ACTM Regional Calculus Competition
March 3, 2012

Instructions: Select the best choice for each question. Afterward, attempt the tie-breaker questions in sequential order (Do #1 first, followed by #2, and then #3 last). Unless otherwise stated, assume all variables are real and all functions are continuous over relevant domains. Assume all angles are in Radians. Good luck!

1. Calculate the following limit:

$$\lim_{x\to\infty} \frac{2x+11}{\sqrt{x+1}}$$

   a. 2
   b. 11
   c. 13
   d. -1
   e. No limit exists.

2. Determine the slope of the normal line to the function $h(x) = x^5 + 4x^3 + 2x$ at the point (1,7).

   a. 1
   b. 7
   c. $-1/7$
   d. 19
   e. $-1/19$

3. Calculate and simplify the derivative of the following function: $g(x) = \sin(\cos^{-1}(x))$

   a. $g'(x) = \cos\left(-\sin(x)\right)$
   b. $g'(x) = -\frac{x}{\sqrt{1-x^2}}$
   c. $g'(x) = \cos\left(-\frac{1}{\sqrt{1-x^2}}\right)$
   d. $g'(x) = \cos\left(\cos^{-1}(x)\right) \cdot \frac{1}{\sqrt{1-x^2}}$
   e. No derivative exists.

4. Find the volume of the solid of revolution bounded by $y = x^2 + 1$, the x–axis, the y–axis, and $x = 2$, rotated around the x–axis, using the Disk/Washer Method.

   a. $\frac{14}{3}\pi$
   b. $\frac{28}{3}\pi$
   c. $\frac{206}{15}\pi$
   d. $\frac{412}{15}\pi$
   e. No volume is generated.

5. Evaluate the following definite integral:

$$\int_{-2\pi}^{2\pi} |\sin x|\, dx$$

   a. 0
   b. 2
   c. 4
   d. 8
   e. Not able to be integrated.

6. For the given limit and ϵ, determine the largest value of δ such that if $0 < |x - a| < \delta$, then $|f(x) - L| < \epsilon$. Round your value of δ to the nearest thousandth.

$$\lim_{x \to 4} x^2 = 16 \qquad \epsilon = 0.1$$

   a. $\delta = .013$
   b. $\delta = .1$
   c. $\delta = .316$
   d. $\delta = .5$
   e. Not enough information given.

7. Determine the following limit:

$$\lim_{x \to 0+} \frac{\cos\sqrt{x} - 1}{\sqrt{x}}$$

   a. 0
   b. 1
   c. π
   d. $\cos - 1$
   e. No limit exists.

8. Determine the value of the following limit:

$$\lim_{x \to 5} \frac{x + 5}{x - 5}$$

   a. 0
   b. 1
   c. ∞
   d. $-\infty$
   e. No limit exists

9. For which value(s) of x is/are $f(x)$ **not** differentiable?

$$f(x) = 3 + |x^2 - 4|$$

   a. $x = 2$
   b. $x = -2$
   c. $x = 2$ & $x = -2$
   d. $x = 0$
   e. $f(x)$ is differentiable for all values of x.

10. Find the equation of the tangent line to the curve $f(x) = \ln(x)$ and passes through the point (1,0).

a. $y = -x$
b. $y = -x + 1$
c. $y = x$
d. $y = x - 1$
e. No tangent line exists.

11. The cost of producing a cell phone part is given by the function $C(x) = 1000 + 2x + .005x^2$. Calculate the marginal cost for producing the 2000th part.

a. \$21.99
b. \$22.50
c. \$24,978.01
d. \$25,000
e. Not enough information given

12. A rock is thrown vertically upward from the top of a building. It reaches a height of $s = 20 + 14t - 4.9t^2$ meters in t seconds. At what time does the rock have a velocity of $10m/s$?

a. $t = .4\ seconds$
b. $t = 1.4\ seconds$
c. $t = 2.4\ seconds$
d. $t = 3.9\ seconds$
e. It never reaches this velocity.

13. What is/are the inflection points point(s) to the function $(x) = x^4 - 4x^3 + 3$?

a. $x = 1$
b. $x = 0$ and $x = 2$
c. $x = 0$ and $x = 3$
d. $x = 1$ and $x \approx 4$
e. The function has no inflection points.

14. Determine the anti-derivative to the following function: $\mathrm{f(x)} = \ln(\mathrm{x})dx$

a. $\frac{1}{x} + C$
b. $x\ln(x) + C$
c. $x\ln(x) - x + C$
d. $\ln(x^2) + C$
e. No anti-derivative exists.

15. Determine the x-coordinate(s) where the following function has a vertical tangent line.

$$f(x) = \sqrt[3]{x} * (4 - x)$$

a. $x = -1, 1$
b. $x = 0$
c. $x = \frac{4}{3}$
d. $x = 4$
e. $f(x)$ has no vertical tangent lines.

16. Calculate the anti-derivative of the function $g(x) = \cos(x) \cdot \sin^4(x)$.

a. $G(x) = -\sin^5(x) + 4\sin^3(x) \cdot \cos^2(x) + C$
b. $G(x) = 4\sin^4(x) \cdot \cos(x) + \mathrm{C}$
c. $G(x) = -\sin(x) \cdot \frac{\cos^5 x}{5} + C$
d. $G(x) = \frac{\sin^5(x)}{5} + C$
e. No anti-derivative exists.

17. Determine the sum of the following:

$$\sum_{i=1}^{6} (-1)^n \cdot \left(\frac{1}{n}\right)$$

a. $-37/60$
b. $-49/20$
c. $49/20$
d. $37/60$
e. The summation has no value

18. Calculate the area under the curve $f(x) = x^2$ between $x = 1$ and $x = 4$ using 6 rectangles. Use left hand end points and a regular partition.

a. 6.375
b. 16.875
c. 17.375
d. 24.875
e. 35

19. If $f(x) = x^2 + x + 1$, find a number c that satisfies the conclusion of the mean value theorem on the interval [0,4].

a. $c = 0$
b. $c = 2$
c. $c = 4$
d. $c = 5$
e. No value for c exists.

20. Characterize the discontinuity of the following function:

$$f(x) = f(x) = \begin{cases} x^2 + 1, & x < 1 \\ 1, & x = 1 \\ x + 1, & x > 1 \end{cases}$$

a. Removable discontinuity at $x = 1$.
b. Non-removable discontinuity at $x = 1$.
c. There is an horizontal asymptote at $x = 1$.
d. There is a vertical asymptote at $x = 1$.
e. The function is continuous at $x = 1$.

21. Find $\frac{dy}{dx}$ by implicit differentiation: $x^3 + y^2 = 2$

a. $\frac{dy}{dx} = \frac{x^4}{4} + \frac{y^2}{2} + C$
b. $\frac{dy}{dx} = 3x^2$
c. $\frac{dy}{dx} = 3x^2 + 2y$
d. $\frac{dy}{dx} = -\frac{3x^2}{2y}$
e. $\frac{dy}{dx} = -\frac{2y}{3x^2}$

22. The position of a point on a coordinate line is given by $s(t) = \frac{t^3+3t+1}{t^2+1}$. Find the average velocity between times $t = 1$ and 2 seconds.

a. $v_{avg} = \frac{1}{2}$
b. $v_{avg} = \frac{5}{2}$
c. $v_{avg} = 3$
d. $v_{avg} = 1$
e. $v_{avg} = 2$

23. Use L'Hôpital's rule to determine the following limit (if it exists).

$$\lim_{x \to 0} \frac{x^2}{e^x - e^{-x}}$$

a. 0
b. 1
c. 1/2
d. ∞
e. No limit exists.

24. Determine where the following function is increasing.

$$f(x) = x^3 - 3x$$

a. $(-\infty, \infty)$
b. $(-\infty, 0)$
c. $(0, \infty)$
d. $(-\infty, -1) \cup (1, \infty)$
e. $(-\infty, 0) \cup (0, \infty)$

25. Determine domain of the following function:

$$G(x) = \frac{x^2 - 9}{x^2 - 4}$$

a. $\{x | x \in \mathbb{R}\}$
b. $\{x | x \neq 4/9\}$
c. $\{x | x \neq 4\}$
d. $\{x | x \neq 2, x \neq -2\}$
e. $\{x | x \neq 3, x \neq -3\}$

ACTM Regional Calculus Competition
Tie Breaker Questions
March 3, 2012

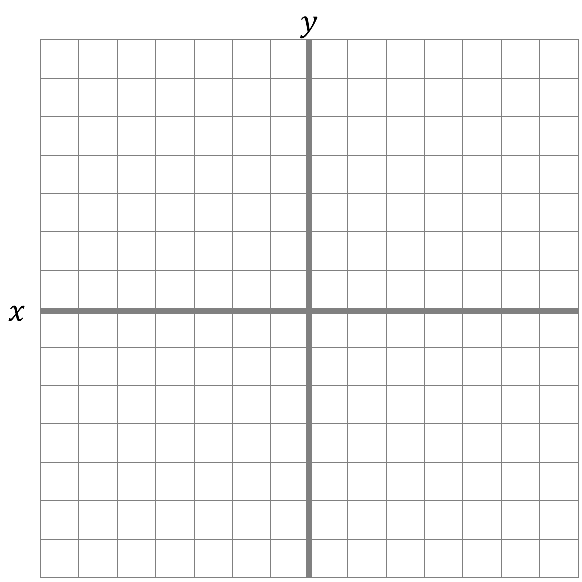
Name ____________________

School/Teacher ____________________

Reminder: Attempt the tie-breaker questions in sequential order (Do #1 first, followed by #2, and then #3 last).

1. Sketch a curve that fits the following characteristics. The curve's first derivative and second derivative are given on a number line below. In addition, $f(0) = 3,\ f(-2) = -1, and\ f(2) = 5$.

$f''(x)$	+	+	+	+	0	−	−	−	−
$f'(x)$	−	−	0	+	+	+	0	−	−
x			-2		0		2		



ACTM Regional Calculus Competition **Name**
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2. Let $g(x)$ be a differentiable function on the closed interval $[a, b]$. For a value $c \in (a, b)$, $g(c) = 5$ and $g'(c) = 3$. Determine the value of $\frac{d}{dx}\left(\frac{1}{g(x)}\right)$ evaluated at the point $x = c$.

ACTM Regional Calculus Competition
Tie Breaker Questions
March 3, 2012

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3. A knitting supply company determines that the cost of manufacturing and packaging x knitting kits per day is given by

$$C(x) = 500 + .02x + .002x^2$$

Each knitting kit is sold for \$13.00.

   a. Determine an equation that gives the revenue
   b. Determine an equation that gives the profit.
   c. Determine the rate of production that will maximize the profit.
   d. Determine the maximum profit.

ACTM Regional Calculus Competition
SOLUTIONS
March 3, 2012

Multiple Choice Answers

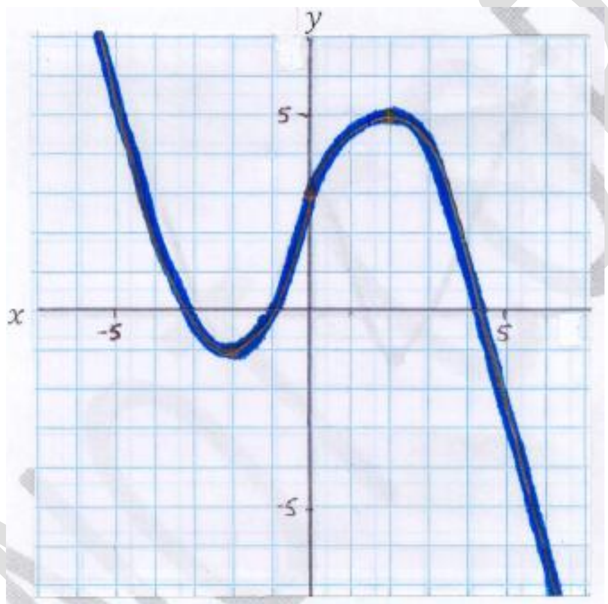
1. ...... E
2. ...... E
3. ...... B
4. C
5. ...... D
6. ...... A
7. ...... A
8. ...... E
9. C
10. .... D
11. .... A
12. .... A
13. .... B
14. C
15. .... B
16. .... D
17. .... A
18. C
19. .... B
20. .... A
21. .... D
22. .... A
23. .... A
24. .... D
25. .... D

Tie Breaker Question 1 Solution

1. Sketch a curve that fits the following characteristics. The curve's first derivative and second derivative are given on a number line below. In addition, $f(0) = 3$, $f(-2) = -1$, and $f(2) = 5$.

$f''(x)$	+	+	+	+	0	–	–	–	–
$f'(x)$	–	–	0	+	+	+	0	–	–
x			-2		0		2		

Student responses should resemble something like this graph. Answers may vary somewhat.



Tie Breaker Question 2 Solution

2. Let $g(x)$ be a differentiable function on the closed interval $[a, b]$. For a value $c \in (a, b)$, $g(c) = 5$ and $g'(c) = 3$. Determine the value of $\frac{d}{dx}\left(\frac{1}{g(x)}\right)$ evaluated at the point $x = c$.

Use the quotient rule on this formula.

$$\frac{d}{dx}\left(\frac{1}{g(x)}\right) = \frac{g(x) \cdot 0 - 1 \cdot g'(x)}{g^2(x)} = \frac{-g'(x)}{g^2(x)}$$

Now, use the values as given in the original problem,

$$\frac{-g'(c)}{g^2(c)} = \frac{-3}{5^2} = \frac{-3}{25}$$

Tie Breaker Question 3 Solution

3. A knitting supply company determines that the cost of manufacturing and packaging x knitting kits per day is given by

$$C(x) = 500 + .02x + .002x^2$$

Each knitting kit is sold for \$13.00 .

a. Determine an equation that gives the revenue

Revenue is determined by price per item times number of items.

$$\boldsymbol{R(x) = 13x}$$

b. Determine an equation that gives the profit.

Profit is Revenue minus cost.

$$\boldsymbol{P(x) = 13x - (500 + .02x + .002x^2)}$$
$$\boldsymbol{P(x) = 13x - 500 - .02x - .002x^2}$$
$$\boldsymbol{P(x) = -.002x^2 + 12.98x - 500}$$

c. Determine the rate of production that will maximize the profit.

To find the maximum profit, find the derivative and set it equal to zero and solve.

$$\boldsymbol{P'(x) = -.004x + 12.98 = 0}$$
$$\boldsymbol{x = \frac{-12.98}{-.004} = 3245}$$

The company should produce 3245 items.

d. Determine the maximum profit.

Using the 3245 item production level, the max profit is

$$\boldsymbol{P(3245) = -.002(3245)^2 + 12.98(3245) - 500}$$

$P(3245)$ =20,560.05

The maximum profit is \$20,560.05.