

Arkansas Council of Teachers of Math Contest 2009
Statistics

1. Identify which, if any, of the following are true:
 - a. If events A and B are independent then they are also disjoint.
 - b. If events A and B are independent then $P(A \cap B) < P(A) P(B)$.
 - c. If events A and B are independent then they can't simultaneously occur.
 - d. None of the statements a. – c. are true.

2. A particular data collection experiment uses the following selection method. Each member of the population is assigned a number and then a table of random numbers is used to select $n=100$ participants for the experiment. The sampling method is an example of:
 - a. a convenience sample
 - b. a simple random sample
 - c. a stratified random sample
 - d. none of the above methods

3. A large multi-national corporation has 20,000 employees in salary grade 7 and 10,000 employees in salary grade 8. It obtains a sample of these employees of size $n=3000$ by randomly selecting 2000 employees from grade 7 and 1000 employees from grade 8. The sampling method is an example of:
 - a. a convenience sample
 - b. a simple random sample
 - c. a stratified random sample
 - d. none of the above methods

4. A “two-tailed” hypothesis test regarding a population mean would be used when the null hypothesis is $\mu = 50$ and the alternate hypothesis is:
 - a. $\mu > 50$
 - b. $\mu < 50$
 - c. $\mu \neq 50$
 - d. none of these

5. In hypothesis testing a Type II error refers to:
 - a. rejecting a null hypothesis when it is true
 - b. an inappropriate appeal to the Central Limit Theorem
 - c. using the wrong alternate hypothesis
 - d. none of these

6. In hypothesis testing a the probability of a Type I error is also referred to as
 - a. the level of significance
 - b. α
 - c. size of the rejection region
 - d. none of a. – c.
 - e. all of a. – c.

7. The partially filled contingency table gives the frequencies of the data on age (in years) and sex from the residents of a retirement home.

	60-69	70-79	Over 79	total
Male	11	9	5	
Female	9	2	4	
total				

What is the proportion of male residents in the age group 60-69?

- a. 0.44 b. 0.55 c. 0.35 d. 0.60 e. 0.50
8. Using advertised prices for used Ford Escorts a linear model for the relationship between a car's age and its price is found. The regression has an $R^2 = 87.1\%$. Describe the relationship
- a. Negative, strong linear relationship. As the age increases the price goes down.
b. Negative, weak linear relationship. As the age decreases the price goes down.
c. Positive, strong linear relationship. As the age increases the price goes up.
d. Positive, weak linear relationship. As the age increases the price goes down.
e. No association
9. In the scatterplot below the relationship shown is a
- a. Strong linear, negative association.
b. Weak linear, negative association.
c. Strong linear, positive association.
d. Weak linear, positive association.
e. No association.



10. A 95% confidence interval is to be constructed for the unknown proportion of American voters that favor a particular constitutional issue. If no prior estimate is known and the margin of error in the resulting interval is to be no more than 2%, which of the following is correct?
- A sample of 1,000 randomly chosen voters will be sufficient
 - A sample of 1,500 randomly chosen voters will be sufficient
 - The sample should exceed 2,400.
 - Any of the above will be sufficient.

11. If an experiment has outcomes A, B, C, and D, find the probability of D using the table below.

Outcomes	A	B	C	D
Probability	1/8	1/8	1/8	

- 5/8
 - 1/8
 - 1/4
 - 3/8
 - none of these choices.
12. When a quarter is tossed four times, 16 outcomes are possible.

HHHH	HHHT	HHTH	HHTT
HTHH	HTHT	HTTH	HTTT
THHH	THHT	THTH	THTT
TTHH	TTHT	TTTH	TTTT

Here, for example, HTTH represents the outcome that the first toss is heads, the next two tosses are tails, and the fourth toss is heads. List the outcomes that comprise the following event.

A = event exactly three tails are tossed

- TTTH
 - HTTT, THTT, TTHT, TTTH
 - HTTT, THTT, TTHT, TTTH, TTTT
 - HTTT, THTT, TTTH
 - THTT, TTHT, TTTH
13. The random variable X is the number that shows up when a loaded die is rolled. Its probability distribution is given in the table. Determine the mean of the distribution of X.

x	P(X = x)
1	0.14
2	0.12
3	0.12
4	0.10
5	0.13
6	0.39

- 0.17
- 4.00
- 3.50
- 4.13

14. The systolic blood pressure of 18-year-old women is normally distributed with a mean of 120 mmHg and a standard deviation of 12 mmHg. What percentage of 18-year-old women have a systolic blood pressure between 96 mmHg and 144 mmHg?
- a. 99.99% b. 99.7% c. 95% d. 68% e. none of these values
15. Find the z-score for which the area under the standard normal curve to its left is 0.96
- a. -1.75 b. 1.82 c. 1.03 d. -1.38 e. 1.75
16. At one school, the average amount of time that tenth-graders spend watching television each week is 21.6 hours. The principal introduces a campaign to encourage the students to watch less television. One year later, the principal wants to perform a hypothesis test to determine whether the average amount of time spent watching television per week has decreased. Which of the following has the correct null and alternative hypotheses?
- a. $H_0 : \mu = 21.6$ hours $H_a : \mu < 21.6$ hours
b. $H_0 : \mu < 21.6$ hours $H_a : \mu = 21.6$ hours
c. $H_0 : \mu = 21.6$ hours $H_a : \mu \leq 21.6$ hours
d. $H_0 : \mu = 21.6$ hours $H_a : \mu > 21.6$ hours
e. $H_0 : \mu < 21.6$ hours $H_a : \mu > 21.6$ hours
17. A nationwide study of American homeowners revealed that 65% have one or more lawn mowers. A lawn equipment manufacturer, located in Charlotte, feels the estimate is too low for households in Charlotte. Find the P-value for a test of the claim that the proportion with lawn mowers in Charlotte is higher than 65%. Among 497 randomly selected homes in Charlotte, 340 had one or more lawn mowers.
- a. 0.0505 b. 0.1118 c. 0.0555 d. 0.0252 e. 0.0500
18. Test the claim that for the population of female college students, the mean weight is given by $\mu = 132$ lb. Sample data are summarized as $n = 20$, $\bar{x} = 137$ lb, and $s = 14.2$ lb. Use a significance level of $\alpha = 0.1$. $H_0: \mu = 132$ $H_a: \mu \neq 132$
- State your conclusion about H_0 .
- a. Do not reject H_0 b. Reject H_0 c. Reject H_a d. Do not reject H_a
19. Suppose we assume that repeated plays at a particular game of skill are independent of each other and the probability of winning on any single play is $p=0.45$. If a selected player is to play ten times, what is the probability that the player wins at least five times.
- a. 0.262 b. 0.738 c. 0.234 d. 0.495 e. none of these is correct

20. With reference to problem #19, if the random variable X is defined to be the number of wins in those ten plays, then the mean of X 's distribution is
- a. 4.5 b. 5.0 c. 0.45 d. 2.48 e. none of these is correct
21. Suppose that A and B are two events from some sample space, $P(A) = .6$, $P(B) = .45$, and the $P(A \cup B) = .83$. Then $P(A \cap B)$ is
- a. 0.27 b. 0.22 c. 0.25 d. none of these is correct

Name: _____

Tie Breaker #1.

Suppose that we take $n=25$ random observations from a normal distribution with an unknown mean and a standard deviation of 25 units. These data are to be used to test the null hypothesis $H_0: \mu=85$ versus the alternative hypothesis $H_a: \mu > 85$ employing a rejection (critical) region $C = \{\bar{X} | \bar{X} \geq 92\}$.

- a. Determine the level of significance (α) for this hypothesis test.
- b. If the true value of $\mu=89$, determine the probability of a Type II error using the same critical region.

Name: _____

Tie Breaker #2.

The table below shows part of a MINITAB printout for the relationship between y = employee productivity and x = Internet use

Internet use and employee productivity

Predictor	Coef	SE Coef	T	P
Constant	24.479	9.258	-2.64	0.012
Internet Use	-0.465	0.1405	5.09	0.000

R-Sq = 66.7

- Find the predicted productivity for a an employee whose Internet use is rated as (i) $x = 50$, (ii) $x = 60$. Describe the effect of a 10-unit increase in x on the productivity measure and show how you could use the slope to get this information.
- Report r^2 , and interpret its value.
- What sign does the correlation have? Why? Find it.

Name: _____

Tie Breaker #3.

An urn contains 10 balls, 9 are black and 1 is red. You and another person will take turns drawing a ball from this urn until the red ball is drawn. The person drawing the red ball is the “winner” of this game.

- a. If the drawing is without replacement and you draw first, what is the probability that you win?
- b. If the drawing is with replacement and you draw first, what is the probability that you win?

ANSWERS:

1. d
2. b
3. c
4. c
5. d
6. e
7. b
8. a
9. a
10. c
11. a
12. b
13. d
14. c
15. e
16. c
17. c
18. a
19. d
20. a
21. b

Tie Breaker #1

- a. The probability of a type I error (level of significance) is computed as the normal distribution probability that a random variable from the distribution having mean 85 and standard deviation 5 exceeds 92 and the value is .0808
- b. The probability of this type II error is computed as the normal distribution probability that a random variable from the distribution having mean 89 and standard deviation 5 is less than 92 and the value is .7258

Tie Breaker #2

- a. (i) 1.229 and (ii) -3.421 The 10 unit increase results in a change in productivity of -4.65. Since slope is change in productivity per unit of internet usage this amount is 10 times the slope (-.465).
- b. $R^2 = .667$ means that 66.7% of the change in productivity may be attributed to change in internet usage.
- c. The coefficient of correlation is negative because the slope is negative and $r = -.8167$

Tie Breaker #3

- a. You (first to draw) win on either the first draw ($1/10$), third draw $(9/10)(8/9)(1/8)=1/10$, fifth draw with similar probability of $1/10$, seventh draw with probability $1/10$, or ninth draw with probability $1/10$. Hence the probability that you win is $1/2$.
- b. $P(\text{you win}) = (1/10) + (9/10)^2(1/10) + (9/10)^4(1/10) = (1/10)(1 + 81/100 + (81/100)^2 + \dots) = 10/19$