ACTM Regional Math Contest
Pre-Calculus/Trig 2011

Select the best answer for each of the following questions and mark it on the answer sheet provided. Be sure to read all of the answer choices before making your selection. When you are finished with the multiple-choice, attempt the tiebreaker questions.

1. Find the point whose rectangular coordinates are the same as the point whose polar coordinates are $(-1, -\pi/3)$

A. $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ B. $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ C. $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ D. $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

2. The terminal side of an angle passes through the point $\left(-3, \sqrt{6}\right)$. Find the cosecant of the angle.

A. $\frac{\sqrt{10}}{2}$ B. $-\frac{\sqrt{15}}{3}$ C. $-\frac{\sqrt{6}}{3}$ D. $\frac{\sqrt{10}}{5}$

3. Identify the reference angle for an angle whose measure is $\frac{94\pi}{12}$.

A. $\frac{\pi}{3}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{6}$ D. $\frac{\pi}{12}$

4. An angle in the III quadrant has a sine of $-\frac{3}{4}$. Find the secant of the angle.

A. $-\frac{3\sqrt{7}}{7}$ B. $-\frac{\sqrt{7}}{4}$ C. $-\frac{4}{3}$ D. $-\frac{4\sqrt{7}}{7}$

5. Mari wanted to measure the height of a large tree. She used a transom to find the angle of elevation between the ground and the top of the tree to be 48.2°. The transom was positioned 52 feet from the base of the tree. What is the height of the tree?

A. 46 feet B. 70 feet C. 58 feet D. 78 feet

6. Two position vectors are defined as **u** =$\langle 2,5 \rangle$ and **v** = $\langle 1,-3 \rangle$. Find 2**u** + **v**.

A. $\langle 5,2 \rangle$ B. $\langle 5,13 \rangle$ C. $\langle 4,-1 \rangle$ D. $\langle 5,7 \rangle$

7. Triangle ABD has the following characteristics: measure of Angle A = $\frac{\pi}{2}$ and the measure of Angle B = $\frac{\pi}{3}$. Find the exact value of the sinC.

A. $\frac{1}{2}$ B. $\frac{\sqrt{3}}{2}$ C. $\frac{\sqrt{2}}{2}$ D. 1

8. A farmer wants to buy a triangular plot of land. He wants to find the area of the land that is for sale. The lengths of the three sides of the land are 450 ft, 380 ft and 520 feet. What is the area of the land in square feet?

A. 989,038 B. 83,334 C. 85,500 D. 117,000

9. Find the asymptotes of the following function: $f(x) = \frac{x^2-2x-8}{x-3}$.

A. y = x+1 B. y = 1 C. x = 3 D. A & B E. A & C

10. The function f(x) yielded the following table of values.

x	-2	0	1	4	10
f(x)	-4	3	0	-2	6

Identify the location of the possible zeros.

A. x=1 B. $-2 < x < 0$ C. $4 < x < 10$ D. A, B & C E. None of these

11. To the thousandths place, evaluate: $\log_4 15$

A. 0.512 B. 1.396 C. 1.953 D. 0.532 E. None of these

12. 50 specimens of a non-native fish were introduced into a local lake. A biologist found that the population growth of these fish could be modeled using $f(x) = -x^4 + 30x + 50$ where x is measured in years. She said that the fish would eventually destroy the habitat in the lake and the species would die off completely. How long will it take for the fish to be completely gone in the lake?

A. 3.5 years B. 3.9 years C. – 1.5 years D. Both A & C E. They never die off

13. Write in terms of simplest logarithmic form: $\log_4\left(\frac{\sqrt{6x^3}}{3}\right)$

A. $\frac{1}{4} + \frac{3}{2}\log_4(x) - \frac{1}{2}\log_4(3)$ B. $\log_4\left(\sqrt{6x^3}\right) - \log_4(3)$

C. $\frac{1}{2}\log_4(6) + \frac{3}{2}\log_4(x) - \log_4(3)$ D. $\log_2(6x^3) - \log_4(3)$

14. Solve for *t* using logarithms with base *a*: $4 = xa^{3t} + y$

A. $\frac{1}{3}\log_a(4 - y - x)$ B. $\frac{1}{3}\log_a\left(\frac{4-y}{x}\right)$

C. $\log_{3a}\frac{4-y}{x}$ D. $\frac{1}{3} \cdot \frac{\log_a 4-y}{x}$

15. A candy shop offers 3 types of candy. One customer buys ½ pound of candy A, ¼ pound of candy B and 1 pound of candy C for a total price of $9.55. A second customer buys 1 pound of candy A and ½ pound of candy C for a total price of $7.50. A third customer buys a 2 pounds of candy A, 1 pound of candy B and 3 pounds of candy C for a total price of $32.20. Find the price of candy A, B and C.

A. A = $4.50, B = $5.20, C = $6.00 B. A = $5.00, B = $8.20, C = $5.00
C. A = $10.50, B = $19.60, C = – $ 6.00 D. Not enough information

16. Given the sequence: $\frac{1}{2}, 1, \frac{5}{4}, \frac{7}{5}, \frac{3}{2}, \ldots$, define the sequence recursively.

A. $a_1 = \frac{1}{2}$ $a_n = \frac{n+2}{a_{n-1}+1}$

B. $a_1 = 1$ $a_n = \frac{2n-1}{a_{n-1}+1}$

C. $a_1 = \frac{1}{2}$ $a_n = \frac{2n-1}{n+1}$

D. $a_1 = \frac{1}{2}$ $a_n = \frac{2n-1}{n^2-1}$

17. A large bowl is created so that the cross section is a parabola. If the vertex of the parabola is set at the origin and the width of the top of the bowl is 20 inches and the height from the vertex to the top of the bowl is 6 inches, then find the equation of the parabola.

A. $y = \frac{3}{25}x^2 - \frac{3}{5}x$ B. $\frac{3}{200}x^2 = y$ C. $\frac{5}{18}x^2 = y$ D. $\frac{3}{50}x^2 = y$

18. Find the sum of $\sum_{k=0}^{8} 3\left(\frac{1}{2}\right)^k$

A. $\frac{3}{256}$ B. $\frac{1533}{256}$ C. $\frac{765}{32768}$ D. $\frac{192}{255}$

19. One angle of a triangle is given to be 40°. Another angle is given to be $\frac{11\pi}{15}$ radians. What can be said of the triangle.

A. The triangle is not possible. B. The missing angle is $\frac{2\pi}{45}$ radians

C. The triangle is acute D. The missing angle is 10° E. None of these

20. A hot air balloon is tethered to the ground by 2 ropes. One rope is 25 feet long and the other is 20 feet long. The angle between the ropes is 8°. How far apart are the ropes at the point where they are anchored to the ground?

A. 34 feet B. 5.9 feet C. 45 feet D. 32 feet

21. Find the $tan\frac{\theta}{2}$ if the $sin\theta = \frac{3}{7}$ and θ is in quadrant II.

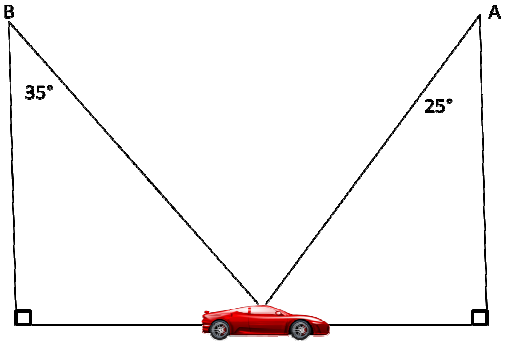
A. $\frac{7-2\sqrt{10}}{3}$ B. $-\frac{3\sqrt{10}}{20}$ C. $\frac{7+2\sqrt{10}}{3}$ D. $\sqrt{\frac{1-\frac{3}{7}}{2}}$

22. Simplify the following expression for all values of θ as completely as possible.

$$sin^4\theta - cos^4\theta + 2cos^2\theta + sin\theta$$

A. $1 + sin\theta$
B. $sin^2\theta - cos^2\theta + 2cos^2\theta = sin\theta$
C. $sin\theta(sin\theta + 1)(sin^2\theta - sin\theta + 1) - cos^2\theta(cos^2\theta - 2)$
D. $sin\theta(sin^3\theta - 1) - cos^2\theta(cos^2\theta - 2)$

23. Two points are located 17 miles apart.
A car is stranded somewhere such that a line from point A forms the 25° angle shown and a line from point B forms the 35° angle shown. How far is the car from point A?



A. 18 miles B. 11 miles C. 8 miles D. 16 miles

24. Find the $sin2\theta$ if $cos\theta = -\frac{1}{6}$ and θ is in quadrant III.

A. $-\frac{\sqrt{35}}{18}$ B. $-\frac{17}{18}$ C. $-\frac{\sqrt{35}}{3}$ D. $\frac{\sqrt{35}}{18}$

25. Convert the following rectangular equation into a polar equation. $y = x^2$

A. $r = cot\theta csc\theta$ B. $r = sin\theta + rsin^2\theta$
C. $r = tan\theta sec\theta$ D. not enough information

Tie Breaker #1

An ellipse is described by $3x^2 + 2y^2 - 18x + 8x + 23 = 0$.

A) Find the vertices of the ellipse.

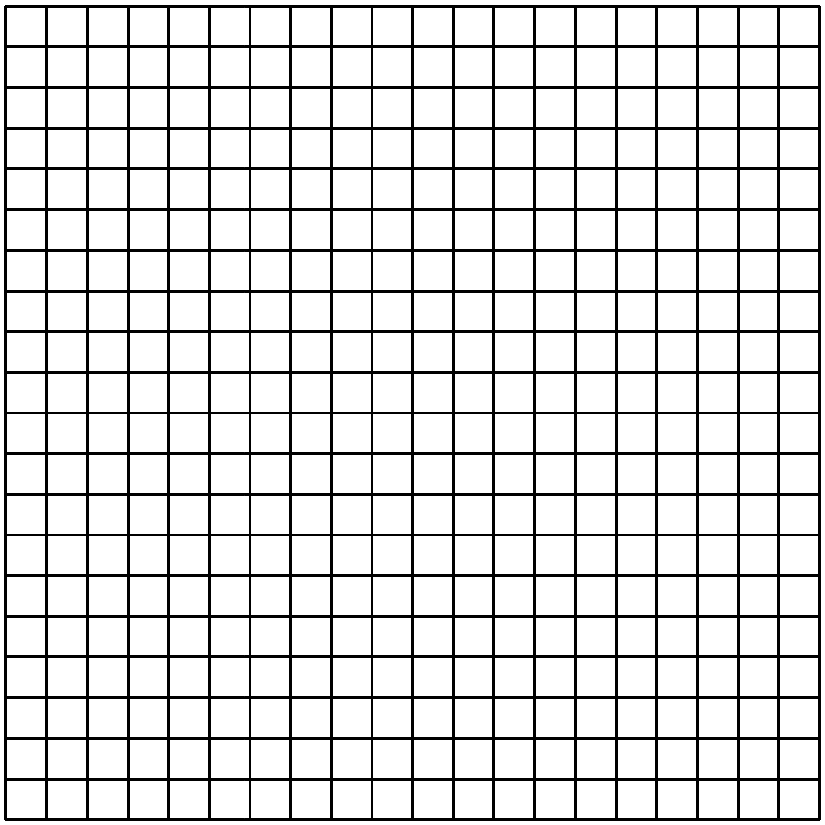
B) Find the foci of the ellipse.

Tie Breaker #2:

A) Sketch the graph of a sin function with the following characteristics:

- The amplitude is half of the regular sine function
- The period is four times the period of the regular sine function
- The function has been shifted vertically such that it is 3 units greater than the sine function

B) Write an equation that describes this function in function notation.



Tie Breaker #3

A pilot wishes to fly from Little Rock to New York City in 2 hours 45 minutes. The distance between the cities is 1100 miles. There is a constant 40 mph wind blowing at a bearing of 138°. If New York is at a bearing of 30° from Little Rock, then at what airspeed and bearing should the plane fly?

Answers:

1. A
2. D
3. C
4. D
5. C
6. D
7. A
8. B
9. E
10. D
11. C
12. A
13. C
14. B
15. A
16. C
17. D
18. B
19. B
20. B
21. C
22. A
23. D
24. D
25. C

Tie Breaker #1:

$3x^2 - 18x + 2y^2 + 8y + 23 = 0$

$3(x^2 - 6x) + 2(y^2 + 4y) = -23$

$3(x^2 - 6x + 9) + 2(y^2 + 4y + 4) = -23 + 27 + 8$

$3(x - 3)^2 + 2(y + 2)^2 = 12$

$\frac{(x-3)^2}{4} + \frac{(y+2)^2}{6} = 1$

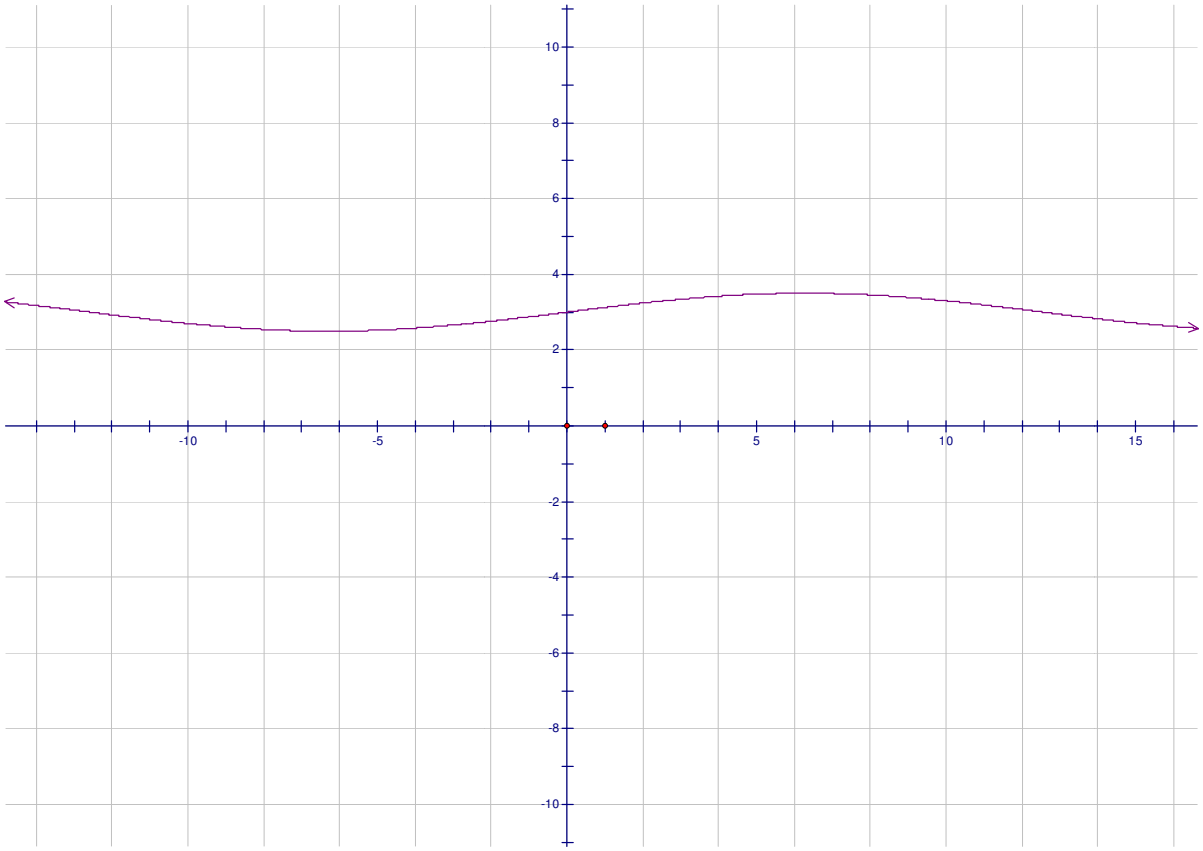
a =$\sqrt{6}$ and b = 2

Center is at (3, – 2)

Vertices are at $\left(3, -2 \pm \sqrt{6}\right)$

Foci are at (3, – 2 ± 2) or at (3, 0) and (3, – 4)

Tie Breaker #2



b) $f(x) = \frac{1}{2}sin\left(\frac{1}{4}x\right) + 3$

Tie Breaker #3

$$\frac{x}{sin108°} = \frac{400mph}{sin42°}$$

So x = 570mph

Then the angle between the resultant and due north is the same as the angle between the resultant and the original direction vector, so the original direction is due North.