## 2010 ACTM Regional Geometry Exam

In each of the following choose the best answer and bubble the corresponding letter on the answer sheet provided. Be sure to work all 25 questions before attempting the tie-breaker problems. Be aware that the figures are not always drawn to scale.

- 1. Which of the following sets of numbers cannot be the lengths of the sides of a triangle?
  - **A.**  $\{2\frac{1}{2}, 3\frac{1}{4}, 5\frac{1}{3}\}$  **B.**  $\{6\frac{1}{4}, 3\frac{1}{3}, 9\frac{1}{2}\}$  **C.**  $\{3.7, 8.5, 12.3\}$

**D.**  $\{4.3, 7.3, 5.8\}$  **E.** All can be sides of a triangle

- 2. A vertical pair of angles are complementary. The measure of the other pair of angles is
  - A. 135° B. 145° C. 125°
  - D. Not enough information E. None of these
- **3.** Below are four figures made of squares. If the pattern continues, how many squares would make up figure 8?



- A. 16 B. 90 C. 56 D. 72
- E. None of these
- 4. In the figure the coordinate of the point that is two-fifths of the distance from A to B is A Problem 4 B

4



E. None of these

- 5. In  $\triangle ABC$ ,  $\overline{AB} \cong \overline{AC}$ . If m $\angle BAC$  is three times m $\angle ABC$ , then m $\angle BAC$  is
  - A. 36° B. 72° C. 108°
  - D. Not enough information E. None of these
- 6. In  $\triangle ABC$  two of the medians are equal in length. Which of the following statements is always true?
  - **A.**  $\triangle$ **ABC** is an equilateral triangle **B.**  $\triangle$ **ABC** is an acute triangle
  - C.  $\triangle ABC$  is a right triangle D.  $\triangle ABC$  is an obtuse triangle

D

Problem 7

в

C

B

Problem 8

С

10

Ε

30

D٩

- E. None of these is always true
- 7. In  $\triangle ABC$ , D and E are midpoints of the sides AB and AC, respectively. The ratio of the area of  $\triangle ADE$  to the area of the quadrilateral BCED is
  - A. 1 to 4 B. 1 to 2
  - C. 2 to 3 D. 1 to 3
  - E. None of these
- 8.  $\overline{AB}$  and  $\overline{CD}$  intersect at E so that AE = 8, CE = 10, AB = 32 and CD = 40. If AC = 15, then BD =
  - A. 36 B. 45 C. 40

D. Not enough information E. None of these

- 9. In rectangle ABCD the length is twice the width. If the perimeter of the rectangle is 72 inches then the area of the rectangle ABCD is
  - A. 1152 in<sup>2</sup> B. 323 in<sup>2</sup> C. 576 in<sup>2</sup>
  - **D.**  $288 \text{ in}^2$  **E.** None of these

- 10. A 48-inch piece of wire is bent to form an equilateral triangle. A median of the triangle measures
  - **A.** 16 in **B.** 12 in **C.**  $8\sqrt{3}$  in **D.**  $8\sqrt{2}$  in
  - E. None of these
- 11.  $\triangle ABC \sim \triangle DEF$ . If AB = 10, DE = 15 and the perimeter of  $\triangle ABC$  is 35, then the perimeter of  $\triangle DEF$  is
  - A. 78.75 B. 62.5 C. 75.25
  - D. 52.5 E. None of these

12. Opposite angles of a quadrilateral inscribed in a circle are supplementary. If the measures of two of the angles of an inscribed quadrilateral are 115° and 75°, then the measures of the other two angles are

A. 65° and 95° B. 65° and 105° C. 105° and 55°

D. Not enough information E. None of these

- 13. In the figure  $I \parallel m \parallel n$  and  $s \parallel t$ . With the angle measures as indicated determine the value of x.
  - A. 40 B. 50 C. 60
  - D. 80 E. None of these



14. Two wires are attached to a pole and then to a point on the ground that is 8 feet from the base of the pole. The lengths of the wires are 10 ft and 17 ft. Assuming the pole is perpendicular to the ground, the distance between the points the two wires are attached to the pole is



- A. 9 ft B. 15 ft C.  $3\sqrt{21}$  ft D. 11 ft
- E. None of these

**15.**  $\triangle$ **ABC** is a right triangle with right angle at C and CD is the altitude to the hypotenuse. If the sides of the triangle are 15, 20, 25, then CD =

**A.** 12 **B.**  $5\sqrt{3}$  **C.**  $4\sqrt{5}$  **D.**  $5\sqrt{5}$  **E.** None of these

- 16. The sum of the measures of the interior angles of a regular polygon is 1080°. An exterior angle of the polygon has a measure of
  - A.
     60°
     B.
     45°
     C.
     72°
     D.
     135°

     E.
     None of these
- 17. ABCD is a parallelogram with one interior angle measuring 42°. The exterior angles of ABCD measure
  - A. Not enough information B. 42° and 128°
  - C. 42° and 128° D. 42° and 138° E. None of these
- 18. The lengths of the three sides of  $\triangle ABC$  are 132, 475, 500.  $\triangle ABC$  is a(n) \_\_\_\_\_\_ triangle
  - A. acute B. right C. obtuse D. oblique E. None of these
- Δ In  $\triangle ABC$ , m $\angle ABC = 60^{\circ}$  and m $\angle ACB = 45^{\circ}$ , AB = 20 19. and  $\overline{AD}$  is an altitude. The perimeter of  $\triangle ABC$ , to the nearest hundredth is 20 Α. 69.68 В. 71.80 C. 71.81 в<u>∕6</u>0° 45° С D Problem 19 D. 71.82 Ε. None of these Q In trapezoid PQRS,  $\overline{UV} \parallel \overline{RS}$ . If QV = 9, QR = 15 20. and PS = 20 then SU = U



E. None of these

S

Problem 20

R

21. The slope of the line that is perpendicular to the line 2x + 3y = 6 is

**A.**  $-\frac{2}{3}$  **B.**  $\frac{2}{3}$  **C.**  $-\frac{3}{2}$  **D.** 3 **E.** None of these

- 22. The length of the segment determine by the x-intercept and y-intercept of 2x 3y = 12 is
  - **A.**  $2\sqrt{5}$  **B.**  $2\sqrt{13}$  **C.**  $5\sqrt{2}$ **D.**  $4\sqrt{5}$  **E.** None of these
- 23. Let A = (1,2) and B = (5,8) be points in the rectangular coordinate system. The equation of the circle whose diameter is  $\overline{AB}$  is
  - A.  $(x-3)^2 + (y-5)^2 = 34$  B.  $(x-3)^2 + (y-5)^2 = 13$
  - C.  $(x-2)^2 + (y-3)^2 = 34$  D.  $(x-2)^2 + (y-3)^2 = 13$

#### E. None of these

24.  $\triangle XYZ$  is a right triangle with right angle at Z. If m $\angle XYZ$  = 40° and the hypotenuse is 15, then to the nearest thousandth XZ =



Problem 25

E. None of these

## **Tie Breaker Questions - Geometry**

Name\_\_\_

[Please print]

School

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Be sure you have answer questions 1 - 25 before attempting the tie breaker questions. These questions will be used only in the event there is a tie for first, second or third place. The questions will be read in the given order; i.e. #1 will be used first to break a tie, then #2, and then #3.

Tie Breaker #1

In  $\triangle ABC$ ,  $\overline{AB} \cong \overline{AC}$  and  $\overline{AD}$  bisects  $\angle BAC$ . Prove:  $\angle ABC \cong \angle ACB$ . Be sure to give reasons for your conclusions.



Tie Breaker #2 In the rectangular coordinate system P = (7, 0) and Q = (-1, 10). Find the y-intercept of the perpendicular bisector of  $\overline{PQ}$ .

| Name           | School       |     |
|----------------|--------------|-----|
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| Tie Breaker #3 |              |     |

In the figure  $\overline{AD} \perp \overline{AB}$ ,  $\overline{BC} \perp \overline{AB}$ , and  $\overline{EF} \perp \overline{AB}$ . If BC = 10, AB = AD = 20 determine the ratio  $\frac{AF}{BF}$ .



## Key Regional Geometry 2010

| 1.  | C | 13. | С |
|-----|---|-----|---|
| 2.  | Α | 14. | A |
| 3.  | D | 15. | A |
| 4.  | Α | 16. | В |
| 5.  | C | 17. | D |
| 6.  | E | 18. | С |
| 7.  | D | 19. | D |
| 8.  | В | 20. | A |
| 9.  | D | 21. | E |
| 10. | C | 22. | В |
| 11. | D | 23. | В |
| 12. | В | 24. | D |
| 25. | В |     |   |

## Tie Breaker #1 – Key

In  $\triangle ABC$ ,  $\overline{AB} \cong \overline{AC}$  and  $\overline{AD}$  bisects  $\angle BAC$ . Prove:  $\angle ABC \cong \angle ACB$ . Be sure to give reasons for your conclusions. B D Tie Breaker #1

Since  $\overline{AD}$  bisects  $\angle BAC$ , then  $\angle BAD \cong \angle CAD$ .  $\overline{AD} \cong \overline{AD}$  and  $\overline{AB} \cong \overline{AC}$ . Therefore by SAS

 $\triangle ABD \cong \triangle ACD$ . Therefore  $\angle ABD \cong \angle ACD$  (by corresponding parts of congruent triangles0. So  $\angle ABC \cong \angle ACB$ . [Note: The student should not just state the Isosceles Triangle theorem. It is important that the student give the proof of this theorem.]

## Tie Breaker #2 In the rectangular coordinate system P = (7, 0) and Q = (-1, 10). Find the y-intercept of the perpendicular bisector of $\overline{PQ}$ .

Slope( $\overline{PQ}$ ) =  $\frac{-10}{8} = \frac{-5}{4}$ . Slope(Perpendicular) =  $\frac{4}{5}$ . Midpoint of  $\overline{PQ}$  = (3,5). So equation of perpendicular bisector is y =  $\frac{4}{5}$ x + b. So 5 =  $\frac{4}{5}$ (3) + b. Thus b =  $\frac{13}{5}$ .

# Tie Breaker #3 In the figure $\overline{AD} \perp \overline{AB}$ , $\overline{BC} \perp \overline{AB}$ , and $\overline{EF} \perp \overline{AB}$ . If BC = 10, AB = AD = 20 determine the ratio $\frac{AF}{BF}$ . $m\angle BAC = 30^{\circ}$ and $m\angle ABD = 45^{\circ}$ . Let AF = x and BF = y. Then EF = $\frac{x}{2}$ = y. Therefore, $\frac{x}{y} = 2$ . So $\frac{AF}{BF} = 2$ .

20 F Tie Breaker #3