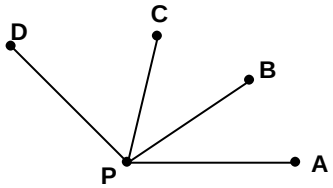
ACTM Regional Geometry Exam – 2009

In each of the following questions choose the best answer and bubble the corresponding letter on the answer sheet. Note: The geometric figures on this exam are not necessarily drawn to scale. When you have completed the first 25 questions please work on the tie-breaker questions. These will be used to break ties for determining first, second and third place, should a tie occur.

1. In the figure a pair of adjacent angles is



Problem 1

A. $\angle BPD, \angle APC$ B. $\angle APB, \angle DPC$

C. $\angle APB, \angle BPC$ D. $\angle BPC, \angle BPD$ E. None of these

2. $\angle A$ and $\angle B$ are complementary angles and $m\angle B$ is 20° more than the $m\angle A$. What is the measure of the angle supplementary to $\angle A$?

A. 125° B. 55° C. 155° D. 145°

E. None of these

3. Consider the following two statements:
If Jana wins the contest, then Jana will get two tickets to the concert.
Jana doesn't win the contest.

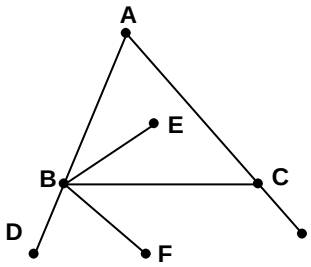
What valid conclusion can be made from these two statements?

A. Jana cannot go to the concert.
B. Jana wins the contest.
C. Jana doesn't go to the concert.
D. Jana wins the contest and goes to the concert.
E. None of these.

4. In the accompanying figure $\overrightarrow{BE}$ bisects $\angle ABC$ and $\overrightarrow{BF}$ bisects $\angle CBD$. The measure of $\angle EBF$ is

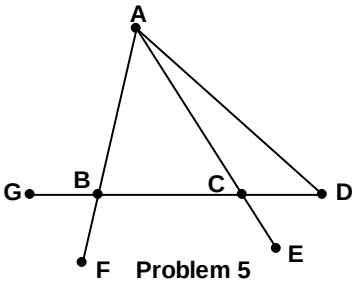
A. 100° B. 95° C. 80°

D. 120° E. None of these



Problem 4

5. In the figure an exterior angle of ΔABC is



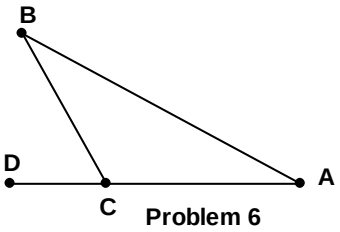
Problem 5

A. $\angle DBF$ B. $\angle ADC$ C. $\angle GBF$

D. $\angle FBE$ E. None of these

6. In the figure, which statement(s) best describes $\angle BCD$?

P: $m\angle BCD = m\angle CBA + m\angle CAB$
Q. $\angle DCB$ is the complement of $\angle BCA$
R. $m\angle DCB > m\angle BAC$



Problem 6

A. P, Q B. P, R C. Q, R

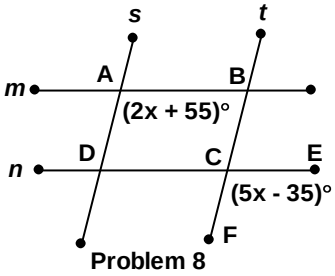
D. P, Q, R E. None of these

7. ΔABC is a right triangle with right angle at C. If AC = 10 and BC = 24, then the length of the median from C is

A. 13 B. 14 C. 17

D. Not enough information E. None of these

8. In the figure m || n and s || t. If the $m\angle BAD = (2x + 55)^\circ$ and $m\angle ECF = (5x - 35)^\circ$, then $m\angle ADC$ is



Problem 8

A. 30° B. 115° C. 108.33°

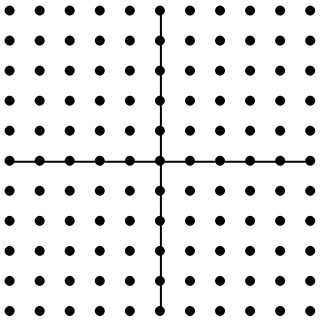
D. 65° E. None of these

9. A line that is parallel to the line $y = \frac{2}{3}x + 1$ and passes through (6,8) is

A. $3x - 2y = 2$ B. $2x - 3y = -12$ C. $2x - 3y = 12$

D. $3x + 2y = 34$ E. None of these

10. The vertices of ΔABC in the rectangular coordinate plane are A = (1, -2), B = (1,3), and C = (3,5). The area of ΔABC is


Problem 10

A. 10 B. 5 C. 14
D. 7 E. None of these

11. A prism and a pyramid have the same base, but the altitude of the pyramid is twice the altitude of the prism. The volume of the pyramid is

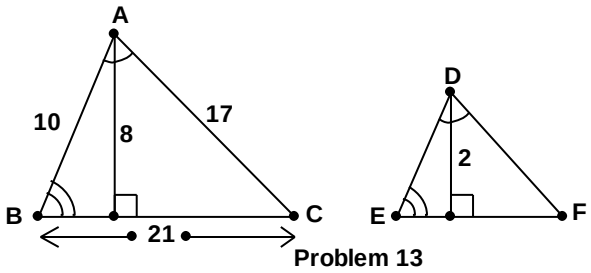
A. equal to the volume of the prism.
B. is one-third the volume of the prism.
C. is one-half the volume of the prism.
D. is two-thirds the volume of the prism.
E. twice the volume of the prism.

12. If one pint of paint is needed to paint a statue that is 4 feet tall, then the number of pints needed to paint (to the same thickness) 400 statues similar to the original, but only 1 foot tall is

A. 100 pints B. 50 pints C. 25 pints
D. 12½ pints E. None of these

13. Consider ΔABC and ΔDEF with $\angle$BAC $\cong$ $\angle$EDF and $\angle$ABC $\cong$ $\angle$DEF. The area of ΔDEF is

A. 21
B. 5.25
C. 10.5
D. Not enough information
E. None of these



Problem 13

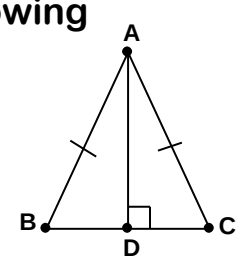
14. A transformation that preserves lengths is an isometry. Which of the following transformations is/are not an isometry?

P: Translation Q: Glide reflection R: Dilatation

A: P B. Q C. R D. Q, R
E. P, Q, R

15. In the figure, $\overline{AD} \perp \overline{BC}$ and $\overline{AB} \cong \overline{AC}$. Which of the following statements is/are true?

P: $\overline{AD}$ bisects $\angle$BAC
Q: $\overline{AD}$ is a median of ΔABC
R: ΔABD ~ ΔACD



Problem 15

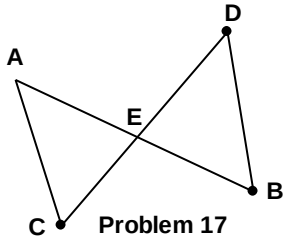
A. P, Q B. P, R C. Q, R
D. P, Q, R E. None of these

16. The diagonals of the parallelogram PQRS are perpendicular. Which of the following statements is the most accurate?

A. PQRS is a kite
B. PQRS is a rectangle
C. PQRS is a rhombus
D. PQRS is a square
E. None of these

17. Which of the following, if given for the figure at the right, would allow the conclusion that $\overline{AC} \parallel \overline{BD}$?

P: $\angle CAE \cong \angle BDE$
Q: $\angle CAE \cong \angle DBE$
R: E is the midpoint of $\overline{AB}$ and $\overline{CD}$.

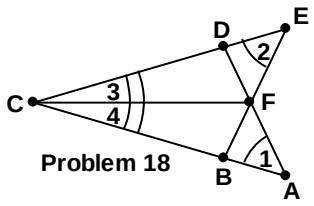


Problem 17

A. P, R
B. Q, R
C. P, Q, R
D. Q
E. None of these

18. In the figure $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$. Which of the following statements is/are true?

P: $\Delta ABF \cong \Delta EDF$
Q: $\Delta BCF \cong \Delta DCF$
R: $\Delta ACD \cong \Delta ECB$



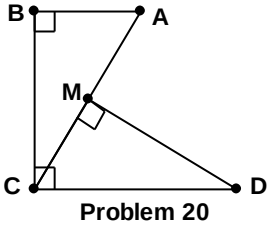
Problem 18

A. P, Q
B. Q, R
C. P, R
D. P, Q, R
E. None of these

19. The triangle with sides 10, 15, 18 is classified as a(n) _(?)_ triangle

A. acute
B. right
C. obtuse
D. oblique
E. None of these

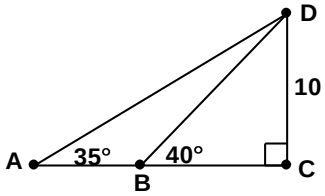
20. In the figure $\overline{AB} \perp \overline{BC}$, $\overline{BC} \perp \overline{CD}$, $\overline{DM} \perp \overline{AC}$, M is the midpoint of $\overline{AC}$ and AB = AM. If BC = 12, then DM =



Problem 20

A. 12
B. $8\sqrt{3}$
C. 6
D. $4\sqrt{3}$
E. None of these

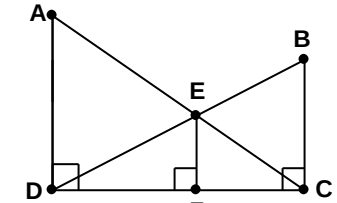
21. In the figure, what is the value of AB to 3 decimal places?



Problem 21

A. 11.918 B. 14.281
C. 2.364 D. 2.375 E. None of these

22. In the figure $\overline{AD} \perp \overline{CD}$, $\overline{BC} \perp \overline{CD}$, $\overline{EF} \perp \overline{CD}$, AD= 40 and BC = 30. To the nearest tenth, EF =



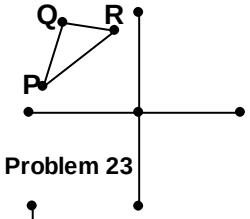
Problem 22

A. 17.0 B. 17.1

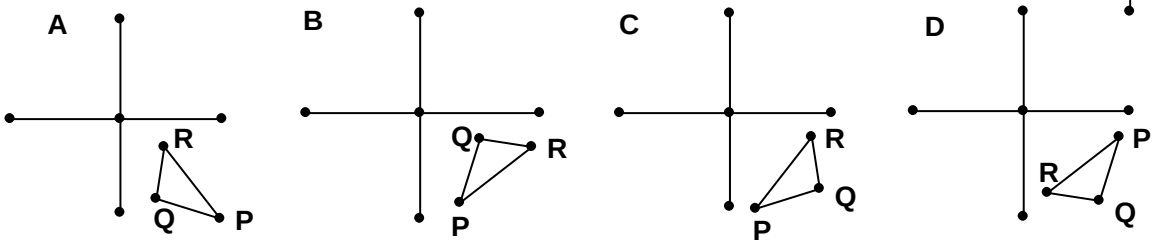
C. 17.2 D. Not enough information

E. None of these

23. Which of the following represent the rotation of ΔPQR 180° around the origin?



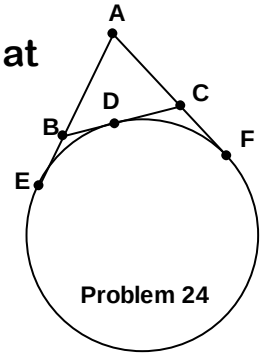
Problem 23



E. None of these

24. In the figure $\overline{AE}$, $\overline{AF}$, and $\overline{BC}$ are tangent to the circle at E, F and D, respectively. Which of the following statements is/are true?

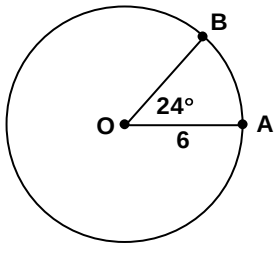
P: AB + BD is one-half the perimeter of ΔABC.
Q: AB + BE = AC + CF
R: AB + BD = AC + CD



Problem 24

A. Q, R B. P, R C. P, Q
D. P, Q, R E. None of these

25. The central angle $\angle AOB$ measures 24°. If the radius of the circle is 6, then the length of the arc intercepted by the angle is



Problem 25

A. $\frac{4\pi}{5}$ B. $\frac{2\pi}{5}$ C. $\frac{12\pi}{5}$
D. 24° E. None of these

Tie Breaker Questions

Name_____ **School**_____

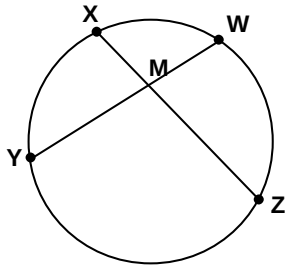
Please Print **Please Print**

The following three questions will be graded in order to break any ties for first, second, and/or third that results in the first 25 questions. Please offer a complete solution for the questions in the order they appear.

Tie Breaker #1

Let $\overline{XZ}$ and $\overline{YW}$ be chords of the circle intersecting at M.

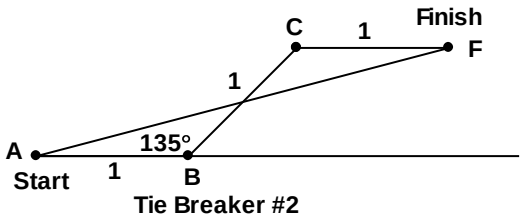
Prove: $XM \times MZ = YM \times MW$



Tie Breaker #1

Tie Breaker #2

A man walks one mile east, then he walks one mile northeast and then he walks one mile east. How far is he from his initial position? See the picture. [Hint: Draw $\overline{FD} \perp \overrightarrow{AB}$ and $\overline{FE} \parallel \overline{BC}$.



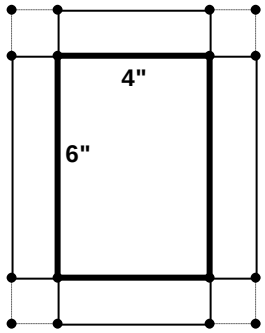
Tie Breaker #2

Name_____ **School**_____

Please Print **Please Print**

Tie Breaker #3

From a rectangular sheet of paper, Thomas makes an open rectangular box by cutting squares from the four corners and then folding up the sides. If the base of the box is 4 inches by 6 inches and its volume is 56 cubic inches, what is the area of the original sheet of paper?



Tie Breaker #3

Key – 2009 Regional Geometry Exam

1. C
2. D
3. E
4. E
5. A
6. B
7. A
8. D
9. B
10. B
11. D
12. C
13. B
14. C
15. D
16. C
17. B
18. D
19. A
20. A
21. C
22. B
23. D
24. D
25. A

Tie Breaker Questions

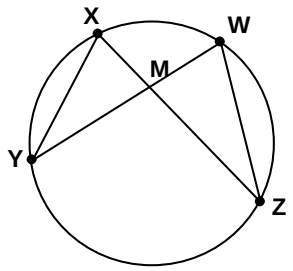
Name_____ **School**_____

Please Print **Please Print**

The following three questions will be graded in order to break any ties for first, second, and/or third that results in the first 25 questions. Please offer a complete solution for the questions in the order they appear.

Tie Breaker #1

Let $\overline{XZ}$ **and** $\overline{YW}$ **be chords of the circle intersecting at M.**



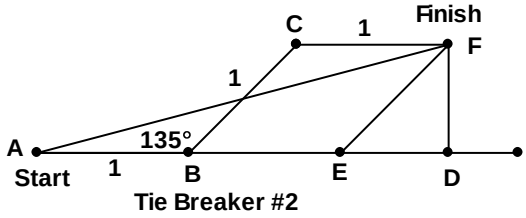
Tie Breaker #1

Prove: $XM \times MZ = YM \times MW$

Draw segments XY and WZ.
$\angle$XMY $\cong$ $\angle$WMZ by vertical angle theorem.
$\angle$XYM $\cong$ $\angle$WZM since both subtend the arc XW.
Therefore, ΔXYM $\sim$ ΔWZM.
So, $\frac{XM}{WM} = \frac{YM}{ZM}$**. Hence,** $XM \times MZ = YM \times MW$**.**

Tie Breaker #2

A man walks one mile east, then he walks one mile northeast and then he walks one mile east. How far is he from his initial position? See the picture. [Hint: Draw $\overline{FD} \perp \overrightarrow{AB}$ **and** $\overline{FE} \parallel \overline{BC}$**.**



Tie Breaker #2

$\angle$CBE = $\angle$FED = 45° and FE = BC = 1 mile.

Therefore ED = FD = $\frac{1}{2}\sqrt{2}$**. Therefore,**

AD = 2 + $\frac{1}{2}\sqrt{2}$ **and using Pythagorean Theorem**

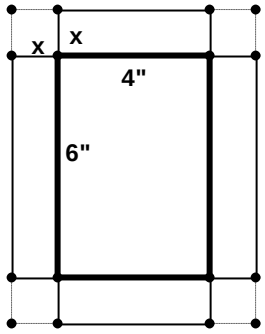
AF = $\sqrt{(AD)^2 + (FD)^2}$ **=** $\sqrt{5 + 2\sqrt{2}}$

Name_____ **School**_____

Please Print **Please Print**

Tie Breaker #3

From a rectangular sheet of paper, Thomas makes an open rectangular box by cutting squares from the four corners and then folding up the sides. If the base of the box is 4 inches by 6 inches and its volume is 56 cubic inches, what is the area of the original sheet of paper?



Tie Breaker #3

Volume = 4 (6)x = 24 x

24x = 56

x = $\frac{7}{3}$ **.**

Thus, 4 + 2($\frac{7}{3}$**) = 8**$\frac{2}{3}$ **and 6 + 2(**$\frac{7}{3}$**) = 10**$\frac{2}{3}$ **are the dimensions of the sheet of paper.**

Therefore, Area = 8$\frac{2}{3}$**(10**$\frac{2}{3}$**) = 92**$\frac{4}{9}$ **.**