2013 – ACTM Regional
Geometry Exam

In each of the following choose the <u>BEST</u> answer and record your choice on the answer sheet provided. To insure correct scoring, be sure to make all erasures completely. The tie-breaker questions at the end will only be used to resolve ties in first, second and/or third place. They will be used in the order given. Complete the first 25 questions before attempting the tie-breaker questions. Figures are not necessarily drawn to scale.

1.   Consider the statement "If Jacob studies geometry, then he will appreciate logic."
        Which of the following statements are logically equivalent to this statement?

        A.   Jacob doesn't study geometry or he will appreciate logic.
        B.   Jacob studies geometry but he doesn't appreciate logic.
        C.   If Jacob appreciates logic, then he will study geometry.
        D.   If Jacob doesn't study geometry, then he will not appreciate logic.
        E.   None of these

2.   A line contains the points (-2,1) and (5,3). The slope of the line is

        A. $-\dfrac{2}{3}$ B. $\dfrac{2}{7}$ C. $\dfrac{4}{3}$ D. $\dfrac{7}{2}$
        E.   None of these

3.   The line 3x + 2y = 6 is perpendicular to a second line in the plane. The slope of the second line is
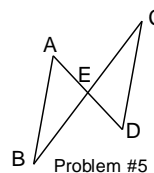
        A. $\dfrac{3}{2}$ B. $\dfrac{2}{3}$ C. $-\dfrac{2}{3}$ D. $-\dfrac{3}{2}$
        E.   None of these

4.   Which of the following is not an abbreviation for theorems used to prove the congruence of triangles?

        A.   SSS   B.   AAS   C.   ASA   D.   SSA
        E.   None of these

5.   In the figure $\overline{AD}$ bisects $\overline{BC}$ and $\angle$ABE $\cong$ $\angle$DCE. Which triangle congruence
        theorem assures that $\triangle ABE \cong \triangle DCE$?

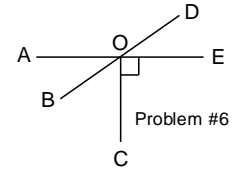   A.   SAS   B.   SSS   C.   ASA
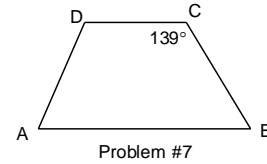   D.   SSA   E.   None of these

Problem #5

6.   Suppose $\overleftrightarrow{AE}$ and $\overleftrightarrow{BD}$ intersect at O and $\overline{CO} \perp \overleftrightarrow{AE}$. Which of the following is false?

   A. $\angle$AOB and $\angle$EOB are supplementary angles.
   B. $\angle$DOE and $\angle$BOC are complementary angles.
   C. $\angle$AOB and $\angle$DOE are congruent angles.
   D. $\angle$AOD and $\angle$COA are adjacent angles.
   E.   All are false.

Problem #6

7.   ABCD is a trapezoid with $m\angle$BCD = 139°. Then $m\angle$ABC is

   A.   31°   B.   51°   C.   41°

   D.   39°   E.   None of these

Problem #7

8.   The supplement of an angle is five times the complement of the angle. What is the measure of the angle?

   A.   23½°   B.   22½°   C.   67½°   D.   112½°
   E.   None of these

9.   The centroid of a triangle is determined by

   A.   angle bisectors of the angles of the triangle.
   B.   the intersections of the mid-lines of the triangle.
   C.   the intersection of the perpendicular bisectors of the sides of the triangle.
   D.   the intersection of the altitudes of the triangle.
   E.   None of these

10.   G is the centroid of the ΔABC. If line segments $\overline{DE}, \overline{FH}$, and $\overline{JK}$ are drawn parallel to the sides of the triangle, then DE + FH + JK is

   A.   one-half the perimeter of ΔABC.
   B.   three-fourths the perimeter of ΔABC.
   C.   two-thirds the perimeter of ΔABC.
   D.   cannot be determined.
   E.   None of these.
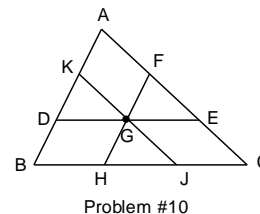
Problem #10

11. An X-scale and a Y-scale are marked adjacently as shown. A reading of 72 on the
Y-scale would correspond to what reading on the X-scale.

A.	108	B.	68	C. 88
D.	128	E.	None of these	

A.   108   B.   68   C. 88

D.   128   E.   None of these

X   Y
170 ──●──100

?──●──72

20 ──●── 0
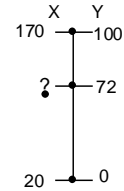
Problem #11

12. The area of a rectangle and the area of a square are equal. The length of the rectangle is twice
the width of the square. If the perimeter of the rectangle is 40 inches, then the perimeter of the
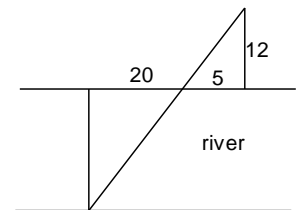square is

A.   40 inches   B.   32 inches   C.   44 inches

D.   64 square inches   E.   None of these

13. The distance between (1, -2) and (6, 10) in the coordinate plane is

A. $\sqrt{149}$ B. $\sqrt{119}$ C. $\sqrt{89}$ D.   13

E.   None of these

14. Find the shortest distance across the river shown at the right.

A.   52 ft   B.   65 ft   C.   48 ft

D. $8\frac{1}{3}$ ft   E.   None of these
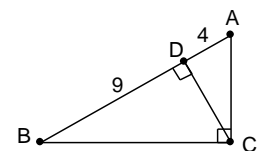
12
20   5
river

Problem #14

15. A point on the x-axis that is a distance of 5 units from (1,4) is

A.   (-2, 0)   B.   (0,0)   C.   (3,0)   D.   (-4, 0)

E.   None of these

16. The altitude from the right angle in $\triangle ABC$ divides the hypotenuse into segments of lengths 4 and
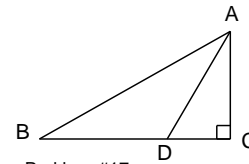9. The length of the altitude is

A.   Not enough information   B.   13

C.   6   D.   5   E.   None of these

A
4
D
9
B
C
Problem #16

17. In right triangle $\triangle ABC$, $\overline{AD}$ bisects $\angle BAC$. If the measure of $\angle BAC = 50°$ then m $\angle ADB =$
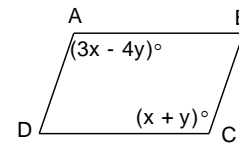
A. 115° B. 80° C. 65°

D. 40° E. None of these

Problem #17

18. In a circle the inscribed angle $\angle BAC$ measures 40°. If the diameter of the circle is 18, then the length of the arc(BC) is

A. 2π B. 4π C. 8π D. 3π

E. Not enough information

19. ABCD is a parallelogram with m$\angle BAD = (3x - 4y)°$ and m$\angle BCD = (x + y)°$. The value of the fraction $\dfrac{x}{y}$ is

A. Not enough information B. $\dfrac{5}{2}$

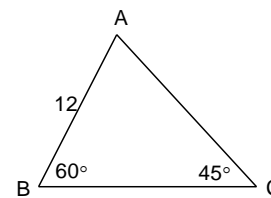C. $\dfrac{3}{4}$ D. $\dfrac{2}{5}$ E. None of these

(3x - 4y)°

(x + y)°

Problem #19

20. A circle in the coordinate plane has A = (-3, 4) and B = (9, -2) as endpoints of its diameter. The center of the circle is

A. (6, -3) B. $\left(\dfrac{1}{2}, \dfrac{7}{2}\right)$ C. (3, 3) D. (3, 1)

E. None of these

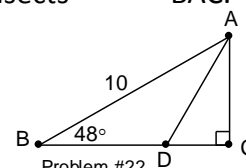21. In $\triangle ABC$, AB = 12, m$\angle ABC = 60°$ and m$\angle ACB = 45°$. Find the area of $\triangle ABC$.

A. $18\sqrt{3} + 18$ B. 49.177

C. $18\sqrt{3} + 54$ D. 85.177

E. None of these

12, 60°, 45°

Problem #21

22. In right triangle $\triangle ABC$ m$\angle ABC = 48°$, AB = 10 and $\overrightarrow{AD}$ bisects $\angle BAC$. Then, to the nearest thousandth, CD =

A. 3.625 B. 3.839 C. 3.264

D. 2.853 E. None of these

10, 48°

Problem #22

23. If the corresponding sides of two triangles are proportional, which of the following are not proportional in the triangles?

A. Corresponding angle bisectors B. Corresponding altitudes

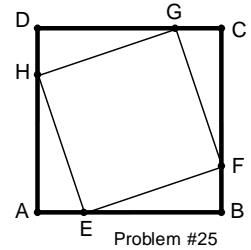C. Corresponding medians D. Areas of the triangles

E. None of these

24. In right triangle $\triangle ABC$ with right angle at C, $\overline{CD}$ is a median. If CD = 10 and AC = 16 then BC =

Problem #24

A.   12   B.   10   C. $10\sqrt{2}$
D.   20   E.   None of these
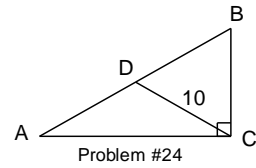
25. ABCD is a square. AE = BF = CG = DH = ¼ AB.
If a point is chosen at random in the square, what if the probability it lies in either of the four corner triangles?

Problem #25

A. $\dfrac{3}{8}$ B. $\dfrac{5}{8}$ C. $\dfrac{2}{5}$
D.   Not enough information   E.   None of these
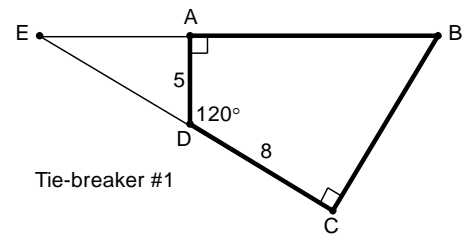
Name_____   School_____

                    Please Print

The following three questions will used to break ties for first, second and/or third place. The will be used in the order they are given. Be sure you have answered the previous 25 questions before proceeding to the tie-breaker questions. Your answers should be written clearly with reasonable justification of your work.

1.   In the figure $\overline{AD} \perp \overline{AB}$, $\overline{BC} \perp \overline{CD}$ and m $\angle$ADC = 120°.

      If AD = 5, and CD = 8, find the perimeter of the quadrilateral ABCD.

      Hint: Extend the segments $\overline{BA}$ and $\overline{CD}$ to intersect at E.

Tie-breaker #1

Name_____   School_____
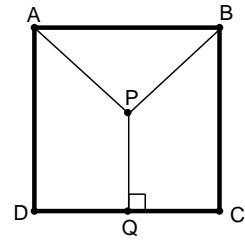
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2.   A plane intersects a sphere and is perpendicular to a diameter of the sphere at a point that is 8 inches from the center of the sphere. The diameter of the sphere is 20 inches. What is the area of the circle that is the intersection of the plane and sphere?

3.   ABCD is a square with a side of 12 inches. P is a point is
        in the interior of the square such that AP = BP = PQ.
        Determine AP.

Tie-breaker #3

Name_____Key_____   School_____

          Please Print

The following three questions will used to break ties for first, second and/or third place. The will be used in the order they are given. Be sure you have answered the previous 25 questions before proceeding to the tie-breaker questions. Your answers should be written clearly with reasonable justification of your work.

1.   In the figure $\overline{AD} \perp \overline{AB}, \overline{BC} \perp \overline{CD}$ and m$\angle$ADC = 120°.
        If AD = 5, and CD = 8, find the perimeter of the quadrilateral ABCD.
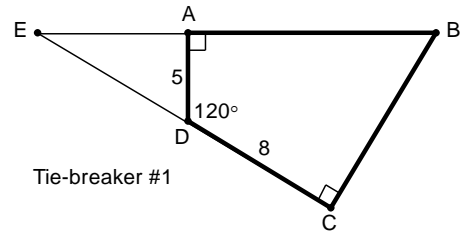   Hint: Extend the segments $\overline{BA}$ and $\overline{CD}$ to intersect at E.

      $\angle$ADE = 60°, $\angle$AED = 30°, $\angle$EBC = 60°.
        AE = $5\sqrt{3}$, and DE = 10.
        In ΔBEC, EC = 18, and BC = $6\sqrt{3}$ so BE = $12\sqrt{3}$.
        Therefore, AB = $7\sqrt{3}$.
        AB + BC + CD + DA = $7\sqrt{3} + 6\sqrt{3} + 8 + 5$
                                      = $13\sqrt{3} + 13$
                                      = $13(\sqrt{3} + 1)$ (optional)
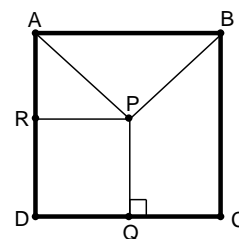

Tie-breaker #1

2.   A plane intersects a sphere and is perpendicular to a diameter of the sphere at a point that is 8 inches from the center of the sphere. The diameter of the sphere is 20 inches. What is the area of the circle that is the intersection of the plane and sphere?

        Since the diameter of the sphere is 20 inches,
        then OA = 10 and OB = 8. Thus AB = 6.
        Therefore the area of the circle of intersection between
        the plane and the sphere is 36π sq. in.

3. ABCD is a square with a side of 12 inches. P is a point is
    in the interior of the square such that AP = BP = PQ.
    Determine AP.

    $\triangle APB$ is isosceles so $\overline{PQ}$ bisects $\overline{DC}$. So DQ = 6.
    Also, PR = DQ = 6.
    Thus, If AP = x, then RD = x and AR = 12 − x.
    Therefore, $(AP)^2 = (12 - x)^2 + 6^2$. Hence, $x^2 = 144 - 24x + x^2 + 36$.
    Therefore, 24x = 180. So x = 7.5. Thus, AP = 7.5.

Tie-breaker #3

Key (25 Questions)

1.	A		14.	C
2.	B		15.	A
3.	B		16.	C
4.	D		17.	A
5.	C		18.	B
6.	E		19.	B
7.	C		20.	D
8.	C		21.	C
9.	E		22.	D
10.	C		23.	E
11.	D		24.	A
12.	B		25.	A
13.	D			