# Arkansas Council of Teachers of Mathematics

## Algebra II: State Contest 2011

For questions 1 through 26, mark your answer choice on the answer sheet provided. After completing items 1 through 26, answer each of the tie-breaker items in sequential order (do #1 first, followed by #2, and then #3 last). Be sure that your name is printed on each of the tie-breaker pages.

1. If $a \neq 0$, then $a^4 a^3 a^{-7} =$

   A. 0 B. 1 C. $a^5$ D. Undefined

2. If $bx^2 + cx + a = 0$, the discriminant is

   A. $a^2 - 4bc$ B. $b^2 - 4ac$ C. $c^2 - 4ab$ D. None of these

3. If $A = \begin{bmatrix} (2-x) & -1 \\ (4-x) & 0 \end{bmatrix}$ and $|A| = 5$, then x =

   A. - 1 B. 0 C. 1.5 D. 2

4. If $x^2 - 3x + 7 = 0$, then *x* =

   A. -1, -7 B. $\frac{3 \pm \sqrt{37}}{2}$ C. $\frac{3 \pm i\sqrt{19}}{2}$ D. -1, -3

5. If $x \neq 0$, then $x^0 + x^{-1} =$

   A. -1 B. $\frac{1}{x}$ C. $\frac{2}{x+1}$ D. $\frac{x+1}{x}$

6. Find the solution to the system

$$\begin{cases} 2x + 4y - 10z = -2 \\ 3x + 9y - 21z = 0 \\ x + 5y - 12z = 1 \end{cases}$$

   A. $(1, 2, -3)$ B. $(3, -1, -2)$ C. $(2, -1, 3)$ D. $(-2, 3, 1)$

7. The domain of $f(x) = \log(x - 2)$ is

   A. $(-\infty, \infty)$ B. $[2, \infty)$ C. $(2, \infty)$ D. $(0,2) \cup (2, \infty)$

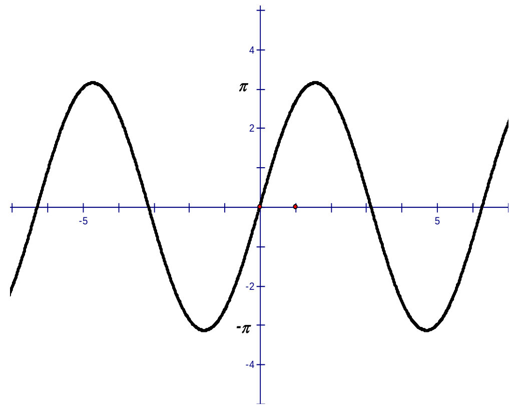
8. Find the range of the function $f(x) = -3 * x^{-2}$

   A. $(-3, \infty)$ B. $(-\infty, 0)$ C. $(-2, \infty)$ D. $(-\infty, -2)$

9. Let $f(x) = \lceil x \rceil$ where $\lceil x \rceil$ = the smallest integer that is greater than or equal to $x$. $f(-2.3) =$

A. $-3$ B. $-2$ C. $-1$ D. Does not exist

10. Find the function with the given graph




A. $f(x) = \pi \cos x$

B. $f(x) = \cos(\pi x)$

C. $f(x) = \pi \sin x$

D. $f(x) = \sin(\pi x)$

11. The regression line associated with the data shown below is

| | | | | |
|---|---|---|---|---|
| Number of hours of cell phone use (*x*) | 2 | 21 | 18 | 9 |
| Grade on Algebra II exam (*y*) | 95 | 61 | 70 | 83 |

A. $y = -1.71x + 98.61$
B. $y = -1.53x + 87.62$
C. $y = -1.37x + 69.78$
D. $y = -0.59x + 47.36$

12. $x^2 + x + 2 = (x+b)^2 + \frac{7}{4}$ where b =

A. $\frac{1}{2}$ B. $\frac{1}{4}$ C. $-\frac{1}{4}$ D. $\frac{1}{8}$

13. $\frac{3}{x-2} - \frac{1}{2-x} =$

A. $-4$ B. $\frac{2}{x^2-4}$ C. $\frac{4(1-x)}{x-2}$ D. $\frac{4}{x-2}$

14. Given $f(x) = x - 3$ and $g(x) = 1 - x^2$, find $(g \circ f)(x)$ and simplify.

A. $-x^2 + 6x - 8$
B. $x^2 - 6x + 10$
C. $-x^2 - 6x + 10$
D. $x^2 - 6x - 8$

15. Find the inverse function of $f(x) = 3x^2 + 1$ is

A. $f^{-1}(x) = \sqrt{\frac{x-1}{3}}$

C. $f^{-1}(x) = \sqrt{3(x+1)}$

B. $f^{-1}(x) = \sqrt[3]{\frac{x-1}{2}}$

D. $f^{-1}$ does not exist

16. $\sqrt{a}\sqrt[3]{b} =$

A. $b\sqrt{ab}$ B. $\sqrt[3]{a^2b}$ C. $\sqrt[4]{ab}$ D. $\sqrt[6]{a^3b^2}$

17. Let $f(x) = \frac{5}{x\sqrt{x^2-9}}$ . The domain of $f$ is

A. $(-\infty, 0) \cup (0, \infty)$

C. $(-3,0) \cup (0,3)$

B. $(-\infty, -3) \cup (3, \infty)$

D. $(-3, 3)$

18. Assume that women's shoe sizes are normally distributed with a mean size of 7 and a standard deviation of 1.5 . In a group of 500 women, approximate the number of women with shoe sizes of 8.5 or less.

A. 420 B. 370 C. 284 D. 170

19. Suppose that you have a system of equations of the form

$$\begin{Bmatrix} y = ax + b \\ y = cx^2 + dx + e \end{Bmatrix}.$$ Then the number of solutions could be at most

A. 0 B. 1 C. 2 D. 3

20. Find the value of $b$ that will give the interval $\left[-\frac{1}{4}, \frac{5}{4}\right]$ as the solution set for the inequality $|2x - 1| \leq b$

A. 1 B. 1.5 C. 2 D. 2.5

21. If $\log_2(x-1)+\log_2(x)=2$, then x =

A. $\frac{1\pm\sqrt{17}}{2}$  B. $\frac{3}{2}$  C. $\frac{5}{2}$  D. $\frac{1+\sqrt{17}}{2}$  E. No solution

22. If a principal *P* is invested at an annual rate *r* compounded continuously, then the amount *A* in the account at the end of *t* years is given by $A=Pe^{rt}$ , where *r* is expressed as a decimal. Using this continuous compound interest formula, how many years, to the nearest year, will it take an initial principal to double if it is invested at an annual rate of 1.20% compounded continuously?

A. 6 years  B. 52 years  C. 58 years  D. 65 years  E. 75 years

23. The area of an ellipse is calculated using the formula $A=\pi * a * b$, where *a* is half the length of the major axis of the ellipse and b is half the length of the minor axis of the ellipse. What is the area enclosed by $9x^2+25y^2+72x-250y+544=0$ ?

A. 9 $\pi$  B. 15 $\pi$  C. 25 $\pi$  D. 60 $\pi$  E. 225 $\pi$

24. Find the area of the region in the first quadrant that is bounded by the lines $x=0,\ y=0,\ x=c, and\ y=kx+b\ where\ c\ >0, b>0, and\ k>0.$

A. $ckx+cb$  C. $\frac{2bc+kc}{2}$  E. $\frac{2b+kc^2}{2}$

B. $\frac{cb+kc}{2}$  D. $\frac{2bc+kc^2}{2}$

25. Which of the following is the sum of the tens digit and the units digit of the solution of the following equation? $15-2\sqrt{x-18}=0.5|x+14|-20$

A. 6  B. 9  C. 10  D. 11  E. 12

26. If $f(x)=\sqrt{x+3}$ and $g(x)=\sqrt{5-x}$, then the domain of $g\big(f(x)\big)$ is:

A. $(-\infty,22]$  B. [-3, $\infty$)  C. $(-\infty,5]$  D. $[-3,5]$  E. $[-3,22]$

ALGEBRA II (Tie-breakers) Student Name ________________________

2011 State Contest Student School ________________________

Note: The tie-breaker questions are graded in order so it is important that you work tie-breaker #1 before going on to #2, etc.

Tie-breaker Question #1: Solve the radical equation $\sqrt{5x+21} = x+3$ .

Tie-breaker Question #2: Write a polynomial in expanded form $a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$ of least degree with zeros of 2 and $3 + i$ .

Student Name ________________________ Student School ________________________

Tie-breaker Question #3:

A daredevil named Hot Rod has decided to strap himself to a rocket that will reach a maximum height and then come crashing down to the earth. Hot Rod has rigged a system that will allow him to eject from the rocket and then parachute to the ground.

a. Hot Rod has tested the rocket appropriately, and has found that the rocket reaches a maximum height of 576 feet above the ground after 6 seconds. Let the quadratic function *d(t)* represent the rocket's height above ground level (in feet) *t* seconds after the rocket is fired. Find *d(t).*

b. Hot Rod determines that he needs to reach a minimum height of 500 feet to parachute safely to the ground and that it takes him a maximum of 3 seconds to safely eject from the rocket. Report the time interval (round values to two decimal places) that the rocket will be at least 500 feet above the ground. Also, mathematically support your conclusion to the following question: Will Hot Rod be at least 500 feet above the ground long enough to safely eject?

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Multiple Choice

1. B
2. C
3. A
4. C
5. D
6. D
7. C
8. B
9. B
10. C
11. A
12. A
13. D
14. A
15. D
16. D
17. B
18. A
19. C
20. B
21. D
22. C
23. B
24. D
25. D
26. E

Tie-breakers

# 1: $\sqrt{5x+21} = x+3$ → 5x+21 = $x^2$ + 6x + 9

0 = $x^2$ + x - 12

0 = (x + 4)(x – 3)

x = - 4 is extraneous

$\boxed{x = 3 \text{ is the only solution}}$

# 2: if 3 + *i* is a zero, then so is 3 - *i* .

So the three zeros are 3 ± *i* and 2.

$$0 = (x - (3+i))(x - (3 - i))(x - 2)$$

$$0 = (x-3-i)(x-3+i)(x-2)$$

$$0 = (x^2 - 3x + ix - 3x + 9 - 3i - ix + 3i + 1)(x-2)$$

$$0 = (x^2 - 6x + 10)(x-2)$$

$$0 = (x^3 - 2x^2 - 6x^2 + 12x + 10x - 20)$$

$$0 = x^3 - 8x^2 + 22x - 20$$

The polynomial in expanded form with zeros of 2, 3 ± *i* is: $\boxed{x^3 - 8x^2 + 22x - 20}$

#3:

**a)** Because the quadratic distance function has a maximum value of 576 at t = 6, the vertex form for $d(t)$ is:

$d(t) = a(t-6)^2 + 576.$

$d(0) = 36a + 576$

Because the rocket has an initial height of 0 ft at time t = 0 seconds, $d(0) = 36a + 576 = 0$

so $36a = -576 \rightarrow a = -16$

Thus, $\boxed{d(t) = -16(t-6)^2 + 576 = -16t^2 + 192t}$

**b)** $-16t^2 + 192t \geq 500$

$-16t^2 + 192t - 500 \geq 0$

$4t^2 - 48t + 125 \leq 0$

Zeros:

$$t = \frac{48 \pm \sqrt{(-48)^2 - 4*4*125}}{2*4}$$

$$\text{t} = \frac{48 \pm \sqrt{304}}{8} = \frac{48 \pm 4\sqrt{19}}{8} = \frac{12 \pm \sqrt{19}}{2}$$

so the zeros are at $t \approx 3.82,\ 8.18$

Using test intervals or a variety of graphical methods, the students should find that:

<u>the rocket will be at least 500 feet above the ground for the time interval 3.82 ≤ t ≤ 8.17 seconds.</u>.

$8.17 - 3.82 = 4.35 > 3\ seconds$; thus, <u>yes, Hot Rod will have enough time to safely eject from the rocket.</u>