &= \int_{t=0}^{\infty} \sum_{n=0}^{\infty} E\left[t + \sum_{i=1}^{n} Y_2\right] Pr\{N(t)=n\} f_S(t)\mathrm{d}t \\ &= \int_{t=0}^{\infty} \sum_{n=0}^{\infty} [t + nE[Y_2]]\, Pr\{N(t)=n\} f_S(t)\mathrm{d}t \end{aligned}$$

Since $N(\cdot)$ is a renewal process with renewal rate α,

$$E[N(t)] = \sum_{n=0}^{\infty} nPr\{N(t)=n\} = \alpha t.$$

Using this and simplifying further, the expression for $E[S']$ simplifies to

$$E[S'] = E[S](1 + \alpha E[Y_2]). \tag{23}$$

Using similar arguments, the second moment of S' can be derived as

$$E[S'^2] = E[S^2](1 + \alpha E[Y_2])^2 + E[S]\eta E[Y_2^2]. \tag{24}$$

Let $E[S'] = 1/\mu'_j$ and since $E[S] = 1/\mu_j$, and $E[Y_2] = 1/\beta$, (23) simplifies to

$$\mu'_j = \frac{\beta}{\alpha+\beta}\mu_j = p_{on}\mu_j.$$

Using (23) and (24), the SCV of S' can be found as follows:

$$\begin{aligned} c_s'^2 = \frac{E[S'^2]}{E[S']^2} &= c_s^2 + \frac{\eta(1 + c_{off}^2)}{E[S]E[Y_2]^2(1 + \eta E[Y_2])^2} \\ &= c_{sj}^2 + \alpha p_{on}^2 \frac{(1 + c_{off}^2)}{\beta^2}, \end{aligned}$$

where $c_s'^2$, c_s^2 and c_{off}^2 are the SCVs of the effective service time, processing time and off time distributions, respectively. □

By replacing μ_j by μ'_j and c_{sj}^2 by $c_{sj}'^2$ in the QNA algorithm given in Appendix A.1, the performance measures of this intermittent model can be computed.

3.2 WSN model with finite buffer nodes

In section 3.1 we derived performance measures of WSN models with infinite buffer nodes analytically. However, almost all real world systems have limitations on the buffer size of sensor nodes. Having finite buffer nodes is therefore a realistic need for WSNs. Many WSNs do not have store and forward mechanism simply because of frequent changes in network topology due to mobility of nodes. Hence if a received packet is not destined to a node, it forwards the packet to neighboring nodes as soon as possible. In this section, we analyze the corresponding finite buffer cases of the models discussed in section 3.1.

In finite buffer models, an artificial node is added to each sensor node model. Whenever the node's buffer is full, the arriving packet at a sensor node is routed to the associated artificial node. We note that the algorithm for the expansion method (see Appendix A.2) is derived on the assumption that after incurring a random/deterministic delay, the packet in the artificial node attempts to rejoin the buffer and if the buffer is full, it is routed back to the artificial node. This process continues for the packets in the artificial node until the artificial node is empty. We modify this assumption so that the packets that join artificial node do not try to join the node's buffer and hence they are considered to be lost due to buffer overflow.

3.2.1 Model with intermittent reception

This model is the same as the one analyzed in section 3.1.1 except that each node has a finite buffer of size N. Let λ'_j denote the average arrival rate of data at node j due to its intermittent reception. Out of these successfully arrived packets, some of the packets are lost due to buffer overflow at node j. Hence let λ''_j be the effective arrival rate of data which successfully joined the buffer of node j. Let c''^2_{aj} be the SCV of the arrival process of packets who joined the buffer of node j.

Theorem 4 *For this model*

$$\lambda''_j = (1 - p_N^{(j)})\lambda'_j \tag{25}$$

$$\textit{and } c''^2_{aj} = y_j[(1 - p_N^{(j)})c'^2_{aj} + p_N^{(j)}] + 1 - y_j, \tag{26}$$

where λ'_j and c'^2_{aj} are given by Theorem 1, and $p_N^{(j)}$ and y_j are given in the expansion method (Appendix A.2).

Proof Let λ_{gj} and λ_{hj} be the average arrival rates of packets at node j from other nodes and from the artificial node h, respectively. Let the corresponding SCVs of these two arrivals be c^2_{gj} and c^2_{hj}. Clearly,

$$\lambda_{gj} = (1 - p_N^{(j)})\lambda'_j, \text{ and } c^2_{gj} = (1 - p_N^{(j)})c'^2_{aj} + p_N^{(j)}.$$

Since $\lambda_{hj} = 0$ we have

$$\lambda''_j = \lambda_{hj} + \lambda_{gj} = (1 - p_N^{(j)})\lambda'_j$$

Since we superpose the arrivals from other nodes and the artificial node of node j, we get [29, (1.2)]:

$$\begin{aligned} c''^2_{aj} &= y_j\left[\left\{\frac{\lambda_{hj}}{\lambda_{hj}+\lambda_{gj}}\right\}c^2_{hj} + \left\{\frac{\lambda_{gj}}{\lambda_{hj}+\lambda_{gj}}\right\}c^2_{gj}\right] + 1 - y_j \\ &= y_j[0 + c^2_{gj}] + 1 - y_j \\ &= y_j\left[(1 - p_N^{(j)})c'^2_{aj} + p_N^{(j)}\right] + 1 - y_j. \end{aligned}$$

□

By replacing λ_j by λ''_j and c^2_{aj} by c''^2_{aj} in the expansion method algorithm given in Appendix A.2, the performance measures of this intermittent model can be computed.

3.2.2 Model with intermittent transmission and/or reception

This model is the same as the one analyzed in section 3.1.2 except that each node has a finite buffer of size N. Let λ'_j denote the average arrival rate of data at node j due to its intermittent transmission and/or reception. Out of these successfully arrived packets, some of the packets are lost due to buffer overflow at node j. Hence let λ''_j be the effective arrival rate of data which successfully joined the buffer of node j. Let c''^2_{aj} be the SCV of the arrival process of packets who joined the buffer of node j.

Theorem 5 *For this model*

$$\lambda''_j = (1 - p_N^{(j)})(\lambda_{0j} + \Sigma_{i=1}^M \lambda'_{ij}) \tag{27}$$

$$\begin{aligned} c''^2_{aj} &= y''_j(1 - p_N^{(j)}) \left[y'_j \sum_{i=1}^{M} \left\{ \frac{\lambda'_{ij}}{\sum_{k=1}^M \lambda'_{kj}} \right\} c'^2_{ij} + 1 - y'_j \right] \\ &\quad + y''_j p_N^{(j)} + 1 - y''_j, \end{aligned} \tag{28}$$

where

$$\begin{aligned} \lambda'_{ij} &= p_{on}\lambda_{ij} \\ c'^2_{ij} &= c^2_{ij} + k\lambda_{ij}. \end{aligned}$$

Here p_{on} and k are given in Theorem 1, y'_j and y''_j are computed using the formula given in the expansion method algorithm (Appendix A.2) using λ'_j and λ''_j, respectively, in place of λ_j.

Proof (Proof of Theorem 5) Due to the presence of intermittent links between the nodes, we use the results of Theorem 2 to get λ'_{ij} and c'^2_{ij}. Since we superpose arrivals from other sensor nodes to node j, we get the following equations due to [29, (1.1) & (1.2)]:

$$\begin{aligned} \lambda'_j &= \lambda_{0j} + \sigma_{i=1}^M \lambda'_{ij} \\ \text{and } c'^2_{aj} &= y'_j \sigma_{i=1}^M \Big\{ \lambda'_{ij} / \sum_{k=1}^M \lambda'_{kj} \Big\} c'^2_{ij} + 1 - y'_j. \end{aligned}$$

Due to finite buffer, we split the arrivals, whenever the buffer is full, to the artificial node and the remaining to the buffer of node j. Let λ_{gj} and λ_{hj} be the arrival rates of packets at node j from other nodes and from the artificial node h, respectively. Let the corresponding SCVs of these two arrivals be c^2_{gj} and c^2_{hj}. Clearly,

$$\lambda_{gj} = (1 - p_N^{(j)})\lambda'_j = (1 - p_N^{(j)})(\lambda_{0j} + \sum_{i=1}^M \lambda'_{ij})$$

and

$$c_{gj}^2 = (1 - p_N^{(j)})c_{aj}^{\prime 2} + p_N^{(j)}$$
$$= (1 - p_N^{(j)})\Big[y_j' \sum_{i=1}^{M} \Big\{\lambda_{ij}' / \sum_{k=1}^{M} \lambda_{kj}'\Big\} c_{ij}^{\prime 2} + 1 - y_j'\Big] + p_N^{(j)}$$

Since $\lambda_{hj} = 0$, we have

$$\lambda_j^{''} = \lambda_{hj} + \lambda_{gj} = (1 - p_N^{(j)})\lambda_j' = (\lambda_{0j} + \sum_{i=1}^{M} \lambda_{ij}').$$

Since we superpose the arrivals from other nodes and the artificial node of node j, we use [29, (1.2)] to get $c_{aj}^{\prime\prime 2}$ as follows:

$$c_{aj}^{''2} = y_j'' \Big[\Big\{\frac{\lambda_{hj}}{\lambda_{hj} + \lambda_{gj}}\Big\} c_{hj}^2 + \Big\{\frac{\lambda_{gj}}{\lambda_{hj} + \lambda_{gj}}\Big\} c_{gj}^2\Big] + 1 - y_j''$$
$$= y_j''[0 + c_{gj}^2] + 1 - y_j''.$$

On substituting for c_{gj}^2 in the last equation we get (28). □

By replacing λ_j by λ_j'' and c_{aj}^2 by $c_{aj}^{\prime\prime 2}$ in the expansion method algorithm given in Appendix A.2, the performance measures of this intermittent model can be computed.

3.2.3 Model with intermittent transmission

This model is the same as the one analyzed in section 3.1.3 except that each node has a finite buffer of size N. Since the finite buffer of the nodes does not influence the intermittency in functionality of the processing unit, the processing time remains unchanged as in infinite buffer case. Hence the results of Theorem 3 hold valid for this finite buffer case.

By replacing μ_j by μ_j' and c_{sj}^2 by $c_{sj}^{\prime 2}$ in the expansion method algorithm given in Appendix A.2, the performance measures of this intermittent model can be computed.

3.3 WSN model with multi-state intermittency

Note that in Theorems 1 - 5, we derived analytical formulas for λ_j and c_{aj}^2 for different models. In this section we provide analytical formulas for these two quantities for a WSN model with multi-state intermittency. Then these formulas can be substituted in these theorems to get the respective performance measures for models with different types of intermittencies.

Since it is impossible to derive analytical formulas for a general multi-state intermittent models, we derive for the following special (specific) intermittent model: We model the intermittency using a process with K states and in each state the process spends a random amount of time s_k (sojourn time) and changes to another state $m \in \mathcal{S}$ according to discrete probability mass values $t_0, t_1, \ldots, t_K$ such that

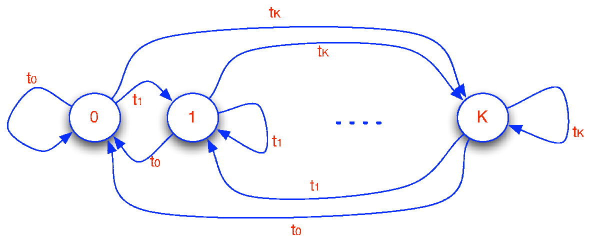


Fig. 5 State diagram of the model for intermittency with independent state transition function

$0 \leq t_k \leq 1$ and $\sum_{k=0}^{K} t_k = 1$ (see Figure 5). We note that this intermittent model is different from the one depicted in Figure 4 where the transition functions depend on the previous state. In the current model the intermittent state transitions take place independent of the past state.

Theorem 6 *For a WSN model with $K+1$-state (multi-) intermittent communication link governed by the mathematical model as shown in Figure 5, we have*

$$\lambda'_j = p_{succ}\lambda_j, \tag{29}$$

$$c'^2_{aj} = p_{succ}c^2_{aj}, \tag{30}$$

where

$$p_{succ} = \frac{\sum_{k=0}^{K} p_k s_k}{\sum_{k=0}^{K} s_k}. \tag{31}$$

Proof At node j, the average arrival rate of packets (both routed and generated) is given by λ_j if there is no intermittency in the communication channels. The communication channel is governed by a $K+1$-state intermittent model such that if the channel is state k, the probability of successful transmission of a packet is p_k and average amount of time the channel stays in this state is s_k. Hence clearly the average arrival rate of packets at node j, after some packets are lost due to channel conditions, is given by

$$\begin{aligned}\lambda'_j &= \frac{p_0 s_0 \lambda_0 + p_1 s_1 \lambda_1 + \cdots + p_K s_K \lambda_K}{s_0 + s_1 + \cdots + s_K} \\ &= \frac{p_0 s_0 + p_1 s_1 + \cdots + p_K s_K}{s_0 + s_1 + \cdots + s_K} \\ &= p_{succ}\lambda_j.\end{aligned}$$

Since the transition of state of the intermittent model happens independent of the previous state, following the analysis for 2-state model given in [32, (4)], we get the following expression for $c_{aj}'^2$:

$$\begin{aligned} c_{aj}'^2 &= \frac{p_0 \lambda_j c_{aj}^2 s_0}{(s_0 + s_1 + \cdots + s_K)\lambda_j} + \cdots \\ &\qquad + \frac{p_K \lambda_j c_{aj}^2 s_K}{(s_0 + s_1 + \cdots + s_K)\lambda_j} \\ &= \frac{\sum_{k=0}^K p_k s_k}{\sum_{k=0}^K s_k} c_{aj}^2 \\ &= p_{succ} c_{aj}^2. \end{aligned}$$

□

3.4 Average number of hops

In this section, we consider a WSN model with 2-state or multi-state intermittent communication channels and with equal routing probabilities, say $r_{ij} = r,\ \forall i, j = 1, 2, \ldots, M$. This assumption is valid for scenarios whereby all sensor nodes are in the communication ranges among themselves and hence any sensor node has a equal routing probability of routing the packets to any other node. For this model, we provide the average number of hops in the following theorem.

Theorem 7 *For the WSN model described above, the average number of hops H_A a packet goes through the WSN before it reaches the sink is given by*

$$H_A = \frac{1}{1 - Mr}, \tag{32}$$

where M is the number of sensor nodes.

Proof Let H denote the number of hops a packet, generated by a sensor nodes, goes through the WSN before it reaches the sink. Then

$$\begin{aligned} Pr\{H = h\} &= \left(\sum_{j=1}^M r_{ij} \right)^{h-1} \left(1 - \sum_{j=1}^M r_{ij} \right) \\ &= (Mr)^{h-1}(1 - Mr). \end{aligned}$$

Note that the above formula is valid only for the case where r_{ij} are same for all $i, j = 1, 2, \ldots, M$. The average number of hops H_A is then given by

$$H_A = \sum_{h=1}^{\infty} h Pr\{H = h\} = \frac{1}{1 - Mr}.$$

□

In the next section we provide numerical results for the performance measures of the intermittent network models with infinite and finite buffer nodes for different intermittent, arrival and processing time distributions.

4 Numerical Examples

In this section we produce some numerical results for the performance measures of the intermittent network models for different values of N, M, K, and different distributions for intermittent, arrival and processing times. Since infinite buffer models are special cases of the finite buffer model with multi-state intermittent links, we provide numerical results of this generic finite buffer model with multi-state intermittent links.

As our main focus of our paper is to analyze the performance measures such as packet loss probability and average end-to-end delay in the network due to intermittency in the communication channel, we vary the following four major parameters which play a crucial role in terms of network connectivity: (i) number of intermittent states K of a communication channel, (ii) average number of hops H_A, (iii) buffer size N, and (iv) data sample time of a node. We note that the density of a given WSN, for a fixed topology and radio range of all nodes, can be captured in terms of H_A: the smaller H_A values correspond to dense networks and its larger values correspond to sparse network. This is because when a network is dense, the packets would reach the sink in less number of hops whereas for a sparse network, the packets are routed through other nodes before they reach the sink. We consider three numerical examples to get insight of the performance measures of three WSNs for different values of these four parameters. In the last example we consider a large WSN with 50 nodes in order to show how stable our analytical results even for large networks.

In [21], the authors have developed a random walk based mobility model for a mobile ad hoc network and have derived the probability distributions of link availability between any two nodes to be Rayleigh distributions. Hence in all the three examples we use Rayleigh distributions with suitable parameter values to capture the intermittency in the communication channels.

Example 1 We consider a WSN model with 10 single-buffer nodes and multi-state communication links with Rayleigh distributed intermittent transmission and/or reception. We assume that at each node, packets are generated according a Poisson process with average generation rate of 5 packets per second and deterministic processing time of 0.04 seconds per packet. In Figures 6, 7, and 8, we plot the performance measures total packet loss probability, packet loss due to intermittency and finite buffer, and average end-to-end delay, respectively, for different values of links intermittent state K and average number of hops H_A a generated packet goes through before reaches the sink. It is clear from Figure 6, the number of state K of the intermittent state increases, the packet loss probability P_L decreases and reaches the minimum when $K = 10$. This is the gain we wanted to achieve by introducing multi-state link due to its practicality when compared to 2-state intermittency. As the average number of hops H_A increases, P_L increases due to intermittency in the link which is evident from Figure 7. When the number of hops increases, packets are being forwarded to intermediate nodes before they reach the sink. During this forwarding process, some packets are lost due to intermittent links and hence loss due to intermittency $P_{L_{il}}$ increases as H_A increases. Since the buffer size is 1, the loss due to buffer overflow $P_{L_{buf}}$ is large when compared to the loss due to intermit-

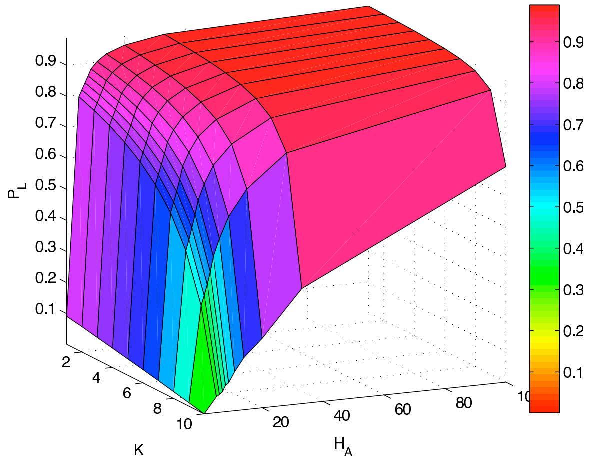


Fig. 6 Total packet loss probability of a WSN with 10 single-buffer nodes for different K and H_A

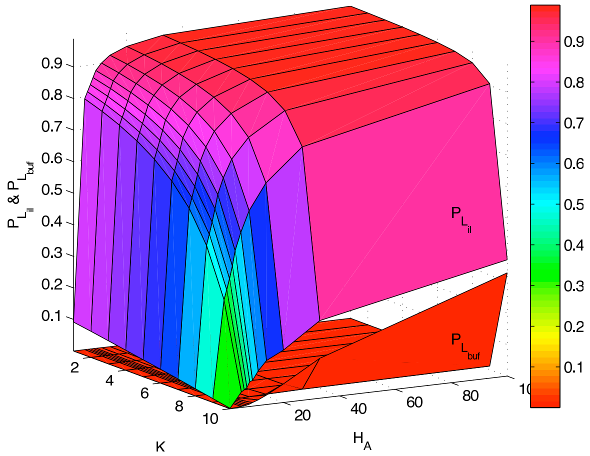


Fig. 7 Packet loss probability due to intermittent links and finite buffer of a WSN with 10 single-buffer nodes for different K and H_A

tency $P_{L_{il}}$. It is interesting to get insight of this WSN by able to answer questions like what should be the number of intermittent states and average number of hops in order to achieve not more than 50% packet loss by referring Figure 8. When the loss of packets decreases, the number of packets in the network increases and as a result the end-to-end delay increases. This is clear from Figure 8.

Example 2 We consider a WSN model with 10 finite-buffer nodes and 10-state communication links with Rayleigh distributed intermittent transmission and/or reception. We assume that at each node, packets are generated according a Poisson process with average generation rate of 5 packets per second and deterministic processing time of 0.04 seconds per packet. In Figures 9, 10, and 11, we plot the performance measures total packet loss probability, packet loss due to intermittency and finite

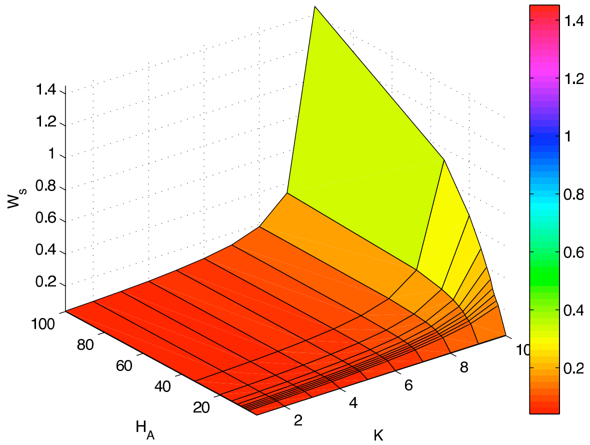


Fig. 8 Average end-to-end delay of a WSN with 10 single-buffer nodes for different K and H_A

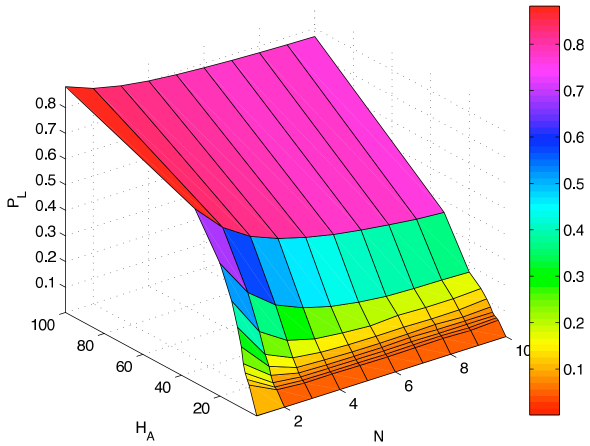


Fig. 9 Total packet loss probability of a WSN with 10 finite-buffer nodes for different N and H_A

buffer, and average end-to-end delay, respectively, for different buffer size N of nodes and average number of hops H_A a generated packet goes through before reaches the sink. To see the effect of intermittency in packet loss, the packet loss probability is plotted for different averages of number of hops in Figure 9. The P_L increases from 0 to 0.88 as the average number of hops increases from 1 to 100 when the buffer size N is 1. The loss remains relatively high though the buffer size increases from 1 to 10. This is due to 100% load in the network which is nothing but the average of the ratios $\frac{\lambda_j'}{\mu_j}$ over all j. Due to this heavy traffic, the P_L gradually reduces as N increases from 1 to 10. It is evident from Figure 10, as the H_A increases, the packet loss due to intermittency $P_{L_{il}}$ stabilizes (around 0.25) whereas the loss due to finite buffer $P_{L_{buf}}$ keeps increasing linearly due to more and more packets are being routed. The $P_{L_{il}}$ increases exponentially as H_A increases from 1 to 30 and stabilizes whereas $P_{L_{buf}}$ remains zero initially and starts piercing through the $P_{L_{il}}$ graph gradually as H_A and

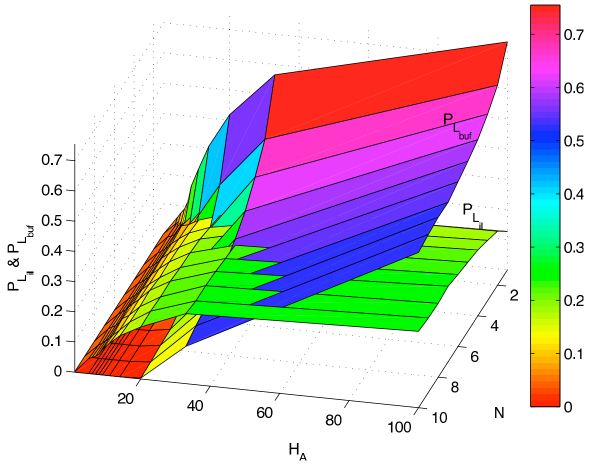


Fig. 10 Packet loss probability due to intermittent links and finite buffer of a WSN with 10 finite-buffer nodes for different N and H_A

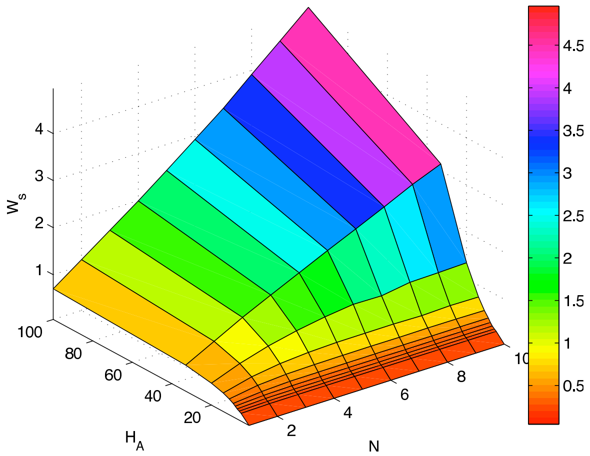


Fig. 11 Average end-to-end delay of a WSN with 10 finite-buffer nodes for different N and H_A

N increase. From Figure 11, we understand that though we loose more packets due to intermittency as H_A increases, the average end-to-end delay W_s increases exponentially for large fixed N. This is because as the buffer size increases, packets are being buffered due to 100% traffic load in the network and hence causes more delay. Whereas when $N = 1$, W_s reaches steady-state as H_A increases.

Example 3 We consider a WSN model with 50 finite-buffer nodes, 10-state communication links with Rayleigh distributed intermittent transmission and/or reception and with average number of hops set to 10. We assume that at each node, packets are generated according a Poisson process with average generation rate of 5 packets per second and deterministic processing time of 0.04 seconds per packet. In Figures 12, 13, and 13, we plot the performance measures total packet loss probability, packet

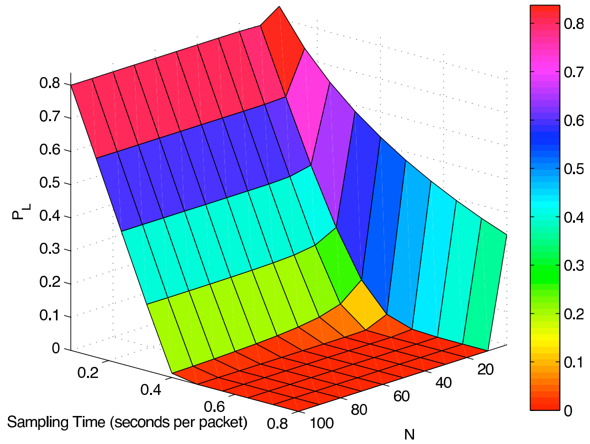


Fig. 12 Total packet loss probability of a WSN with 50 finite-buffer nodes for different N and sensing frequencies

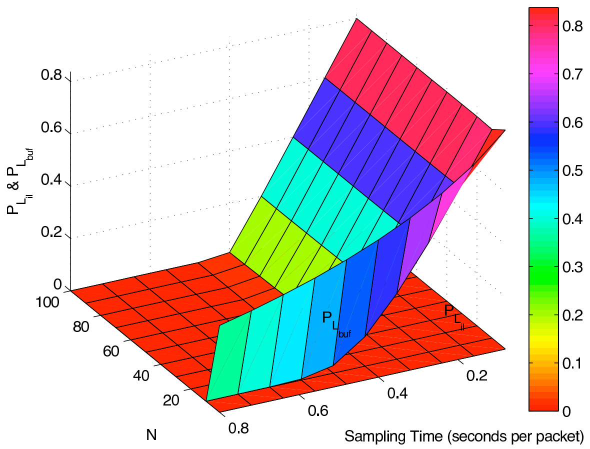


Fig. 13 Packet loss probability due to intermittent links and finite buffer of a WSN with 50 finite-buffer nodes for different N and sensing frequencies

loss due to intermittency and finite buffer, and average end-to-end delay, respectively, for different buffer size N of nodes and different data sensing frequencies. The effect of sampling time and buffer size on the packet loss P_L is depicted in Figure 12. When $N = 1$, the P_L is high when compared to other values of N and it increases from 0.33 to 0.87 as the sampling time decreases. We note that sampling time is inversely proportional to the load of the network. For this model, as the sampling time increases from 0.08 to 0.8, the load of the network decreases from 250% to 25%. Also, in the model, we have chosen the parameter values so that the loss due to intermittency is almost zero. It is evident from Figure 13. This is done to see the effect of buffer size and sampling times on the performance measures. In the sampling time region [0.1 0.4], the P_L is more when compared to its value in the sampling time

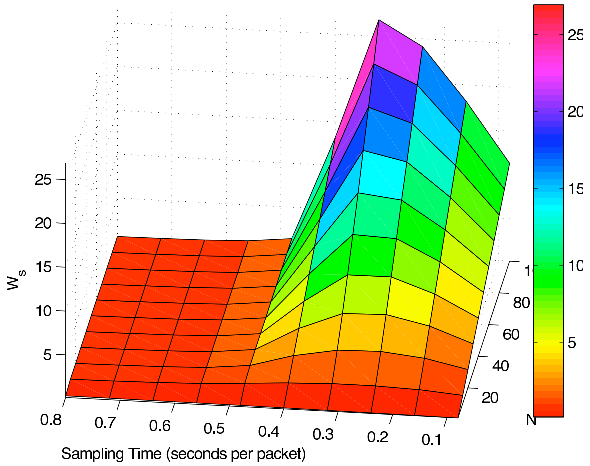


Fig. 14 Average end-to-end delay of a WSN with 50 finite-buffer nodes for different N and sensing frequencies

region [0.4,0.8] where it is almost zero. This is because in the sampling time region [0.1,0.4], the load of the network is more than 100%. This effect can be seen in the average end-to-end delay W_s's graph in Figure 14. During the heavy traffic sampling time region [0.1,0.4], the W_s increases linearly as the buffer size increases as more and more packets join the buffer. In this region, the W_s increases as the sampling time increases and reaches respective local maximums when the sampling time is 0.32 and starts decreasing to 1 around 0.5. It reaches the maximum value of 27 seconds when $N = 100$.

In this example, we have shown that the performance measures can be computed without much effort even for a WSN with large number of sensor nodes.

5 Conclusion and Future Work

We modeled WSN using suitable stochastic networks known as open queuing networks and derived analytical formulas for the performance measures such as throughput, average end-to-end delay, and packet loss probability. These mathematical models are amenable to use intermittent distributions of communication links based on mobility models. Extensive numerical results are carried out to illustrate the efficiency of the analytical results

Mathematical models are available in the literature which can possibly be used to model an individual wireless sensor node but as far as we are aware, this is the first attempt to model entire mobile WSN by using existing mobility models.

The model does not put any limitations on the parameter values nor on the distributions to capture packet generation, processing time and intermittency.

As we noted in Section 1, we did not consider details like nature of microcontroller and antenna, battery life, collision of packets, and retransmission of packets. Our focus was mainly on the intermittency in network communication among sensor nodes due to mobility. We assumed the rest are assured to function well. The subject of further interests include modeling the retransmission of packets and medium access control algorithms protocols and compare the analytical results with the simulation results of WSN simulation package known as Castalia - the WSN framework of OMNeT++ [28].

A QNA and Expansion Method

For completeness purposes, we present the QNA and expansion methods here.

A.1 QNA

The QNA is an approximation algorithm developed at Bell Laboratories to calculate the average queuing delay at each node of open queuing networks without intermittency and with large number of infinite buffer nodes.

A.2 Expansion Method

The expansion method calculates the average delay at each node of open queuing networks without intermittency and with large number of finite buffer nodes. In this approach, an artificial node is added to each finite buffer node. Whenever a node's buffer is full, the packet is routed to the associated artificial node. After incurring a random/deterministic delay, the packet attempts to rejoin the buffer and if the buffer is full, it is routed back to the artificial node. This process continues for the packets in the artificial node until the artificial node is empty.

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QNA algorithm [25]:
Input:

1. M,
2. $\lambda_i, c_{0i}^2, \rho_i$, and c_{si}^2 for $i,\ i = 1, 2, \ldots, M$
3. $\lambda_{ij}, r_{ij},\ i, j = 1, 2, \ldots, M$

Output: $W_{q_j},\ j = 1, 2, \ldots, M$
for *j = 1 to M* **do**

$$q_{ij} = \frac{\lambda_{ij}}{\lambda_j};$$

$$v_j = \left[\sum_{i=0}^{m} q_{ij}\right]^{-1};$$

$$w_j = \left[1 + 4(1-\rho_i)^2(v_j - 1)\right]^{-1};$$

$$x_i = \max_{1 \le i \le M}(c_{si}^2, 0.2);$$

$$b_{ij} = w_j r_{ij} q_{ij}(1-\rho_i^2);$$

$$g_j = f(\rho_j, c_{aj}^2, c_{sj}^2) \text{ such that } g_j = 1 \text{ for } c_{aj}^2 \ge 1$$

$$a_j = 1 + w_j\{(q_{0j}c_{0j}^2 - 1) + \sum_{i=1}^{m} q_{ij}\left[1 - r_{ij} + r_{ij}\rho_i^2 x_i\right]\}; \quad (33)$$

$$c_{ij}^2 = q_{ij}[1 + (1-\rho_i^2)(c_{ai}^2 - 1) + \rho_i^2(c_{si}^2 - 1)] + 1 - q_{ij}; \quad (34)$$

$$c_{aj}^2 = a_j + \sum_{i=1}^{M} b_{ij} c_{ai}^2 \quad (35)$$

(or)

$$c_{aj}^2 = 1 - w_j + w_j \sum_{i=1}^{M} r_{ij} c_{ij}^2; \quad (36)$$

$$W_{qj} = \frac{\lambda_j (c_{aj}^2 + c_{sj}^2) g_j}{2(1-\rho_j)}; \quad (37)$$

end

11. Schoeneman, J., Smartt, H., and Hofer, D. (2000) Wipp transparency project - container tracking and monitoring demonstration using the authenticated tracking and monitoring system (atms). *Proc. Waste Management 2000 (UM2K) Conference*, Tucson, AZ, U.S.A., January. US Department of Energy.
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Expansion method algorithm ([29], [30],[31]):
Input:

1. M, N
2. $\lambda_i, c_{0i}^2, \rho_i$, and c_{si}^2 for $i,\ i = 1, 2, \ldots, M$
3. $\lambda_{ij}, r_{ij},\ i, j = 1, 2, \ldots, M$

Output: $p_j^{(N)}$ and $W_{s_i},\ i = 1, 2, \ldots, M$

for *j = 1 to M* **do**

$$p_j = \left[\sum_{i=1}^{M}\left(\frac{\lambda_{ij}}{\sum_{k=1}^{M}\lambda_{kj}}\right)^2\right]^{-1}$$

$y_j = (1 + 2.1(1-\rho_j)^{1.8}p_j)^{-1}$;

Compute c_{ij}^2 using (34);

$$c_{aj}^2 = y_j\sum_{i=1}^{M}\left[\frac{\lambda_{ij}}{\sum_{k=1}^{M}\lambda_{kj}}\right]c_{ij}^2 + 1 - y_j;$$

$$a = \frac{1}{\lambda_j};$$

$$b = \frac{1}{\mu_j};$$

$$a_R = \frac{c_{aj}^2 + 1}{2\lambda_j};$$

$$b_R = \frac{c_{sj}^2 + 1}{2\mu_j};$$

$$p_j^{(N)} = (1-\rho_j)\Big[\Big(\frac{a_R}{a_R + b - a}\Big)\Big(1 + \frac{a - b}{a_R + b_R - a}\Big)^{N-1} - \rho_j\Big]^{-1};$$

$$L_{sj} = \frac{\lambda_j(a_R + b_R - a)}{1-\rho_j} + \frac{b - b_R}{a_R} + \Big[\frac{\rho_j(b - b_R)}{(1-\rho_j)a_R} - \frac{\lambda(a_R + b_R - a) + \rho_j N}{1-\rho_j}\Big]p_{Nj} + (1 - p_{Nj})(1 - \lambda_j a_R);$$

$$W_{sj} = \frac{L_{sj}}{\lambda_j(1 - p_N^{(j)})};$$

end

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