

ACTM State Mathematics Contest
Pre-calculus/Trigonometry

1. Solve the following equation for x: $\frac{x-1}{4x} + 4 = \frac{x+2}{x}$

a. $x = \frac{9}{16}$ b. $x = \frac{9}{13}$ c. $x = \frac{3}{16}$ d. $x = \frac{-5}{3}$

2. Solve the polynomial equation for all solutions of x: $x^3 + 6x^2 - 14x + 16 = 0$

a. $\{1+i, 1-i, -8\}$ b. $\{1+i, 1-i, 8i\}$ c. $\{-8, 8\}$ d. $\{1+i, 1-i, 8\}$

3. Use properties of logarithms to rewrite the expression $7\ln(x-9) - 11\ln x$ into a single logarithm.

a. $\ln\left(\frac{(x-9)^7}{x^{11}}\right)$ b. $\ln\frac{7(x-9)}{11x}$ c. $\ln 77x(x-9)$ d. $\ln x^{11}(x-9)^7$

4. Convert -160° to radians and express the answer as a multiple of π

a. $\frac{-7\pi}{9}$ radians b. $\frac{-8\pi}{9}$ radians c. $\frac{-9\pi}{10}$ radians d. $\frac{-7\pi}{8}$ radians

5. The population of a particular country was 25 million in 1983; in 1993, it was 32 million. The exponential growth function $A = 25e^{kt}$ describes the population of this country t years after 1983. Use the fact that 10 years after 1983 the population increased by 7 million to find k to 3 decimal places.

a. 0.668 b. 0.195 c. 0.035 d. 0.025

6. During a road trip, Tony drove one-third the distance that Lana drove. Mark drove 24 more miles than Lana drove. The total distance they drove on the trip was 577 miles. How many miles did each person drive?

- a. Tony-71, Lana-213, Mark-237
b. Tony-65, Lana-195, Mark-317
c. Tony-79, Lana-237, Mark-261
d. Tony-711, Lana-237, Mark-213

7. Find the domain of the function: $f(x) = \log\left(\frac{x+5}{x-8}\right)$

a. $(-5, 8)$ b. $(8, \infty)$ c. $(-\infty, 5)$ d. $(-\infty, -5) \cup (8, \infty)$

8. A wheel with a 30-inch radius is marked at two points on the rim. The distance between the marks along the wheel is found to be 17 inches. What is the angle (to the nearest tenth degree) between the radii and the two marks?

- a. 28.5° b. 34.5° c. 32.5° d. 30.5°

9. Find the exact value of $\cos\left(\tan^{-1}\frac{7}{6}\right)$

- a. $\frac{\sqrt{85}}{6}$ b. $\frac{6\sqrt{85}}{85}$ c. $\frac{7}{6}$ d. $\frac{6}{85}$

10. The pH of a solution ranges from 0 to 14. An acid has a pH less than 7. Pure water is neutral and has a pH of 7. The pH of a solution is given by $\text{pH} = -\log x$, where x represents the concentration of the hydrogen ions in the solution in moles per liter. Find the pH if the hydrogen ion concentration is 5.4×10^{-14}

- a. 13.73 b. 13.27 c. 14.73 d. 14.27

11. Solve the following equation: $3x^2 = -12x - 2$

a. $\left\{ \frac{-12 - \sqrt{30}}{3}, \frac{-12 + \sqrt{30}}{3} \right\}$

b. $\left\{ \frac{-6 - \sqrt{30}}{6}, \frac{-6 + \sqrt{30}}{6} \right\}$

c. $\left\{ \frac{-6 - \sqrt{30}}{3}, \frac{-6 + \sqrt{30}}{3} \right\}$

d. $\left\{ \frac{-6 - \sqrt{42}}{3}, \frac{-6 + \sqrt{42}}{3} \right\}$

12. Solve the following expression for x : $125^x = \frac{1}{\sqrt{5}}$

- a. $-\frac{1}{3}$ b. -3 c. $-\frac{1}{6}$ d. $\frac{1}{6}$

13. To find the distance AB across a river, a distance BC of 1,257 meters is laid off on one side of the river. It is found that $B = 103.9^\circ$ and $C = 18.6^\circ$. Find AB. Round to the nearest meter.

- a. 475 meters b. 402 meters c. 399 meters d. 478 meters

14. Solve the following equation on the interval $[0, 2\pi)$: $\tan 2x - \tan x = 0$

- a. $\{0, \pi\}$ b. $\left\{\frac{\pi}{12}, \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{5\pi}{3}\right\}$ c. 0 d. $\left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$

15. Find the values for the variables so that the matrices are equal for: $\begin{bmatrix} x & y+8 \\ 2z & 1 \end{bmatrix} = \begin{bmatrix} 4 & 17 \\ 20 & 1 \end{bmatrix}$

- a. $x = 4, y = 9, z = 10$
b. $x = 1, y = 25, z = 40$
c. $x = 17, y = 1, z = 4$
d. $x = 4, y = 17, z = 20$

16. A painter needs to cover a triangular region 61 meters by 67 meters by 71 meters. A can of paint can cover 70 square meters. How many cans will be needed to paint the entire region?

- a. 308 cans b. 3 cans c. 14 cans d. 27 cans

17. Solve the following equation for x : $\ln \sqrt{x+5} = 2$

- a. $e^2 - 5$ b. $e^4 - 5$ c. $\frac{e^2}{2} + 5$ d. $e^4 + 5$

18. A straight trail with a uniform inclination of 18° leads from a lodge at an elevation of 1000 feet to a mountain lake at an elevation of 8800 feet. What is the length of the trail to the nearest foot?

- a. 28,477 feet b. 8,201 feet c. 9,253 feet d. 25,241 feet

19. From a boat on a lake, the angle of elevation to the top of the cliff is $35^\circ 38'$. If the base of the cliff is 1153 feet from the boat, how high is the cliff to the nearest foot?

- a. 826 b. 830 feet c. 840 feet d. 837 feet

20. A ceramics workshop makes wreaths, trees, and sleighs for sale at Christmas. A wreath takes 3 hours to prepare, 2 hours to paint, and 9 hours to fire. A tree takes 15 hours to prepare, 3 hours to paint, and 4 hours to fire. A sleigh takes 4 hours to prepare, 15 hours to paint, and 7 hours to fire. If the workshop has 123 hours of prep time, 77 hours for painting and 108 hours for firing available, how many of each item can be made?

- a. 7 wreaths, 3 trees, 7 sleighs
b. 8 wreaths, 7 trees, 4 sleighs
c. 3 wreaths, 7 trees, 6 sleighs
d. 7 wreaths, 6 trees, 3 sleighs

21. If $f(x) = \frac{2x+1}{x-3}$, find $f^{-1}(x)$

a. $f^{-1}(x) = \frac{3x+1}{x-2}$

b. $f^{-1}(x) = \frac{3x-1}{x+2}$

c. $f^{-1}(x) = \frac{2x-1}{x+3}$

d. $f^{-1}(x) = \frac{2x-1}{x-3}$

22. Perform the indicated operations and leave answer in polar form: $\left[2(\cos 10^\circ + i\sin 10^\circ)\right]^5$

a. $10\cos 10^\circ + 10i\sin 10^\circ$

b. $32\cos 50^\circ + 32i\sin 50^\circ$

c. $32\cos 10^\circ + 32i\sin 10^\circ$

d. $10\cos 50^\circ + 10i\sin 50^\circ$

23. A child is pulling a wagon with a force of 40 pounds. How much work is done in moving the wagon 60 feet if the handle makes an angle of 35° with the ground? Round answer to the nearest foot-pound.

a. 1255 ft-lb

b. 1966 ft-lb

c. 2066 ft-lb

d. 1746 ft-lb

24. Find the exact value of $\cos 22.5^\circ$.

a. $\frac{\sqrt{\sqrt{5}+5}}{2}$

b. $\frac{\sqrt{\sqrt{3}+3}}{3}$

c. $\frac{\sqrt{\sqrt{2}+3}}{2}$

d. $\frac{\sqrt{\sqrt{2}+2}}{2}$

25. The average cost per unit, y , of producing x units of a product is modeled by

$y = \frac{1,950,000 + 0.35x}{x}$. Describe the company's production level so that the average cost of producing each unit does not exceed \$6.85.

a. At least 300,000 units

b. At least 400,000 units

c. Not more than 400,000 units

d. Not more than 300,000 units

Statistics

Name: _____

Bonus Question #1:

1. Find the partial fraction decomposition for $\frac{x-18}{x(x-3)^2}$

Name: _____

Bonus Question #2

2. Psychologists can measure the amount learned, L , at time t using the model $L = A(1 - e^{-kt})$ where A represents the total amount to be learned and k is the learning rate. A student preparing for the SAT has 300 new vocabulary words to learn: $A = 300$. This particular student can learn 20 vocabulary words after 5 minutes: if $t = 5$, $L = 20$.

a. Find k , the learning rate, correct to 3 decimals

b. Approximately how many words will the student have learned in 20 minutes?

c. How long will it take the student to learn 260 words?

Name: _____

Bonus Question #3

3. Two fire-lookout stations are 10 miles apart, with station B directly east of station A. Both stations spot a fire. The bearing of the fire from station A is N 25° E and the bearing of the fire from station B is N 56° W. How far, to the nearest mile, is the fire from each lookout station?

Answers:

1. B

2. A

3. A

4. B

5. D

6. C

7. D

8. C

9. B

10. B

11. C

12. C

13. A

14. A

15. A

16. D

17. B

18. D

19. A

20. D

21. A

22. B

23. B

24. D

25. A

Bonus Question Solutions:

1. $-\frac{2}{x} + \frac{2}{x-3} - \frac{5}{(x-3)^2}$

2. a. 0.014

b. 73 words

c. about 144

3. Station A is about 6 miles from the fire and station B is about 9 miles from the fire.