

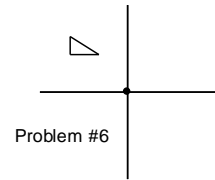
Arkansas Council of Teachers of Mathematics

2013 State Contest

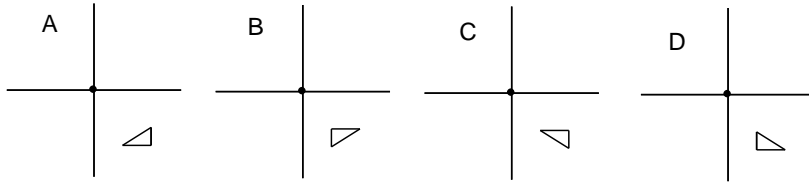
Geometry Exam

In each of the following choose the BEST answer and shade the corresponding letter on the Scantron Sheet. Answer all 25 multiple choice questions before attempting the tie-breaker questions. The tie-breaker questions at the end are to be used to resolve any ties between 1<sup>st</sup>, 2<sup>nd</sup>, and/or 3<sup>rd</sup> place. Be sure that your name is printed on each of the tiebreaker pages. The figures are not necessarily drawn to scale. Good Luck!

- Two lines intersect to form an angle that has a measure of  $53^\circ$ . Another angle formed by the two lines has a measure of
  - $180^\circ$
  - $37^\circ$
  - $127^\circ$
  - $137^\circ$
  - None of these
- An angle is two-thirds its supplement. The complement of the angle is
  - $72^\circ$
  - $28^\circ$
  - $18^\circ$
  - $108^\circ$
  - None of these
- The center of a circle is the point  $(2, -3)$ . The circle has a point of tangency at  $(5,1)$ . The slope of this tangent line is
  - $\frac{4}{3}$
  - $-\frac{4}{3}$
  - $\frac{3}{4}$
  - $-\frac{3}{4}$
  - None of these
- The slope of a line with a y-intercept at  $(0, -2)$  is  $\frac{2}{5}$ . Another point on the line is
  - $(10, 2)$
  - $(2,3)$
  - $(4,8)$
  - $(-2, -7)$
  - None of these
- The area of a circle with the equation  $(x + 3)^2 + (y - 4)^2 = 10$  is
  - $5\pi$
  - $100\pi$
  - $20\pi$
  - $10\pi$
  - None of these



6. If the right triangle in the figure at the right is rotated  $180^\circ$  about the origin, which of the following will be the rotated figure?



E. None of these

7. The diagonals of a quadrilateral are perpendicular bisectors of each other. The quadrilateral is best described as a

- A. square      B. rectangle      C. kite  
D. rhombus      E. None of these

8. Each interior angle of a regular polygon measures  $156^\circ$ . The number of sides of the polygon is

- A. 12      B. 16      C. 10      D. 15  
E. None of these

9. A particular median of a triangle divides the triangle into two congruent triangles. The triangle is best described as an

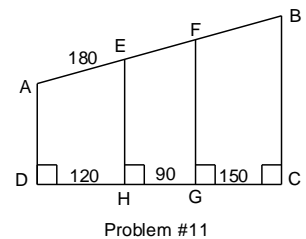
- A. equilateral triangle      B. isosceles triangle  
C. obtuse triangle      D. acute triangle      E. None of these

10. The centroid of a triangle is

- A. the point of intersection of the angle bisectors of the triangle  
B. the point of intersection of the perpendicular bisectors of the sides of the triangle  
C. the center of the circle that is tangent to the 3 sides of the triangle  
D. the center of the circle passing through the 3 vertices of the triangle  
E. None of these

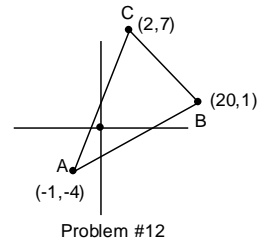
11. ABCD is a trapezoid with the indicated lengths. The length of BE is

- A. 340      B. 360      C. 540  
D. 240      E. None of these



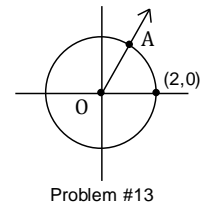
12. In the figure a median is drawn from A.  
How long is this median?

- A.  $4\sqrt{13}$       B. 10      C.  $\sqrt{149}$   
D. Not enough information      E. None of these



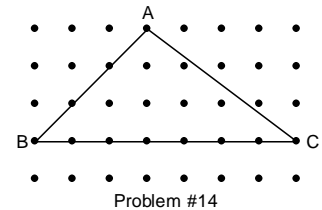
13. In the figure the ray  $\overrightarrow{OA}$  makes a  $60^\circ$  angle with the positive x-axis. The point of intersection of the ray and the circle is

- A.  $(2, 2\sqrt{3})$       B.  $(1, \sqrt{3})$       C.  $(\frac{1}{2}, \frac{\sqrt{3}}{2})$   
D.  $(\frac{\sqrt{3}}{2}, \frac{1}{2})$       E. None of these



14. In the figure at the right the vertical and horizontal distances between the dots is one-unit. Which of the following measurements is irrational?

- A. AC      B. BC      C. Area( $\Delta ABC$ )  
D. Perimeter( $\Delta ABC$ )      E. All are rational



15. The altitude of an equilateral triangle is 12. To the nearest hundredth, the perimeter of a square that has the same area as the equilateral triangle is

- A. 9.12      B. 83.14      C. 20.78      D. 36.47  
E. None of these

16.  $\Delta ABC$  is an isosceles triangle with vertex angle A. If the measure of the exterior angle ACD is  $148^\circ$ . What is the measure of the angle BAC?

- A.  $74^\circ$       B.  $116^\circ$       C.  $32^\circ$       D.  $106^\circ$   
E. None of these

17. A prism has 30 edges. The number of sides of the polygonal base is

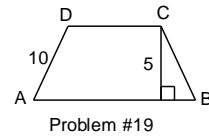
- A. 15      B. 14      C. 10      D. 9  
E. None of these

18. A clock has a circular face and a minute hand that is 10 inches in length. To the nearest hundredth, what is the area swept out by the minute hand in 8 minutes?

- A. 20.11 sq. in.      B. 4.19 sq. in.      C. 41.89 sq. in.  
 D. 25.13 sq. in      E. None of these

19. ABCD is an isosceles trapezoid with the measurements shown. The angles of the trapezoid have the measures of

- A.  $60^\circ, 60^\circ, 120^\circ, 120^\circ$       B.  $30^\circ, 30^\circ, 150^\circ, 150^\circ$   
 C.  $45^\circ, 45^\circ, 135^\circ, 135^\circ$       D.  $72^\circ, 72^\circ, 108^\circ, 108^\circ$   
 E. None of these

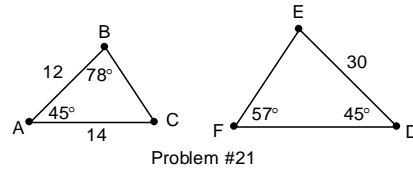


20. Which of the following geometric figures has exactly 2 lines of symmetry?

- A. equilateral triangle      B. square      C. isosceles triangle  
 D. isosceles trapezoid      E. None of these

21. Consider the triangles as shown at the right. Which of the following statements is true?

- A.  $FD = 35$       B.  $FE = 12$   
 C. Angle C has a measure greater than Angle E  
 D.  $\text{Area}(\triangle DEF) = 4\text{Area}(\triangle ABC)$



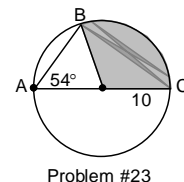
- E. None of these

22. A circle with a diameter of 30 has a central angle that intercepts an arc that has a length of  $6\pi$ . The measure of the central angle is

- A.  $36^\circ$       B.  $72^\circ$       C.  $108^\circ$       D.  $144^\circ$   
 E. None of these

23. The radius of the circle is 10 and the inscribed angle (BAC) measures  $54^\circ$ . The area of the shaded region is

- A.  $30\pi$       B.  $15\pi$       C.  $20\pi$   
 D.  $60\pi$       E. None of these

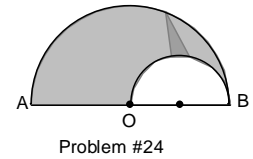


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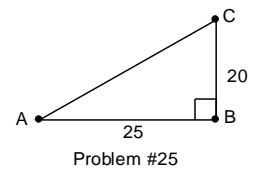
24. The figure consists of two semicircles with their diameters on  $\overline{AB}$ . If  $AB = 20$  and  $O$  is the midpoint of  $\overline{AB}$ , what is the area of the shaded region, to the nearest hundredth?

- A. 47.10      B. 57.10      C. 117.81  
 D. 117.75      E. None of these



25. What is the angle of elevation, to the nearest minute, of a person at  $A$ , to see an object at  $C$ ?

- A.  $38^\circ 40'$       B.  $36^\circ 52'$       C.  $38^\circ 30'$   
 D.  $36^\circ 42'$       E. None of these



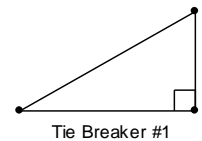
**Tie-Breaker Questions**

Name \_\_\_\_\_  
[Please Print]

School \_\_\_\_\_  
[Please Print]

**In each of the following you must show supporting work for your answers to receive credit. The questions will be used in the order given to resolve ties for 1<sup>st</sup>, 2<sup>nd</sup>, and/or 3<sup>rd</sup> place. Be sure that your name is printed on each of the tiebreaker pages.**

1. A right triangle is created from a piece of wire that is 24 inches in length. If one leg is twice the shorter leg, what is the area of the triangle, to the nearest thousandth of a square inch?



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Tie-Breaker Questions

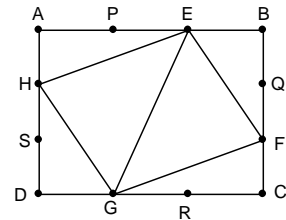
Name \_\_\_\_\_  
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School \_\_\_\_\_  
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2. ABCD is a rectangle.

$$AP = PE = GR = RC = BE = DG = \frac{1}{3} AB \text{ and } AH = HS = SD = BQ = QF = FC = \frac{1}{3} AD.$$

a. Explain why EFGH is a parallelogram.



b. Find EG.

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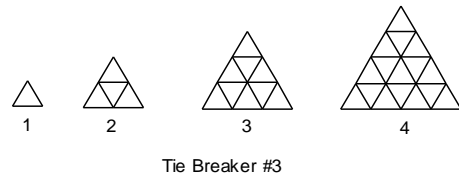
Tie-Breaker Questions

Name \_\_\_\_\_  
[Please Print]

School \_\_\_\_\_  
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3. Consider the following sequence of pictures.

- a. How many small triangles [  $\Delta$  ] are in the bottom row of the 35<sup>th</sup> picture?  
Explain



- b. How many small triangles [  $\Delta$  ] are in the 35<sup>th</sup> picture? Explain!



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KEY  
Multiple Choice

- |    |   |     |   |     |   |
|----|---|-----|---|-----|---|
| 1. | C | 10. | E | 18. | C |
| 2. | C | 11. | B | 19. | B |
| 3. | D | 12. | A | 20. | E |
| 4. | A | 13. | B | 21. | A |
| 5. | D | 14. | D | 22. | B |
| 6. | C | 15. | D | 23. | A |
| 7. | D | 16. | B | 24. | C |
| 8. | D | 17. | C | 25. | A |
| 9. | B |     |   |     |   |

KEY  
Tie Breakers

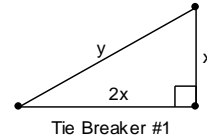
1. A right triangle is created from a piece of wire that is 24 inches in length. If one leg is twice the shorter leg, what is the area of the triangle, to the nearest thousandth of a square inch?

$$y = \sqrt{x^2 + 4x^2} = x\sqrt{5}$$

$$x + 2x + x\sqrt{5} = 24$$

$$x = \frac{24}{3 + \sqrt{5}}$$

$$A = \left(\frac{24}{3 + \sqrt{5}}\right)^2 = 21.009$$



2. ABCD is a rectangle.

$$AP = PE = GR = RC = BE = DG = \frac{1}{3}AB \text{ and } AH = HS = SD = BQ = QF = FC = \frac{1}{3}AD.$$

- a. Explain why EFGH is a parallelogram.

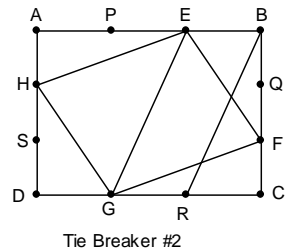
Use a slope argument or find HE, GF and EF, HD

$$\text{Slope (EH)} = \frac{\frac{1}{3}AD}{\frac{2}{3}AB} = \frac{AD}{2AB}$$

$$\text{Slope(FG)} = \frac{\frac{1}{3}AD}{\frac{2}{3}AB} = \frac{AD}{2AB}$$

$$\text{Slope (HG)} = \frac{-\frac{2}{3}AD}{\frac{1}{3}AB} = -\frac{2AD}{AB}$$

$$\text{Slope(EF)} = \frac{-\frac{2}{3}AD}{\frac{1}{3}AB} = -\frac{2AD}{AB}$$



Therefore,  $\overline{HE} \parallel \overline{FG}$  and  $\overline{HG} \parallel \overline{EF}$

So, HEFG is a parallelogram because the opposite pairs of sides are parallel.

$$\text{Distance argument: } HE = \frac{1}{3}\sqrt{AD^2 + 4AE^2}; FG = \frac{1}{3}\sqrt{AD^2 + 4AE^2} \quad HE = FG$$

$$HG = \frac{1}{3}\sqrt{AB^2 + 4AD^2}; EF = \frac{1}{3}\sqrt{AB^2 + 4AD^2}; HG = EF.$$

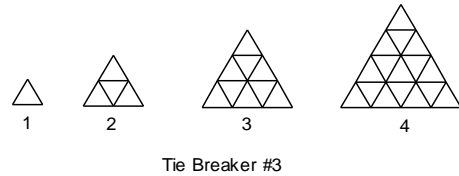
So, HEFG is a parallelogram because opposite pairs of sides are equal in length.

- b. Find EG.

$$\text{Note: } EG = BR. \quad EG = \frac{1}{3}\sqrt{AB^2 + 9BC^2}$$

3. Consider the following sequence of pictures.

- a. How many small triangles [  $\Delta$  ] are in the bottom row of the 35<sup>th</sup> picture? Explain!



1<sup>st</sup> row  $1 = 2(1) - 1$

2<sup>nd</sup> row  $3 = 2(2) - 1$

3<sup>rd</sup> row  $5 = 2(3) - 1$

4<sup>th</sup> row  $7 = 2(4) - 1$

Continuing the pattern, 35<sup>th</sup> row  $2(35) - 1 = 69$ .

- b. How many small triangles [  $\Delta$  ] are in the 35<sup>th</sup> picture? Explain!

1<sup>st</sup> row  $1 = 1^2$

1<sup>st</sup> and 2<sup>nd</sup> row -  $4 = 2^2$

1<sup>st</sup>, 2<sup>nd</sup>, and 3<sup>rd</sup> row  $9 = 3^2$

1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup>, and 4<sup>th</sup> row  $16 = 4^2$

Continuing the pattern the number of triangles in the 35<sup>th</sup> picture is  $35^2 = 1225$