

2014 Regional Exam

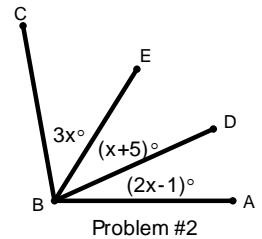
Geometry

In each of the following please select the appropriate answer for each question and record the corresponding letter on the answer sheet. Any erasure on the answer sheet should be completely erased to avoid errors in scoring the exam. Answer each of the 25 questions prior to working on the Tie Breaker questions as they will be used only to break ties for first, second and/or third place. Figures are not necessarily drawn to scale. In some of the questions you may want to draw a figure when none is given.

- The statement “Jack gets his weekly allowance if he takes the garbage out” is true. Which of the following statements is also true?
  - Jack does not take the garbage out or he gets his weekly allowance.
  - Jack does not take the garbage out and he gets his weekly allowance.
  - Jack does not get his weekly allowance or he takes the garbage out.
  - Jack takes the garbage out and he does not get his weekly allowance.
  - None of these.

- In the figure at the right the measure of angle(ABC) is  $100^\circ$ . If the measures of  $\angle ABD = (2x - 1)^\circ$ ,  $\angle DBE = (x + 5)^\circ$  and  $\angle CBE = 3x^\circ$  then the measure of  $\angle ABE$  is

- $52^\circ$
- $56^\circ$
- $62.6^\circ$
- $66.4^\circ$
- None of these



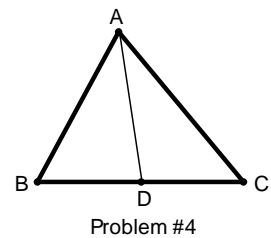
- What is the supplement of the angle whose complement is  $38^\circ$ ?

- $142^\circ$
- $138^\circ$
- $118^\circ$
- $132^\circ$
- None of these

- In the figure D is the midpoint of  $\overline{BC}$ . Which of the following statement(s) is/are true?

- P:  $\overline{BD} \cong \overline{DC}$   
 Q:  $\angle BAD \cong \angle CAD$   
 R:  $\text{Area}(\triangle ABD) = \text{Area}(\triangle ACD)$

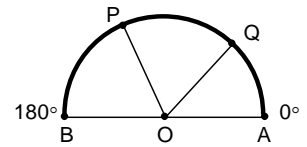
- P, Q
- P, R
- Q, R
- P
- None of these



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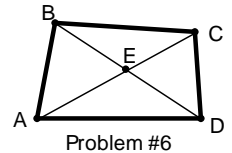
5. In the figure  $m\angle AOP = (2x + 10)^\circ$  and  $m\angle BOQ = (290 - 3x)^\circ$ . In terms of  $x$ , what is  $m\angle POQ$ ?

- A.  $(-x + 120)^\circ$       B.  $(x - 120)^\circ$       C.  $(-x + 300)^\circ$   
 D.  $(300 - x)^\circ$       E. None of these



6. In the quadrilateral ABCD, the diagonals bisect each other at E. Which of the following statements is/are true?

- P.  $\overline{AB} \parallel \overline{CD}$       Q:  $\overline{AD} \cong \overline{BC}$       R: ABCD is a parallelogram



- A. Only P and R      B. Only Q, R      C. Only P and Q  
 D. P, Q, R      E. None of these

7. Which of the following statements is/are true about a parallelogram ABCD. Draw a figure.

P:  $\angle BAD$  and  $\angle ADB$  are complementary angles.

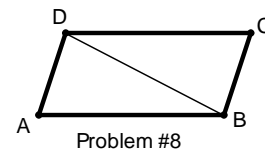
Q:  $\angle DAB \cong \angle ADC$

R:  $\overline{AC}$  bisects  $\overline{BD}$

- A. P      B. Q      C. R      D. P, Q, R  
 E. None of these

8. In the figure  $\overline{AB} \parallel \overline{CD}$  and  $\overline{AB} \cong \overline{CD}$ . Which of the following statements has to be true?

- A.  $\angle ABD \cong \angle BAD$       B.  $\triangle ABD \cong \triangle CDB$   
 C.  $\overline{AB} \cong \overline{BC}$       D.  $\angle CDB \cong \angle CBD$   
 E. None of these

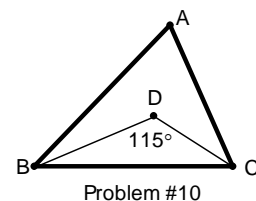


9. The orthocenter of a triangle is the point of intersection of the

- A. medians      B.  $\angle$  bisectors      C. altitudes  
 D. Perpendicular bisectors of the sides      E. None of these

10. In  $\triangle ABC$ ,  $\overline{BD}$  bisects  $\angle ABC$  and  $\overline{CD}$  bisects  $\angle ACB$ . If  $m\angle BDC = 115^\circ$ , then  $m\angle BAC =$

- A. Not enough information      B.  $30^\circ$   
 C.  $40^\circ$       D.  $50^\circ$   
 E. None of these



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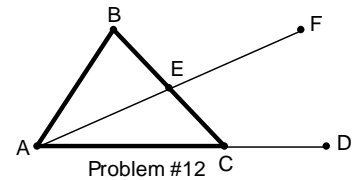
11. It is given that  $\angle ACD$  is an exterior angle of  $\triangle ABC$ . Which of the following **has** to be true?  
[Draw a figure]

- A.  $m\angle ACD > m\angle ACB$                       B.  $m\angle ABC > m\angle ACD$   
 C.  $m\angle ABC + m\angle ACB = m\angle ACD$       D.  $m\angle ABC + m\angle ACB > m\angle ACD$   
 E. None of these

12. In the figure  $\overline{AF}$  bisects  $\overline{BC}$  and  $\overline{AE} \cong \overline{EF}$ . Which of the following is/are true?

- P:  $\triangle ACE \cong \triangle FCE$                       Q:  $\overline{FC} \cong \overline{AB}$                       R:  $\angle FCD \cong \angle CAB$

- A. P, Q, R                      B. P, Q                      C. P, R  
 D. Q, R                      E. None of these



13. A triangle has an area of 20 sq ft. If the altitude is halved and the corresponding base is tripled the new triangle has an area of

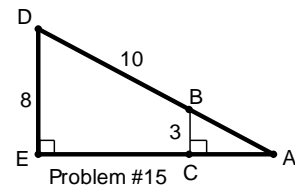
- A. 15 sq ft                      B. 36 sq ft                      C. 30 sq ft  
 D. 24 sq ft                      E. None of these

14.  $\triangle ABC \sim \triangle DEF$  and  $DE = 2.5AB$ . If the area of  $\triangle DEF = 325 \text{ in}^2$  then the area of  $\triangle ABC$  is

- A.  $130 \text{ in}^2$                       B.  $12.5 \text{ in}^2$                       C.  $203.25 \text{ in}^2$   
 D.  $52 \text{ in}^2$                       E. None of these

15. In the figure  $BC = 3$ ,  $DE = 8$  and  $BD = 10$ . Determine  $AB$ .

- A. 12                      B. 9                      C. 6  
 D. 5                      E. None of these



16. The diagonals of a rhombus measure 10 cm and 24 cm. The perimeter of the rhombus is [Draw a figure]

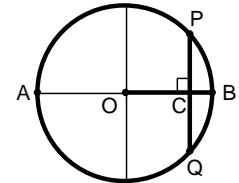
- A. Not enough information                      B. 68 cm                      C. 60 cm  
 D. 52 cm                      E. None of these

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17. The diagonal of a square is 10 ft. The area of the square is
- A.  $25 \text{ ft}^2$                       B.  $100 \text{ ft}^2$                       C.  $25\sqrt{2} \text{ ft}^2$   
D.  $20\sqrt{2} \text{ ft}^2$                       E. None of these

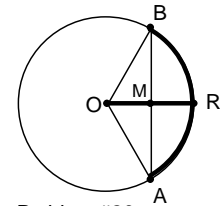
18. The perimeter of a rectangle is 84 cm. If the length of the rectangle is 6 more than the width, the length of the **diagonal** (to the nearest tenth of a centimeter) is [Draw a figure]
- A. 30.0 cm                      B. 36.5 cm                      C. 59.5 cm  
D. 32.1 cm                      E. None of these

19. The circle at the right is a unit-circle centered at the origin. If  $\overline{PQ} \perp \overline{AB}$  at C and the coordinates of C are  $(\frac{3}{4}, 0)$ , then the coordinates of P are
- A.  $(\frac{3}{4}, \frac{1}{4})$                       B.  $(\frac{3}{4}, \frac{7}{16})$                       C.  $(\frac{3}{4}, \frac{\sqrt{7}}{4})$   
D.  $(\frac{3}{4}, \frac{1}{2})$                       E. None of these



Problem #19

20. In the circle at the right the chord  $\overline{AB}$  is the perpendicular bisector of the radius  $\overline{OR}$  at M. If the diameter of the circle is 12, then the length of the arc(ARB) is
- A.  $3\pi$                       B.  $4\pi$                       C.  $6\pi$   
D. Not enough information                      E. None of these



Problem #20

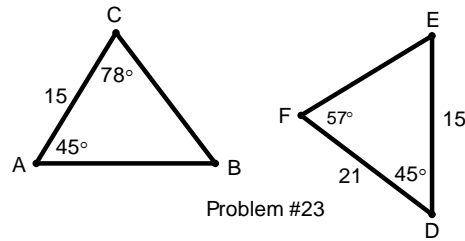
21. The point (a,b) is reflected in both the x-axis and the y-axis. The distance between the two images of these reflections is [Draw a figure]
- A.  $2a + 2b$                       B.  $2\sqrt{a^2 + b^2}$                       C.  $2a$                       D.  $2b$   
E. None of these

22. The point (4,0) is rotated  $60^\circ$  in a counterclockwise direction about the origin. The coordinates of the image are [Draw a figure]
- A.  $(2\sqrt{3}, 2)$                       B.  $(2, 2\sqrt{3})$                       C.  $(-2, 2\sqrt{3})$   
D.  $(-2\sqrt{3}, 2)$                       E. None of these

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23. Consider  $\triangle ABC$  and  $\triangle DEF$  as shown. Which of the following statements is true?

- A. The triangles are right triangles
- B.  $m\angle C > m\angle E$
- C.  $AB = 21$
- D.  $BC = 21$
- E. None of these

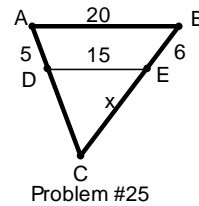


24. The parallel sides of an isosceles trapezoid measure 20 and 50. The area of the trapezoid is  $175\sqrt{3}$ . What is the perimeter of the trapezoid to the nearest hundredth? [Draw a figure]

- A. 104.64      B. 101.22      C. 99.54
- D. 106.23      E. None of these

25. In the figure if  $\overline{DE} \parallel \overline{AB}$ , then determine the value of  $x = CE$ .

- A. 4.5      B. 3.75      C. 15
- D. 18      E. None of these



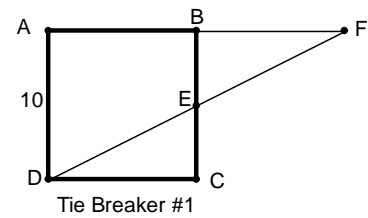
Be sure you have answered each of the previous 25 questions before proceeding to the tie-breaker questions. The tie-breakers will only be used to break ties for 1<sup>st</sup>, 2<sup>nd</sup>, and/or 3<sup>rd</sup> place. They will be used in the order given to break the ties so begin with the first tie-breaker question.

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Name \_\_\_\_\_

Tie-Breaker 1

ABCD is a square with a side of 10. If E is the midpoint of  $\overline{BC}$  and A, B and F are collinear find DF.



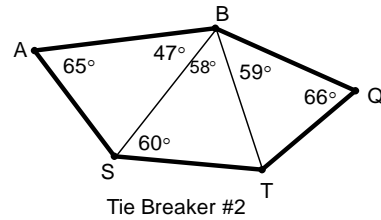
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Tie-Breaker 2

In the figure at the right determine which sign ( $<$ ,  $=$ ,  $>$ ) replaces “?” in the following. Give all the necessary reasons for your conclusions.

AB ? BQ

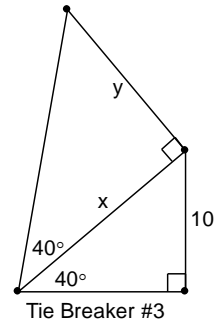


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Tie-Breaker 3

Use right triangle trigonometry to determine  $y$  to the nearest hundredth.





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Key – Geometry 2014

- |    |   |     |   |     |   |     |   |     |   |
|----|---|-----|---|-----|---|-----|---|-----|---|
| 1. | A | 6.  | D | 11. | E | 16. | D | 21. | B |
| 2. | A | 7.  | C | 12. | D | 17. | E | 22. | B |
| 3. | E | 8.  | B | 13. | C | 18. | A | 23. | C |
| 4. | B | 9.  | C | 14. | D | 19. | C | 24. | A |
| 5. | A | 10. | D | 15. | C | 20. | B | 25. | D |

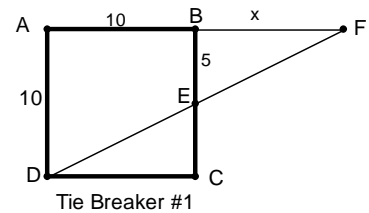
KEY – Tie-Breaker Questions

1. ABCD is a square with a side of 10. If E is the midpoint of  $\overline{BC}$  and A, B and F are collinear find DF.

Because  $\triangle FBE \sim \triangle FAD$ ,  $\frac{FB}{5} = \frac{FB+10}{10}$ . Therefore,  $FB = 10$ .

Since  $(AF)^2 + (AD)^2 = (DF)^2$ , then  $DF = \sqrt{20^2 + 10^2} = \sqrt{500}$

So,  $DF = 10\sqrt{5}$ .



2. In the figure at the right determine which sign ( $<$ ,  $=$ ,  $>$ ) replaces “?” in the following. Give all the necessary reasons for your conclusions.

$AB$  ?  $BQ$

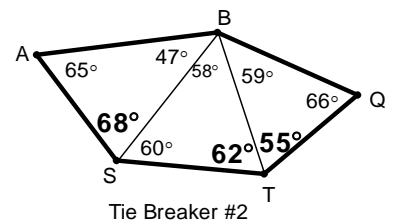
Compute remaining angles in the figure.

$AB > BS$  since  $\angle BSA$  is the larger angle in its triangle.

$BS > BT$  since  $\angle BTS$  is the larger angle in its triangle.

$BT > BQ$  since  $\angle BTQ$  is the larger angle in its triangle.

Therefore,  $AB > BQ$



3. Use right triangle trigonometry to determine  $y$  to the nearest hundredth.

$$\sin 40^\circ = \frac{10}{x} \text{ and } \tan 40^\circ = \frac{y}{x}$$

$$\text{Therefore, } \frac{10}{\sin 40^\circ} = \frac{y}{\tan 40^\circ}. \text{ So, } y = \frac{10 \tan 40^\circ}{\sin 40^\circ}.$$

So  $y = 13.05407289$ . Thus rounding,  $y = 13.05$ .

Watch for premature rounding to hundredths.

