ACTM Regional Algebra II Exam

March 3, 2012

For questions 1 through 25, mark your answer choice on the answer sheet provided. After completing items 1 through 25, answer each of the tie-breaker items in sequential order (do #1 first, followed by #2, and then #3 last). Be sure that your name is printed on each of the tie-breaker pages.

1. Find a value for *c* so that $\frac{a^6 a^c a^{-3}}{a^2 a^{-4}} = a^3$.

A. 8 B. -2 C. 2 D. -5 E. None of these

2. Determine *f(x)* if $f^{-1}(x) = 4x - 7$.

A. $4x + 7$ B. $\frac{1}{4}x + 7$ C. $\frac{1}{4}x + \frac{7}{4}$ D. $\frac{1}{4}x - \frac{7}{4}$ E. None of these

3. Suppose a jar contains five blue marbles, four red marbles and three green marbles. Suppose you choose two marbles from the jar. What is the probability that both marbles are the same color?

A. $\frac{1}{2}$ B. $\frac{25}{144}$ C. $\frac{3}{8}$ D. $\frac{19}{66}$ E. None of these

4. Multiply the following product: $(3 - 4\sqrt{2})(5 - 6\sqrt{2})$

A. $63 - 38\sqrt{2}$ B. $15 + 24\sqrt{2}$ C. $24 - 15\sqrt{2}$
D. $15 - 38\sqrt{2}$ E. None of these

5. Find the coefficient of the x^4y^2 term of the binomial expansion of $(x - 2y)^6$.

A. 64 B. -192 C. -160 D. 240 E. None of these

6. Which expression is the factored form of $x^3 + 2x^2 - 5x - 6$?

A. (x+1)(x+1)(x-6) B. (x+2)(2x-5)(x-6) C. (x+3)(x+2)(x-2)

D. (x-3)(x-1)(x+2) E. None of these

7. Which binomial is NOT a factor of $2x^4 + 9x^3 - 20x^2 - 57x + 90$?

A. (x+5) B. (x+3) C. (x+2) D. (2x-3) E. All are factors

8. Since $3^4 = 81$ then

A. $\log_{81} 4 = 3$ B. $\log_4 81 = 3$ C. $\log_3 81 = 4$
D. $\log_3 4 = 81$ E. None of these

9. Which of the following expressions is/are true?

I. $\left(\sqrt[4]{\sqrt{5}}\right)^8 = 5$ II. $\left(\sqrt[3]{\sqrt{5}}\right)^5 = 5$ III. $\frac{1}{\left(\frac{2}{\sqrt{5}}\right)^1} = \frac{5}{2}$

A. All are correct B. I only C. I and III only
D. II and III only E. III only

10. Let $X = \begin{bmatrix} -2 & 4 & 1 \\ 3 & 0 & -1 \\ 1 & a & 1 \end{bmatrix}$. Find the value of *a* so that $|X| = -14$.

A. 2 B. 14 C. -1 D. -30 E. None of these

11. Give the ordered pair (in terms of *a*) that satisfies the system of equations

$$\begin{cases} 2x + 4y = a \\ -3x + 2y = 1 \end{cases}$$

A. $\left(-\frac{a+2}{4}, -\frac{3a+2}{8}\right)$ B. $\left(\frac{2-a}{4}, \frac{3a-2}{8}\right)$ C. $\left(\frac{a+2}{8}, \frac{3a+12}{16}\right)$

D. $\left(\frac{a-2}{8}, \frac{3a+2}{16}\right)$ E. None of these

12. Suppose a rectangle has an area of $x^2 - 2x - 48$. What is one possible side length of the rectangle?

A. (x-4) B. (x+6) C. (x-6) D. (x+8) E. None of these

13. Simplify i^{539}.

A. i B. -1 C. $-i$ D. 1 E. None of these

14. Which expression is equal to $\log_7 x + 3\log_7 y - 2\log_7 z$?

A. $\log_7(-6xyz)$ B. $-\log_7\left(\frac{3xy}{z^2}\right)$ C. $\log_7 \frac{(xy)^3}{z^2}$

D. $\log_7 \frac{xy^3}{z^2}$ E. None of these

15. If, for all values of y, $(y+a)^2 = y^2 - 6y + a^2$, then a =

A. 9 B. 3 C. -3 D. -6 E. None of these

16. If $a > 0$ and $b > 0$, then $\sqrt{125\sqrt{625a^4b^6}}$ =

A. $25ab\sqrt{5b}$ B. $125ab\sqrt{5b}$ C. $125ab\sqrt{b}$

D. $25a^2b^3$ E. None of these

17. If $\log_{10} g = 3$ then g =

A. $\frac{1}{10,000}$ B. $\frac{1}{1,000}$ C. 10

D. 100 E. None of these

18. The inequality $y^2 - 19y < 20$ is equivalent to

A. $-4 < y < 5$ B. $-1 < y < 20$ C. $4 < y < 5$

D. $y < -1$ or $y > 20$ E. None of these

19. The solution set for the equation $x + 4 - \sqrt{14x} = 0$

A. consists of exactly one positive number and one negative number
B. consists of exactly one positive number
C. consists of exactly one negative number
D. consists of exactly two positive numbers
E. has no real solution

20. One of the solutions to the equation $x^2 = 6x - 10$ is

A. -10 B. 2 C. $6 - i$ D. $3+i$ E. None of these

21. What is the vertex of $f(x) = -2x^2 - 8x + 5$?

A. (-2, 13) B. (2, -19) C. (-2, 29) D. (-2, -3) E. None of these

22. A cell phone company charges a flat fee per month plus a charge for each minute used by the consumer. Bill talked for 75 minutes last month and was charged \$56.50, while Linda talked for 120 minutes and was charged \$61. Write a function to represent the cost *C* for a monthly bill in terms of how many minutes *t* spent talking on the phone.

A. C(t) = $0.15t + 50$ B. C(t) = $0.1t + 49$ C. C(t) = $0.1t + 4.5$

D. C(t) = $0.1t + 45$ E. None of these

23. Find the solution to this system of equations: $\begin{cases} y + 2x = 10 \\ -x + y - 2z = -2 \\ 3x - 2y + 4z = 7 \end{cases}$

A. (3, 16, 5/2) B. (3, -4, 3/2) C. (-3, 16, 15/2)

D. (3, 4, 3/2) E. None of these

24. Solve the inequality $4 - \frac{3x}{5} \le 7x - 4$

A. $x \le \frac{20}{19}$ B. $x \ge \frac{19}{20}$ C. $x \le -\frac{20}{19}$

D. $x \ge \frac{20}{19}$ E. None of these

25. Find the inverse of $f(x) = 2x^2 - 5$.

A. $\pm\sqrt{\frac{1}{2}x + \frac{5}{2}}$ B. $\pm\sqrt{\frac{x}{2} + \frac{5}{2}}$ C. $\pm\sqrt{\frac{x}{2} - \frac{5}{2}}$

D. $\pm\sqrt{\frac{1}{2}x - \frac{5}{2}}$ E. None of these

Tiebreakers

Name_____

The tie-breaker questions are graded in order, so it is important that you work tie-breaker #1 before going on to #2, etc. Show all your work to receive credit.

1. Write a polynomial in expanded form $\left(a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0\right)$ that has roots at -5, 6, and 4 + 2*i*.

Name_____

2. A baker purchased the following ingredients (in lbs) during the three given months. Determine the price per pound the baker paid for each ingredient.

	# of lbs of flour	# of lbs of sugar	# of lbs of butter	Total Cost
January	10	7	8	72
February	5	10	10	70
March	6	3	4	38

Name_____

3. Suppose you invest $2,000 in an account that compounds interest annually. What is the minimum interest rate for this account that would allow your investment to double in 10 years? Express your answer as a percent rounded to one decimal place.

Answer Key

1. **B**
2. **C**
3. **D**
4. **A**
5. **E**
6. **E**
7. **C**
8. **C**
9. **B**
10. **A**
11. **D**
12. **B**
13. **C**
14. **D**
15. **C**
16. **A**
17. **E**
18. **B**
19. **E**
20. **D**
21. **A**
22. **B**
23. **D**
24. **D**
25. **E**

Tie-Breaker Solutions

1. $x^4 - 9x^3 - 2x^2 + 220x - 600$

2. flour = \$3/lb; sugar = \$2/lb; butter = \$3.50/lb

3. 7.2%