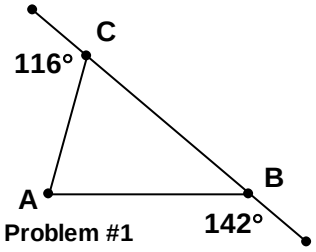
ACTM Geometry Exam
State – 2010

In each of the following select the answer and record the selection on the answer sheet provided. Note: Pictures are not necessarily drawn to scale.

1. The measure of ∠BAC in the given figure is

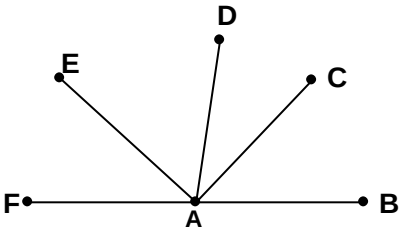


Problem #1

A. 54° **B. 88°** **C. 78°**

D. 38° **E. None of these**

2. In the figure, a linear pair of angles is



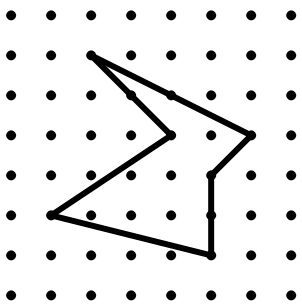
Problem #2

A. ∠BAC, ∠FAD **B. ∠BAE, ∠FAD**

D. ∠BAD, ∠FAE **D. ∠BAD, ∠EAD**

E. None of these

3. The area of the polygon on the grid is


Problem #3

A. 9½ **B. 11** **C. 10½**

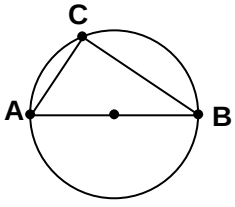
D. 10 **E. None of these**

4. A circle is drawn inside a rectangle so that it is tangent to three sides of the rectangle. If the dimensions of the rectangle are 2 and 3 then the ratio of the circumference of the circle to the perimeter of the rectangle is

A. π **: 5** **B.** π **: 6** **C.** π **: 10**

D. Not enough information **E. None of these**

5. The diameter, $\overline{AB}$**, of the circle is 10. What are the possible lengths of** $\overline{AC}$ **and** $\overline{BC}$**?**



Problem #5

A. $6,\ 4\sqrt{5}$ **B.** $5,\ 5\sqrt{3}$ **C.** $4,\ 3\sqrt{21}$

D. $3,\ 2\sqrt{26}$ **E. None of these**

6. A tangent to a circle is always perpendicular to the radius of the circle drawn to the point of tangency. What is the slope of the tangent to the circle whose equation is $(x-2)^2 + (y-1)^2$ at the point (6,4)?

A. $-\frac{4}{3}$ **B.** $\frac{3}{4}$ **C.** $\frac{4}{3}$ **D.** $-\frac{3}{4}$

E. None of these

7. The equation of the line that is parallel to $2x - 3y = 6$, with a y-intercept that is four more than the y-intercept of the given line is

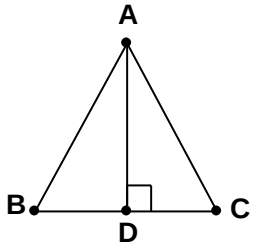
A. $2x - 3y = 10$ **B.** $2x - 3y = 24$ **C.** $2x - 3y = -6$

D. $3x - 2y = -4$ **E.** None of these

8. The points A = (–1, –3), B = (4, –1), C = (7, 3) and D = (2, 1) are the vertices of a parallelogram. The point of intersection of the diagonals of ABCD is

A. (4, 0) **B.** (0, 4) **C.** (0,3) **D.** (3, 0)

E. None of these



Problem #9

9. $\overline{AD}$ is the perpendicular bisector of $\overline{BC}$. Which of the following statements is/are true?

P: $\overline{AB} \cong \overline{AC}$ **Q:** $\Delta ABD \cong \Delta ADC$ **R:** $\angle BAD \cong \angle DAC$

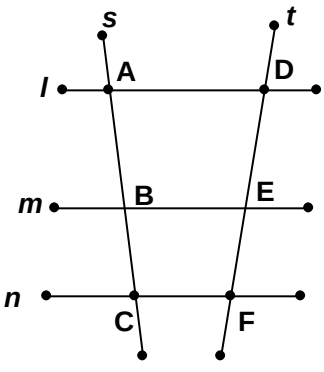
A. P, Q **B.** P, R **C.** Q, R **D.** P, Q, R

E. None of these

10. In the figure $l \parallel m \parallel n$ and s and t are transversals. If AB = 2x + 3, BC = 6x + 1, DE = x + 4 and EF = 3x + 2 then the ratio of AB to AC is

A. $\frac{2}{5}$ **B.** $\frac{3}{5}$ **C.** $\frac{5}{8}$ **D.** $\frac{8}{5}$

E. None of these



Problem #10

11. Consecutive angles of the quadrilateral PQRS are supplementary. Which of the following always describes PQRS?

A. PQRS is a rectangle **B. PQRS is an isosceles trapezoid**

C. PQRS is a kite **D. PQRS is a parallelogram**

E. None of these

12. A quadrilateral ABCD is inscribed in a circle. Which of the following statements is true?

A. ABCD is a parallelogram
B. Opposite pairs of angles are complementary
C. ABCD is an isosceles trapezoid
D. ABCD is a kite
E. None of these

13. PQRSTUVWX is a regular polygon. The measure of the angle $\angle$PRQ is

A. 140° **B. 40°** **C. 20°** **D. 36°**

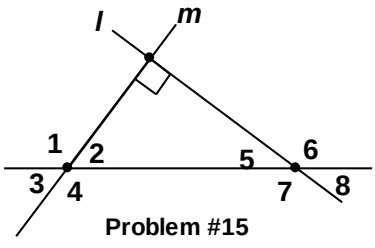
E. None of these

14. An equilateral triangle and a regular hexagon are inscribed in a circle. The ratio of the perimeter of the hexagon to the perimeter of the triangle is

A. 2 : 1 **B. 2 : $\sqrt{3}$** **C. 3 : $\sqrt{2}$**

D. Not enough information **E. None of these**

15. In the figure *l* $\perp$ *m*. A pair of complementary angles is



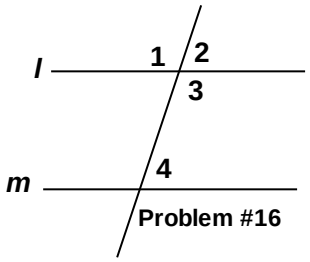
Problem #15

A. $\angle$1, $\angle$5 **B. $\angle$2, $\angle$3**

C. $\angle$3, $\angle$7 **D. $\angle$3, $\angle$8**

E. None of these

16. In the figure if $l \parallel m$, $m\angle 2 = x°$, $m\angle 3 = y°$, and $m\angle 4 = 8x - 3y)°$, then x =

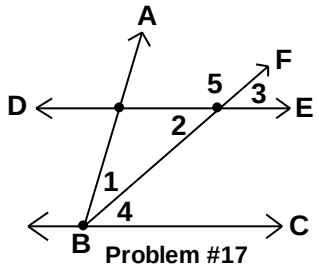


Problem #16

A. 54° B. 135° C. 45°

D. 108° E. None of these

17. Given that $\overleftrightarrow{DE} \parallel \overleftrightarrow{BC}$ and $\angle 1 \cong \angle 3$. Which of the following statements is/are true?



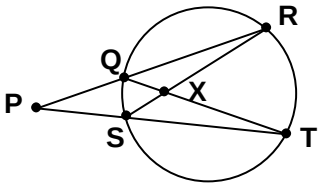
Problem #17

P: $\angle 2 \cong \angle 3$
Q: $\overrightarrow{BF}$ bisects $\angle ABC$
R: $\angle 4$ and $\angle 5$ are supplementary

A. P, Q B. P, R C. Q, R D. P, Q, R

E. None of these

18. In the figure $m\angle QPS = 12°$ and $m\angle PRS = 20°$. Then $m\angle QXS$ =

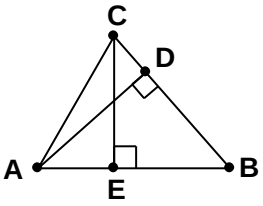


Problem #18

A. 52° B. 20° C. 64°

D. 40° E. None of these

19. In ΔABC $\overline{AD}$ and $\overline{CE}$ are altitudes. If AB = 40, BC = 37, and CE = 12, then, to the nearest hundredth, AD =

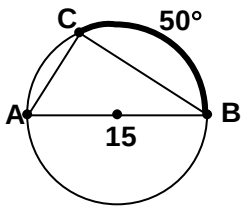


Problem #19

A. 6.49 B. 12.97 C. 11.10

D. 5.55 E. None of these

20. In the circle if $\overline{AB}$ is a diameter, AB = 15 and the measure of the arc BC is 50°, then AC =



Problem #20

A. 12.40 B. 6.34 C. 13.59

D. 12.29 E. None of these

21. A, B, C and D are four distinct points on a line. Using these 4 points, how many segments can be named?

A. 12 **B.** 8 **C.** 3 **D.** 9

E. None of these

22. The number of regular solids that have triangular faces is

A. 5 **B.** 4 **C.** 3 **D.** 2

E. None of these

23. A prism and a pyramid have the same number of edges. If the base of prism is an octagon then the number of vertices in the pyramid is

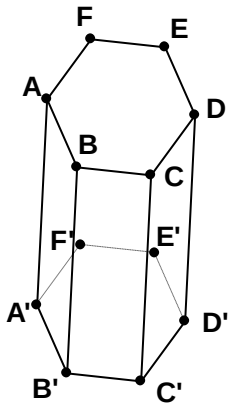
A. 12 **B.** 13 **C.** 14 **D.** 16

E. None of these

24 The right prism in the figure has a regular hexagonal base. If a side of the base is 6 and the height of the pyramid is 5, then AD' =

A. 13 **B.** 12 **C.** $\sqrt{61}$

D. Not enough information **E.** None of these

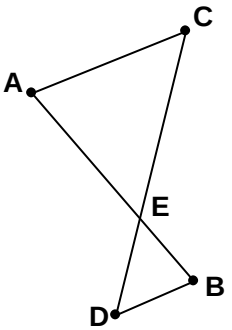


Problem #24

25. In the figure $\angle ACE \cong \angle DBE$, AE = 25 and DE = 10. If AC = 15 then BD =

A. 8 **B.** 9 **C.** 5

D. 6 **E.** None of these



Problem #25

Be sure that you have answered all 25 of the preceding questions before attempting the tie-breaker problems. They will only be used to break ties for first, second and third place. They will be used in the order they are given.

Tie Breaker Problems

Name_____ **School**_____

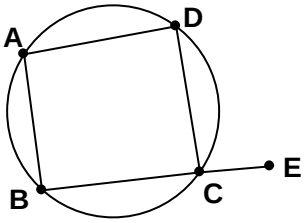
[Please Print] **[Please Print]**

Tie-Breaker #1

ABCD is an inscribed quadrilateral and B, C and E are collinear.

Prove: $\angle$**DAB** $\cong$ $\angle$**DCE**

[Be sure to state clear reasons for each statement in the proof.]

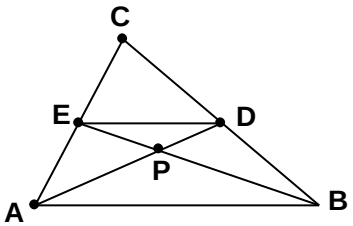


Tie Breaker #1

Tie-Breaker #2

In Δ**ABC,** $\overline{AD}$ **and** $\overline{BE}$ **are medians intersecting at P. Determine the following ratio. Justify your conclusion.**

$$\frac{\text{AREA}(\Delta PED)}{\text{AREA}(\Delta PAB)}$$



Tie Breaker #2

Name_____ **School**_____

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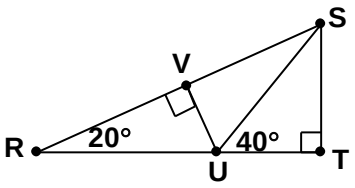
Tie-Breaker #3

In the figure ΔRST is a right triangle, m$\angle$SUT = 40°, m$\angle$SRT = 20° and ST = 45. If $\overline{UV} \perp \overline{RS}$, find UV.



Tie Breaker #3

Geometry – 2010
Key

1. C
2. E
3. A
4. A
5. B
6. A
7. C
8. D
9. B
10. C
11. D
12. E
13. C
14. B
15. D
16. A
17. D
18. A
19. B
20. C
21. A
22. C
23. B
24. A
25. D

KEY

Tie Breaker Problems

Name_____ **School**_____

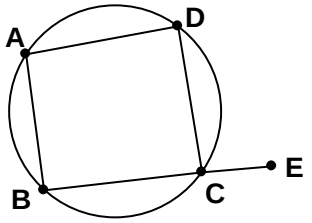
[Please Print] **[Please Print]**

Tie-Breaker #1

ABCD is an inscribed quadrilateral and B, C and E are collinear.

Prove: $\angle DAB \cong \angle DCE$

[Be sure to state clear reasons for each statement in the proof.]



Tie Breaker #1

Since opposite angles of an inscribed quadrilateral are supplementary then $\angle DAB$ and $\angle BCD$ are supplementary [Students may use an inscribed angle argument here]

Since $\angle DCE$ and $\angle BCD$ are a linear pair of angles they are supplementary.

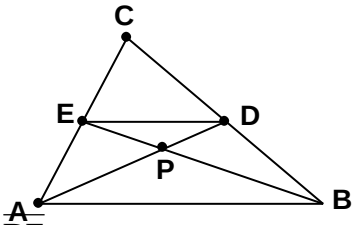
Therefore, $\angle DAB \cong \angle DCE$, since both are supplementary to $\angle BCD$.

Tie-Breaker #2

In ΔABC, $\overline{AD}$ and $\overline{BE}$ are medians intersecting at P.

Determine the following ratio. Justify your conclusion.

$$\frac{\text{AREA}(\Delta PED)}{\text{AREA}(\Delta PAB)}$$

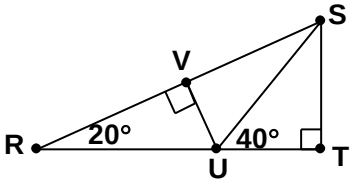


Tie Breaker #2

Since D and E are midpoints of their respective sides $\overline{AB} \parallel \overline{DE}$. So $\angle DEP \cong \angle ABP$ and $\angle EDP \cong \angle BAP$ by alternate interior angle theorem. Thus, $\Delta DEP \sim \Delta ABP$. Since medians intersect at a point that is two-thirds of the distance from the vertex to the midpoint of the opposite side, AP = 2PD. So the proportionality constant between the triangles is 2. Therefore, $\frac{\text{AREA}(\Delta PED)}{\text{AREA}(\Delta PAB)} = \frac{1}{4}$**.**

Tie-Breaker #3

In the figure ΔRST is a right triangle, $m\angle SUT = 40°$, $m\angle SRT = 20°$ and ST = 45. If $\overline{UV} \perp \overline{RS}$, find UV.



Tie Breaker #3

$\tan 20^\circ = \frac{45}{RT}$ **and** $\tan 40^\circ = \frac{45}{UT}$**. Thus,** $RT = \frac{45}{\tan 20^\circ}$ **and** $UT = \frac{45}{\tan 40^\circ}$**.**

RU = RT – UT and UV = RU sin 20°. Therefore, UV = ($\frac{45}{\tan 20^\circ}$ **-** $\frac{45}{\tan 40^\circ}$**)sin 20°.**

Therefore, UV = 23.94.