Arkansas Council of Teachers of Mathematics
2013 State Contest
Statistics Exam

In each of the following choose the BEST answer and shade the corresponding letter on the Scantron Sheet. Answer all 25 multiple choice questions before attempting the tie-breaker questions. The tie-breaker questions at the end are to be used to resolve any ties between 1st, 2nd, and/or 3rd place. Be sure that your name is printed on each of the tiebreaker pages. Good Luck!

1. A group of 40 data points has a mean of 34.7. Another group of 50 data points has a mean of 42.1. What is the mean of 90 points together? Round to the nearest tenth.
   a. 38.8
   b. 76.8
   c. 38.4
   d. 37.3
   e. None of the above

2. The time required for an automotive center to complete an oil change service on an automobile approximately follows a normal distribution, with a mean of 19 minutes and a standard deviation of 2.5 minutes. The automotive center currently guarantees customers that the service will take no longer than 20 minutes. If it does take longer, the customer will receive the service for half-price. If the automotive center does not want to give the discount to more than 2% of its customers, how many minutes should it make the guaranteed time limit?
   a. 34 minutes
   b. 25 minutes
   c. 14 minutes
   d. 22 minutes
   e. None of the above

3. A local bank needs information concerning the savings account balances of its customers. A random sample of 15 accounts resulted in the mean balance of $686.75 with a standard deviation of $256.20. Find a 98% confidence interval for the true mean. Assume that the account balances are normally distributed and round to the nearest dollar.
   a. ($513, $860)
   b. ($487, $564)
   c. ($533, $841)
   d. ($326, $437)
   e. None of the above

4. Health care issues are receiving much attention in both academic and political arenas. A sociologist recently conducted a survey of citizens over 60 years of age whose net worth is too high to qualify for government health care but who have no private health insurance. The ages of 25 uninsured senior citizens were as follows:

   68, 78, 66, 76, 86, 74, 61, 89, 65, 90, 69, 92, 76, 62, 81, 63, 68, 81, 70, 73, 60, 87, 75, 64, and 82

   If we assume that the distribution of ages is bell shaped and use the empirical rule, what percentage of the respondents will be between 64.47 and 93.78 years old?
   a. Approximately 81.5%
   b. Approximately 95%
   c. Approximately 68%
   d. Approximately 83.9%
   e. None of the above

5. The monthly telephone usage (in minutes) of 30 adults is listed below:

   154, 156, 165, 165, 170, 171, 172, 180, 184, 185, 189, 189, 190, 192, 195, 198, 198, 200, 200, 200, 205, 205, 211, 215, 220, 220, 225, 238, 255, and 265

   The interquartile range for the telephone usage of the 30 adults is
   a. 30
   b. 29
   c. 31
   d. 32
   e. None of the above

6. The z-score of a normal random variable $X = 30$ with mean 60 and variance 64 is
   a. 3.75
   b. -0.469
   c. 0.469
   d. -3.75
   e. None of the above

7. A residual is the difference between
   a. the observed value of y and the predicted value of y
   b. the observed value of y and the predicted value of x
   c. the observed value of x and the predicted value of x
   d. the observed value of x and the predicted value of y
   e. None of the above.

8. If 8 schools are all in the same conference, how many soccer games are played during the season if the teams all play each other exactly once?
   a. 40,320
   b. 720
   c. 56
   d. 28
   e. None of the above.

9. If the probability of a certain team winning is 3/4, what is the probability that this team will win at least 3 games out of the 4 games it plays?
   a. 0.105
   b. 0.422
   c. 0.684
   d. 0.738
   e. None of the above

10. The manager of a used car lot took inventory of the automobiles on his lot and constructed the following table based on the age of his car and its make (foreign or domestic).

Make	Age of Car (in years)				
	0 – 2	3 – 5	6 – 10	Over 10	Total
Foreign	38	20	10	32	100
Domestic	45	21	12	22	100
Total	83	41	22	54	200

   A car was randomly selected from the lot. Given that the car selected is older than two years old, the probability that it is not a foreign car is
   a. 0.62
   b. 0.55
   c. 0.53
   d. 0.47
   e. None of the above

11. In a sandwich shop, the following probability distribution was obtained. The random variable x represents the number of condiments used for a hamburger.

X	0	1	2	3	4
P(x)	0.30	0.40	0.20	0.06	0.04

Find the mean and variance for the random variable x.
   a. 1.14; 1.04
   b. 1.14; 1.08
   c. 2; 1.41
   d. 2; 2
   e. None of the above

12. In a hypothesis test, a P-value is
   a. a probability of observing a parameter more extreme than the one observed under the assumption that the null hypothesis is true.
   b. a probability of observing a parameter more extreme than the one observed under the assumption that the null hypothesis is false.
   c. a probability of observing a statistic at least as extreme as the one observed under the assumption that the null hypothesis is true.
   d. a probability of observing a statistic at least as extreme as the one observed under the assumption that the null hypothesis is false.
   e. None of the above.

13. For a linear regression model, how does a confidence interval differ from a prediction interval?
   a. Confidence intervals are used to measure the accuracy of the mean response of all the individuals in the population, while a prediction interval is used to measure the accuracy of a single individual's predicted value.
   b. Confidence intervals are used to measure the accuracy of a single individual's predicted value, while a prediction interval is used to measure the accuracy of the mean response of all the individuals in the population.
   c. Confidence intervals are constructed about the predicted values of y while prediction intervals are constructed about a particular value of x.
   d. Confidence intervals are constructed about the predicted values of x while prediction intervals are constructed about a particular value of y.
   e. None of the above.

14. A certain number of undergraduate students were asked to view a 40-minute television program that included ads for a digital camera. Some students saw a 30-second commercial; others saw a 90-second version. The same commercial was shown either 1, 3, or 5 times during the program. How many factors and treatments are there for this experimental study?
   a. 6 factors and 2 treatments
   b. 2 factors and 6 treatments
   c. 2 factors and 5 treatments
   d. 5 factors and 2 treatments
   e. None of the above

15. _____ is a condition applied to the experimental units involved in an experiment.
   a. The factor level
   b. The design
   c. A treatment
   d. The sampling design
   e. None of the above

16. The amount of soda a dispensing machine pours into a 12 ounce can of soda follows a normal distribution with a mean of 12.33 ounces and a standard deviation of 0.22 ounce. The cans only hold 12.55 ounces of soda. Every can that has more than 12.55 ounces of soda poured into it causes a spill and the can needs to go through a special cleaning process before it can be sold. What is the probability a randomly selected can will need to go through this process?
    a. 0.159
    b. 0.341
    c. 0.659
    d. 0.841
    e. None of the above

17. Of a freshman class, half of the students are enrolled in 15 class hours, most of the remaining freshmen are taking 12 hours with a few students taking 18 hours. Select the statement which is true about this distribution.
    a. The mode is the same as the mean.
    b. The median is less than the mean.
    c. The mean is less than the median.
    d. The mean is greater than the mode.
    e. None of the above.

18. A report indicated that 41% of adults had received a bogus email intended to steal personal information. Suppose a random sample of 900 adults is obtained. What is the probability that no more than 39% had received such an email?
    a. 0.118
    b. 0.889
    c. 0.891
    d. 0.109
    e. None of the above

19. A researcher at a major clinic wishes to estimate the proportion of the adult population of the United States that has sleep deprivation. How large a sample is needed in order to be 95% confident that the sample proportion will not differ from the true proportion by more than 5%?
    a. 10
    b. 385
    c. 769
    d. 271
    e. None of the above

20. The mean starting salary for students who have majored in statistics is $55,000. Determine the null and alternative hypotheses
    a. $H_0 : m = 55,000; H_a : m = 55,000$
    b. $H_0 : m \ne 55,000; H_a : m = 55,000$
    c. $H_0 : m = 55,000; H_a : m \ne 55,000$
    d. $H_0 : m \ne 55,000; H_a : m > 55,000$
    e. None of the above

21. Three men and nine women want to attend a conference. If 4 people are chosen at random from this group, what is the probability that no men will attend the conference?
    a. 0.255
    b. 0.745
    c. 0.218
    d. 0.782
    e. None of the above

22. Find the standardized test statistic, z, to test the hypothesis that $H_a : p_1 < p_2$. The following sample statistics are from independent samples: $n_1 = 550,\ n_2 = 690,\ x_1 = 121,$ and $x_2 = 195.$
    a. -2.513
    b. -2.132
    c. -0.985
    d. 1.116
    e. None of the above

23. Many track hurdlers believe that they have a better chance of winning if they start in the inside lane that is closest to the field. For the data below, the lane closest to the field is Lane 1, the next lane is Lane 2, and so on until the outermost lane, Lane 6. The data lists the number of wins for track hurdlers in the different starting positions. Calculate the chi-square test statistic c^2 to test the claim that the probabilities of winning are the same in the different positions. The results are based on 240 wins.

Starting Position	1	2	3	4	5	6
Number of Wins	36	50	33	44	32	45

    a. 9.326
    b. 15.541
    c. 12.592
    d. 6.750
    e. None of the above

24. In order to be accepted into a program at North Point, a student must score in the top 2% of a standardized test on general knowledge. Historically, the scores for this test are found to be normally distributed with mean score of 70 and variance of 9. What would be the minimum score (rounded to the nearest integer) on this test in order for a student to be accepted into the program?
    a. 83
    b. 89
    c. 81
    d. 76
    e. None of the above

25. In a chi-square test of homogeneity of proportions, we test the claims that
    a. across a single sample the proportion of individuals with the same characteristic is the same as the population.
    b. different populations have the same proportions of individuals with the same characteristics.
    c. the proportion of individuals with a given characteristic does not change over time.
    d. the proportion of a population having a given characteristic is based on the homogeneity of the population.
    e. None of the above.

Tie-Breaker Questions

Name_____   School_____
        [Please Print]   **[Please Print]**

In each of the following you must show supporting work for your answers to receive credit. The questions will be used in the order given to resolve ties for 1st, 2nd, and/or 3rd place. Be sure that your name is printed on each of the tiebreaker pages.

1. A student wanted to determine whether the wait time in the drive-through at McDonald's differed from that at Wendy's. She used a random sample of 12 cars at McDonald's and 12 cars at Wendy's and obtained the following results. Assume that the population data are normally distributed.

Wait Time at McDonald's Drive-Through (seconds)				
151.09	227.38	111.84	131.21	128.75
191.60	126.91	137.90	195.44	246.59
141.78	127.35			

Wait Time at Wendy's Drive-Through (seconds)				
281.90	71.02	204.29	110.55	183.79
187.53	199.86	190.91	182.54	471.62
196.84	233.65			

a. Conduct an appropriate test to check whether there is a difference in wait times at each restaurant's drive-through? Use the 0.1 level of significance. Provide hypotheses, test statistic, P-value, decision, and conclusion.

b. Find 86% CI for the μ_w-μ_m, where μ_w is the average wait time of all Wendy's drive-through customers and μ_m is the average wait time of all McDonald's drive-through customers and interpret your result.

Tie-Breaker Questions

Name_____   **School**_____
          [Please Print]   **[Please Print]**

2. The data in the table are typical prices for a gallon of regular leaded gasoline and a barrel of crude oil for the indicated years.

Year	Gasoline (¢ per gallon)	Crude Oil ($ per barrel)
1975	57	10.38
1976	59	10.89
1977	62	11.96
1978	63	12.46
1979	86	17.72
1980	119	28.07
1981	131	35.24
1982	122	31.87
1983	116	28.99
1984	113	28.63
1985	112	36.75
1986	86	14.55
1987	90	17.9
1988	90	14.67
1989	100	17.97
1990	115	22.23

Use crude oil price as a predictor variable and gasoline price as a response variable to answer the following.
   a. Find the fitted least-squares line.
   b. Find the linear correlation coefficient. Interpret it.
   c. At 0.05 level of significance, conduct a suitable hypothesis test to test whether a linear relation exists between crude oil price and gasoline price. Provide hypotheses, test-statistic, P-value, decision, and conclusion.

Tie-Breaker Questions

Name_____   **School**_____
                    [Please Print]   **[Please Print]**

3. A recent General Social Survey asked the following two questions of a random sample of 1492 adult Americans under the hypothetical scenario that the government suspected that a terrorist act was about to happen:
   a. Do you believe the authorities should have the right to tap people's telephone conversations?
   b. Do you believe the authorities should have the right to stop and search people on the street at random?

   Of 1492 adults, 494 have said 'yes' to both questions; 335 have said 'yes' to the first question and 'no' to the second question; 126 have said 'no' to the first question and 'yes' to the second question; and 537 have said 'no' to both questions. Conduct an appropriate hypothesis test to check the claim that the proportions who said 'yes' to each of the questions above differ significantly? Use 0.05 level of significance. Provide hypotheses, test-statistic, P-value, decision, and conclusion.

KEY:

1. A	14. B
2. B	15. C
3. A	16. A
4. A	17. C
5. C	18. A
6. D	19. B
7. A	20. C
8. D	21. A
9. D	22. A
10. D	23. D
11. B	24. D
12. C	25. B
13. A	

Tie-Breaker #1 Solution:

a.

  i.   **Hypotheses:** $H_0 : m_M = m_W$; $H_a : m_M \, ^1 \, m_W$

  ii. **Test statistic** = -1.6 or 1.6

  iii. **P-value** = 0.13 (allow .14 if using the conservative freedom estimate with df = 14)

  iv. **Decision:** Fail to reject H$_0$

  v.   **Conclusion:** There is not sufficient evidence, at the 0.1 level of significance, that there is a difference in mean wait times at each restaurant's drive-through.

b. 86% Confidence Interval for the difference $m_W - m_M$ is (1.345, 98.098)(or slightly wider for df = 14). We are 86% confident that the difference between the means of wait times at drive-through of all customers of McDonald's and Wendy is between 1.345 seconds and 98.098 seconds.

Tie-Breaker #2 Solution:

a. $\hat{y} = \hat{b}_1 x + \hat{b}_0 = 2.48x + 42.311$

b. r = .907. Crude oil price and gasoline price are positively and strongly correlated.

c.

  i.   **Hypotheses:** $H_0 : b_1 = 0$; $H_a : b_1 \, ^1 \, 0$ (or accept $H_0 : \rho = 0; H_a : \rho \neq 0$)

  ii. **Test statistic =** 8.049

  iii. **P-value** $= 1.276 \, ^\times \, 10^{-6}$

  iv. **Decision:** Reject H$_0$

  v.   **Conclusion:** At 0.05 level of significance, collected data provide significant evidence to support the claim that a linear relation exists between crude oil price and gasoline price.

Tie-Breaker #3 Solution:

i. **Hypotheses:**

H$_0$: $P_T = P_S$ where P_T = proportion of people who said 'yes' for phone tapping

P_S = proportion of people who said 'yes' for random searches

H$_1$: $P_T \neq P_S$

Test statistic = 9.69

ii. **P-value** $\gg 0$

iii. **Decision:** Reject H$_0$.

iv. **Conclusion:** At 0.05 level of significance, that the proportions who said 'yes' with each scenario differ significantly.