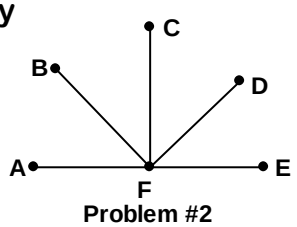
2009 ACTM – Geometry Exam

In each of the following choose the best answer for the question and bubble the appropriate letter on the corresponding answer sheet. The geometric figures are not necessarily drawn to scale. Be sure to answer questions 1 – 25 before working the tie-breaker questions. These questions will be used only in the event of a tie for 1st, 2nd, or 3rd place.

1. **In a quadrilateral ABCD, AB = CD and AD = BC. The most accurate name for ABCD is**

   A. Kite **B.** Rectangle **C.** Parallelogram
   D. Rhombus **E.** None of these

2. **In the figure $\overline{CF} \perp \overline{AE}$ and $\overline{BF} \perp \overline{DF}$. A pair of complementary angles is**



Problem #2

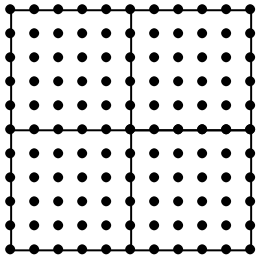
   A. ∠AFD, ∠CFD **B.** ∠EFD, ∠BFC
   C. ∠AFD, ∠DFE **D.** ∠CFE, ∠EFD
   E. None of these

3. **The equation of the line that is perpendicular to the line $y = \frac{2}{3}x + 1$, and passes through the point (-1, 2) is**

   A. $y = -\frac{3}{2}x + \frac{7}{2}$ **B.** $y = -\frac{3}{2}x + \frac{1}{2}$ **C.** $y = \frac{3}{2}x + \frac{7}{2}$
   D. $y = -\frac{2}{3}x + \frac{4}{3}$ **E.** None of these

4. **The vertices of a quadrilateral PQRS in the rectangular coordinate plane are P = (1, -2), Q = (1,3), R = (3,5) and S = (4,1). The area of PQRS is**

   A. 12 **B.** 11 ½
   C. 13 ½ **D.** 12 ½
   E. None of these


Problem #4

5. **Let WXYZ be a quadrilateral. Which of the following statements could be used to complete the statement "If WXYZ is a parallelogram, then ____", to create a theorem.**

   P: $\overline{WX} \cong \overline{XZ}$ **Q:** $\overline{WZ} \parallel \overline{XY}, \overline{WX} \cong \overline{ZY}$
   R: $\overline{WX}$ and $\overline{ZY}$ **bisect each other**

   A. P **B.** Q **C.** R **D.** Q, R
   E. None of these

6. The point of intersection of the lines containing the altitudes of a triangle is called the

A. Incenter B. Circumcenter C. Centroid

D. Supercenter E. None of these

7. The measures of the three angles are a triangle are in the ratio 2 : 4 : 9. The measures of the three angles of the triangle are

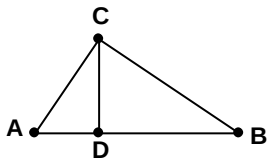
A. 22, 44, 114 B. 24, 48, 108 C. 18, 36, 81

D. 26, 52, 117 E. None of these

8. $\triangle ABC$ is a right triangle with right angle at C. $\overline{CD}$ is an altitude of the triangle. Which of the following statements is/are true?

P: $CD = \sqrt{AD \times BD}$ Q: $CD = \sqrt{(AD)^2 + (BD)^2}$

R: $CD = \sqrt{(AC)^2 - (BC)^2}$



Problem #8

A. P B. Q C. R

D. P, Q, R E. None of these

9. The median to the hypotenuse of a right triangle is 10 units. If one leg is 12 units then the other leg is [answer to the nearest hundredth]

A. 15.62 units B. 16.00 units C. 6.63 units

D. 20.00 units E. None of these

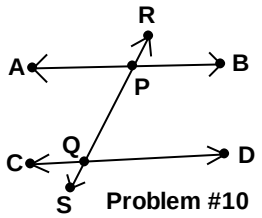
10. In the figure which of the following would imply that $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$?

A. $\angle APQ \cong \angle SQC$ B. $\angle APQ \cong \angle BPQ$

C. $\angle APQ$ and $\angle CQS$ are supplementary

D. $\angle APR$ and $\angle CQP$ are complementary

E. None of these



Problem #10

11. A polygon has 11 diagonals that contain a particular vertex A. The total number of diagonals in the polygon is

A. 77 B. 154 C. 55

D. Not enough information E. None of these

12. A regular polygon has an interior angle that measures 165°. The sum of all the interior angles of the polygon is

A. 360° B. 5400° C. 2700°

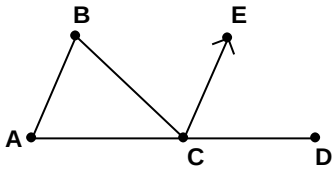
D. 3960° E. None of these

13. In the figure $\overrightarrow{CE}$ bisects ∠BCD and $\overrightarrow{CE} \parallel \overline{AB}$. If m∠BCD = 118°, then ΔABC is

A. equilateral B. scalene

C. isosceles D. obtuse

E. None of these



Problem #13

14. A prism has 12 faces. The ratio of the number of edges of the prism to the number of vertices is

A. 3 : 2 B. 2 : 3 C. 5 : 3 D. 5 : 2

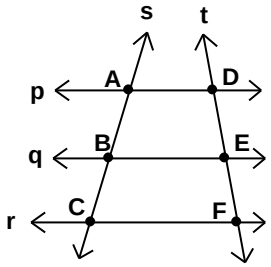
E. None of these

15. In the figure p, q, r are parallel lines and s and t are transversals. If AB = 9, BC = 15 and DF = 30 then DE =

A. 11.25 B. 12

C. 12.5 D. 18

E. None of these



Problem #15

16. Ten gallons of water are needed to fill a container that is 2 feet tall. How much water is needed to fill 32 containers that are similar to the given container but are 6 inches tall?

A. 10 gallons B. 20 gallons C. 5 gallons

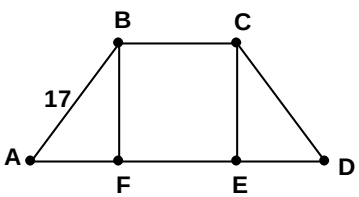
D. 8 ½ gallons E. None of these

17. ABCD is an isosceles trapezoid whose area is 345 and BCEF is a square whose area is 225. If AB = 17 then the perimeter of ABCD is

A. 110 B. 72

C. 80 D. 90

E. None of these



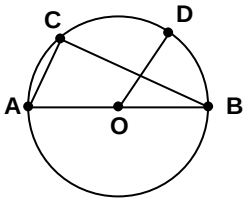
Problem #17

18. A right circular cone and a right circular cylinder have the same height. If the two solids have the same volume then the ratio of the radius of the cone to the radius of the cylinder is

A. 1 : 3 **B.** 3 : 1 **C.** 1 : $\sqrt{3}$ **D.** $\sqrt{3}$: 1

E. None of these

19. In the circle $\overline{AB}$ is a diameter and $\overline{OD}$ bisects $\overline{BC}$. Which of the following conclusions is/are correct?



Problem #19

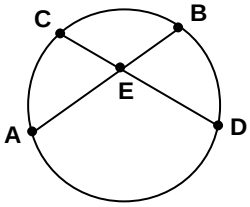
P: $\overline{OD} \parallel \overline{AC}$ **Q:** $\overline{OD} \perp \overline{BC}$

R: $\angle DOC \cong \angle DOB$

A. P, Q **B.** P, R **C.** Q, R

D. P, Q, R **E.** None of these

20. Chords $\overline{AB}$ and $\overline{CD}$ intersect in the circle at E. If AE = 6, BE = 4, CD = 11, then the equation needed to determine the possible values of CE and DE is



Problem #20

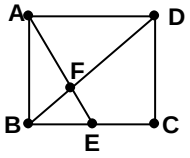
A. x + y = 11 **B.** xy = 24

C. $x^2 - 11x - 24 = 0$ **D.** $x^2 - 11x + 24 = 0$

E. None of these

21. In the rectangle ABCD, $\overline{AE}$ intersects the diagonal $\overline{BD}$ at F so that $BF = \frac{1}{3}BD$.

If the area of ABCD is 30, then the area of ΔABF is

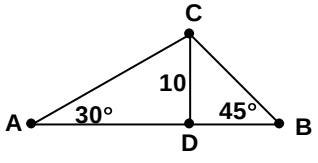


Problem #21

A. 3 **B.** 5 **C.** 6

D. Not enough information **E.** None of these

22. In ΔABC, $\overline{CD}$ is an altitude. If CD = 10, $m\angle CAD = 30°$, and $m\angle CBD = 45°$ then the area of ΔABC (to the nearest hundredth) is



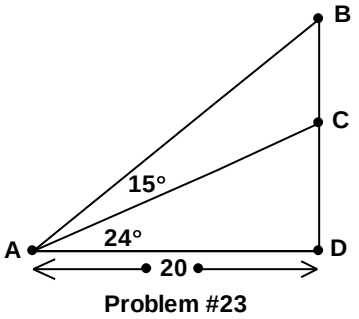
Problem #22

A. 150.00 **B.** 157.31

C. 173.21 **D.** 136.60

E. None of these

23. In the figure m∠DAC = 24° , m∠CAB = 15° and AD = 20. If ∆ABD is a right triangle find BC correct to 3 decimal places.



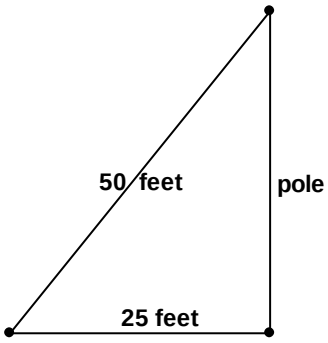
Problem #23

A. 5.359 B. 16.196

C. 7.291 D. 8.905

E. None of these

24. A guy wire from the top of a pole that is perpendicular to the ground is 50 feet long. If the wire is attached to the ground at a point that is 25 feet from the base of the pole, the angle the wire makes with the ground is



Problems #24,#25

A. 30° B. 60°

C. 63.4° D. 90°

E. None of these

25. The flag pole in feet and inches, to the nearest inch, in problem 24 is

A. 43’ 4” B. 43’ 5” C. 43’ 3”

D. 28’ 10” E. None of these

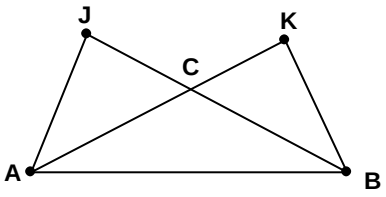
Be sure you have answered questions 1 – 25 before attempting the tie-breaker questions. These questions will only be used to break ties between first, second and third place should a tie exist.

Tie Breaker Questions

Name_____ **School**_____

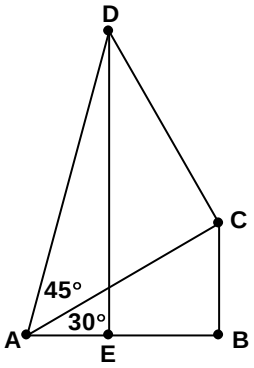
1. In the figure $\angle JAB \cong \angle KBA$ and $\angle CAB \cong \angle CBA$.

Prove: $\Delta JAC \cong \Delta KBC$
[Be sure to give complete steps and reasons]



Tie Breaker #1

2. In the figure $\overline{BC} \perp \overline{AB}$, $\overline{CD} \perp \overline{AC}$ and $\overline{DE} \perp \overline{AB}$. If BC = 10, find the perimeter of ABCD to the nearest thousandth. Show your work and give reasons for each conclusion.



Tie Breaker #2

Name_____ **School**_____

3. The hypotenuse of a right triangle is $x^2 + y^2$ and one of the legs is 2xy. Find the measure of the other leg in terms of x and y.

ACTM Geometry Key

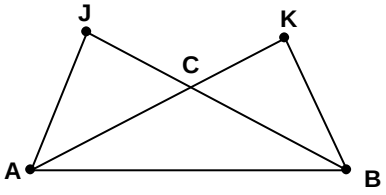
1. C
2. E
3. B
4. D
5. C
6. E
7. B
8. A
9. B
10. A
11. A
12. D
13. C
14. A
15. A
16. C
17. C
18. D
19. D
20. D
21. B
22. D
23. C
24. B
25. A

Tie Breaker Questions

Name KEY **School** _____

1. In the figure $\angle JAB \cong \angle KBA$ and $\angle CAB \cong \angle CBA$.

Prove: $\Delta JAC \cong \Delta KBC$
[Be sure to give complete steps and reasons]



Tie Breaker #1

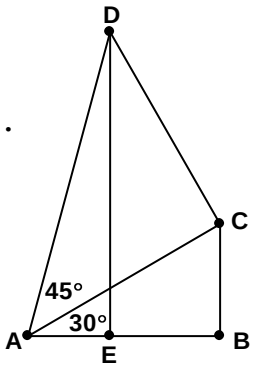
Since $\angle CAB \cong \angle CBA$, $\overline{AC} \cong \overline{BC}$ by the inverse of the isosceles triangle theorem. $\angle JCA \cong \angle KCB$ since they are vertical angles. By angle subtraction $\angle JAC \cong \angle KBC$. Thus, $\Delta JAC \cong \Delta KBC$

There are additional proofs available. The student may prove $\Delta JAB \cong \Delta KBA$ by ASA. This gives $\overline{AJ} \cong \overline{BK}$. Then use the vertical angles and angle subtraction to obtain $\Delta JAC \cong \Delta KBC$ by AAS.

2. In the figure $\overline{BC} \perp \overline{AB}$, $\overline{CD} \perp \overline{AC}$ and $\overline{DE} \perp \overline{AB}$. If BC = 10, find the perimeter of ABCD to the nearest thousandth. Show your work and give reasons for each conclusion.

Since ΔABC is a $30^0 - 60^o - 90^o$ triangle and BC = 10 then AC = 20 and AB = $10\sqrt{3}$.
Since ΔACD is a $45^o - 45^o - 90^o$ triangle then CD = 20 and AD = $20\sqrt{2}$.
Thus the perimeter of ABCD is $10\sqrt{3}$ + 10 + 20 + $20\sqrt{2}$.

Therefore, to the nearest thousandth, the perimeter of ABCD is 75.605.



Tie Breaker #2

3. The hypotenuse of a right triangle is $x^2 + y^2$ and one of the legs is 2xy. Find the measure of the other leg in terms of x and y.

Let $c = x^2 + y^2$ and $a = 2xy$.

Then $b^2 = c^2 - a^2$.

So $b^2 = (x^2 + y^2)^2 - (2xy)^2$.

Then $b^2 = x^4 + 2x^2y^2 + y^4 - 4x^2y^2$.

Therefore $b^2 = x^4 - 2x^2y^2 + y^4$.

So, $b^2 = (x^2 - y^2)^2$.

Thus, $b = x^2 - y^2$.