

2012 ACTM Regional Geometry Exam

In each of the following please select the appropriate answer for each question and record the corresponding letter on the answer sheet. Any erasure on the answer sheet should be completely erased to avoid errors in scoring the exam. Answer each of the 25 questions prior to working on the Tie Breaker questions as they will be used only to break ties for first, second and/or third place. Figures are not necessarily drawn to scale.

1. Which of the following geometric relationships will be preserved by a rotation of the plane?

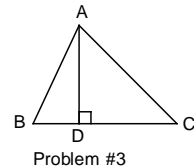
P: Area      Q: Betweenness of Points      R: Congruence  
 A. P, Q      B. P, R      C. Q, R      D. P, Q, R  
 E. None of these

2. Let  $\triangle PQR$  be an equilateral triangle. What rotation will transform  $\triangle PQR$  onto itself?

A. A rotation of  $60^\circ$       B. A rotation of  $120^\circ$       C. A rotation of  $180^\circ$   
 D. A rotation of  $90^\circ$       E. None of these

3. In  $\triangle ABC$ ,  $\overline{AD}$  is an altitude,  $AD = 12$ ,  $AC = 20$  and  $BC = 21$ . Find  $AB$ .

A. 13      B. 20      C.  $\sqrt{119}$   
 D. 16      E. None of these

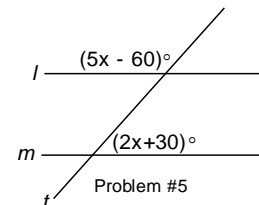


4. The number of lines of symmetry for a regular octagon is

A. 16      B. 4      C. 8      D. 12  
 E. None of these

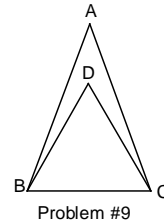
5. In the figure  $l$  and  $m$  are parallel lines and  $t$  is a transversal. The value of  $x$  is

A. 45      B. 30      C. 25  
 D. 15      E. None of these



6. Adjacent angles that are also congruent must be
- A. Right angles                      B. Supplementary angles
- C. Complementary angles            D. Acute angles
- E. None of these
7. Which statement best describes a point on the bisector of an angle of a triangle?
- A. It is interior to the triangle.
- B. It is on the opposite side of the triangle from the given vertex.
- C. It is equidistant from the lines containing the sides of the angle that is bisected.
- D. It lies on the median of the triangle drawn from the vertex.
- E. None of these.
8. To construct a circle that circumscribes a triangle one must first construct
- A. the perpendicular bisectors of two sides of the triangle
- B. two altitudes of the triangle
- C. two medians of the triangle
- D. the angle bisectors of two angles of the triangle
- E. None of these

9. In the figure,  $\overline{AB} \cong \overline{AC}$ , and  $\triangle BCD$  is an equilateral triangle. Which statement is true about the ray  $\overrightarrow{AD}$ ?

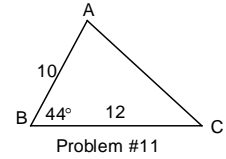


- P: It contains an altitude of  $\triangle ABC$ .
- Q: It contains a median of  $\triangle ABC$ .
- R: It bisects  $\angle BAC$ .

- A. P, Q            B. Q, R            C. P, R            D. P, Q, R
- E. None of these
10. A parallelogram always has
- A. diagonals that are perpendicular
- B. diagonals that are congruent
- C. diagonals that bisect each other
- D. diagonals that form a pair of acute angles and a pair of obtuse angles
- E. None of these

11. The area of the triangle at the right, to the nearest hundredth, is

- A. 41.68      B. 43.16      C. 41.70  
D. 50.02      E. None of these



12. A ladder placed on the side of a building makes an angle of  $62^\circ$  with the sidewalk. If the top of the ladder is 12 feet above the ground, how many feet, to the nearest hundredth, is the foot of the ladder from the building?

- A. 22.57 ft      B. 6.38 ft      C. 13.59 ft  
D. 10.59 ft      E. None of these

13.  $\triangle ABC$  and  $\triangle DEF$  are congruent right triangles. If the two triangles are rearranged so that a pair of congruent sides correspond exactly, which of the following geometric figures can be formed?

- P: rectangle      Q: parallelogram      R: isosceles triangle  
A. P, Q      B. Q, R      C. P, R      D. P, Q, R  
E. None of these

14. Let  $A = (-1, 2)$  and  $B = (5, -6)$  be points in the coordinate plane. Find the equation of the circle that has  $\overline{AB}$  for a diameter.

- A.  $(x - 2)^2 + (y + 2)^2 = 25$       B.  $(x + 2)^2 + (y - 2)^2 = 25$   
C.  $(x - 2)^2 + (y + 2)^2 = 100$       D.  $(x + 1)^2 + (y - 2)^2 = 100$   
E. None of these

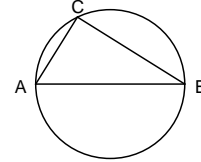
15. The slope of the line joining the points  $A = (2, -3)$  and  $B = (-3, -3)$  is

- A. undefined      B. zero      C.  $\frac{6}{5}$       D.  $\frac{1}{6}$   
E. None of these

16. The line that is perpendicular to and has the same y-intercept as  $2x - 3y = 6$  is

- A.  $2x + 3y = 6$       B.  $2x + 3y = -6$       C.  $4x - 6y = 12$   
 D.  $3x - 2y = -4$       E.  $3x + 2y = -4$

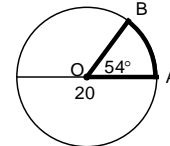
17. In the figure  $\overline{AB}$  is a diameter of the circle and  $\overline{AC}$  and  $\overline{BC}$  are chords. The angles  $\angle CAB$  and  $\angle CBA$  are



Problem #17

- A. Supplementary angles      B. Complementary angles  
 C. Opposite angles      D. Central angles  
 E. None of these

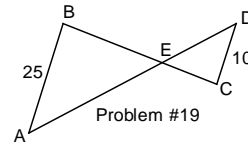
18. In a circle whose diameter is 20 feet, the measure of a central angle is  $54^\circ$ . The area of the sector AOB is



Problem #18

- A.  $60\pi$  sq ft      B.  $30\pi$  sq ft      C.  $15\pi$  sq ft  
 D.  $20\pi$  sq ft      E. None of these

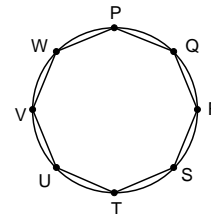
19. In the figure  $\overline{AB} \parallel \overline{CD}$ . If  $AB = 25$ ,  $CD = 10$  and  $BC = 28$  then  $CE =$



Problem #19

- A. 20      B. 12      C. 15      D. 8  
 E. None of these

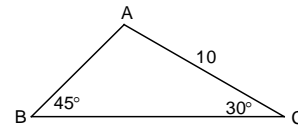
20. PQRSTU VW is a regular octagon inscribed in a circle. The measure of  $\angle PTS$  is



Problem #19

- A.  $135^\circ$       B.  $90^\circ$       C.  $67\frac{1}{2}^\circ$   
 D.  $45^\circ$       E. None of these

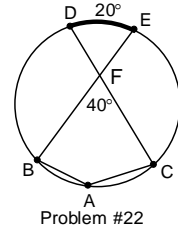
21. In  $\triangle ABC$ ,  $\angle ABC = 45^\circ$  and  $\angle ACB = 30^\circ$ . If  $AC = 10$ , then  $BC =$



Problem #21

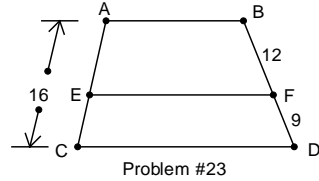
- A. Not enough information      B.  $10\sqrt{3}$       C.  $10 + 10\sqrt{3}$   
 D.  $5 + 5\sqrt{3}$       E. None of these

22. In the figure  $\angle BFC = 40^\circ$  and the minor arc DE measures  $20^\circ$ .  
The measure of  $\angle BAC$  is



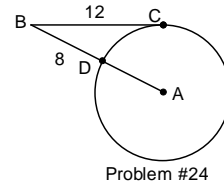
- A.  $100^\circ$
- B.  $150^\circ$
- C.  $120^\circ$
- D.  $140^\circ$
- E. None of these

23. In the figure  $\frac{AE}{CE} = \frac{BF}{DF}$ ,  $AC = 16$ ,  $BF = 12$  and  $DF = 9$ .  
Determine CE.



- A.  $9\frac{3}{5}$
- B.  $9\frac{1}{7}$
- C.  $6\frac{2}{5}$
- D.  $6\frac{6}{7}$
- E. None of these

24. In the figure  $\overline{BC}$  is tangent to the circle whose center is A.  
If  $BC = 12$  and  $BD = 8$ , then the radius, AC, of the circle is



- A. 5
- B. 13
- C. 10
- D. 8
- E. None of these

25. Consider the two statements:  
 “If a number n is divisible by 9, then n is divisible by 3.”  
 “The number n is divisible by 3.”

Based on these two statements, which of the following is a correct conclusion?

- P: n is divisible by 9
- Q: n is a multiple of 9
- R: n has a factor of 9
- A. P
- B. Q
- C. R
- D. P, Q, R
- E. None of these

Before you continue on to work the tie breaker questions determine if you have answered the previous 25 questions. The tie breaker questions will be used in the order given to break ties for first, second and/or third place.

### Tie Breaker Questions

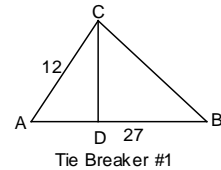
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The tie breaker questions will be used only in the event it is necessary to break ties between first, second and/or third place. Please give a clear concise answer for the question in the order given because they will be used in the given order to resolve ties.

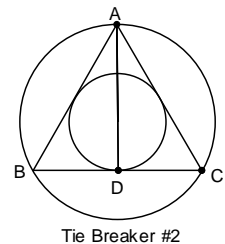
#### Tie Breaker #1

$\triangle ABC$  is a right triangle with right angle at  $C$ .  $\overline{CD}$  is an altitude.  $AC = 12$  and  $AB = 27$ . Find  $AD$ .



#### Tie Breaker #2

$\triangle ABC$  is an equilateral triangle with the inscribed and circumscribed circles as shown. If  $\overline{AD}$  is an altitude of the triangle and  $AD = 18$ , find the difference between the areas of the circumscribed circle and the inscribed circle.

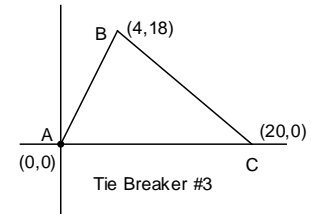


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### Tie Breaker #3

The triangle at the right is on a rectangular coordinate plane. Find the point of intersection of the medians of the triangle.



**Key - 2012**  
**Regional Geometry Exam**

- |     |   |     |   |
|-----|---|-----|---|
| 1.  | D | 14. | A |
| 2.  | B | 15. | B |
| 3.  | A | 16. | E |
| 4.  | C | 17. | B |
| 5.  | B | 18. | C |
| 6.  | E | 19. | D |
| 7.  | C | 20. | C |
| 8.  | A | 21. | D |
| 9.  | D | 22. | B |
| 10. | C | 23. | D |
| 11. | A | 24. | A |
| 12. | B | 25. | E |
| 13. | D |     |   |



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### Tie Breaker #1

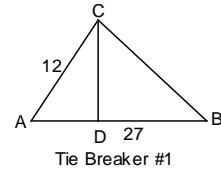
$\triangle ABC$  is a right triangle with right angle at C.  $\overline{CD}$  is an altitude.  $AC = 12$  and  $AB = 27$ . Find AD.

$AC = 12$  is the geometric mean between AD and 27.

$$27(AD) = 144$$

$$AD = 144/27$$

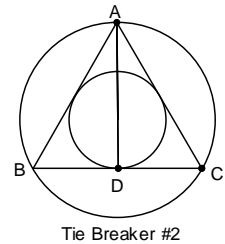
$$AD = 5 \frac{1}{3}$$



### Tie Breaker #2

$\triangle ABC$  is an equilateral triangle with the inscribed and circumscribed circles as shown. If  $\overline{AD}$  is an altitude of the triangle and  $AD = 18$ , find the difference between the areas of the circumscribed circle and the inscribed circle.

The center of the two circles is the centroid of the triangle. So the radius of the inscribed circle is 6 and the radius of the circumscribed circle is 12. So the difference in area is  $144\pi - 36\pi = 108\pi$



### Tie Breaker #3

The triangle at the right is on a rectangular coordinate plane. Find the point of intersection of the medians of the triangle.

Midpoint of BC is (12, 9)

Centroid is  $\frac{2}{3}$  of the distance from (0,0) to (12, 9)

$$x = 0 + (\frac{2}{3})(12) = 8$$

$$y = 0 + (\frac{2}{3})(9) = 6$$

So (8, 6) is the intersection of the medians of the triangle.

