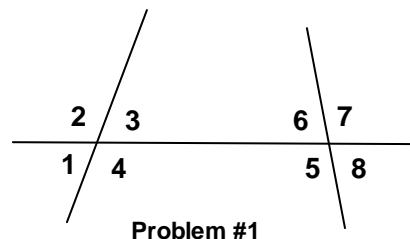


2008 ACTM State Geometry Exam

In each of the following choose the best answer and bubble the corresponding letter on the answer sheet provided. Be sure to work all 25 questions before working the tie-breaker problems. **Be aware that the figures are not always drawn to scale.**

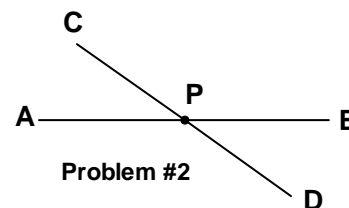
1. In the figure $\angle 3 \cong \angle 6$. A pair of supplementary angles is

- A. $\angle 2, \angle 4$
- B. $\angle 4, \angle 5$
- C. $\angle 1, \angle 6$
- D. $\angle 1, \angle 7$
- E. None of these



2. Let P be the midpoint of \overline{AB} and \overline{CD} . Which of the following statements is true?

- R. $\overline{AC} \parallel \overline{BD}$
- S. $\overline{AD} \cong \overline{BC}$
- T. $AP + PD = BP + PC$



- A. R and S
- B. R and T
- C. S and T
- D. R, S, and T
- E. None of These

3. The complement of an angle is $\frac{7}{25}$ of the supplement of the angle. What is the measure of the complement of the angle?

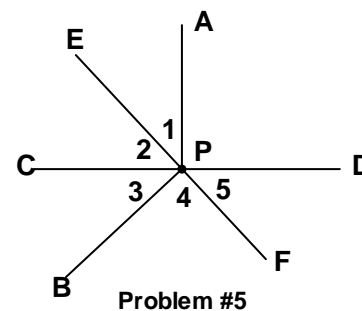
- A. 35°
- B. 145°
- C. 55°
- D. 125°
- E. None of these

4. In an isosceles trapezoid the diagonals

- A. bisect each other
- B. are perpendicular
- C. are congruent
- D. intersect at a 45° angle
- E. None of these

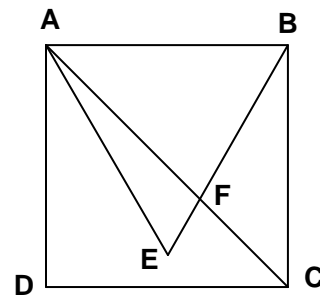
5. In the figure $\overrightarrow{PA} \perp \overrightarrow{CD}$ and $\overrightarrow{PB} \perp \overrightarrow{EF}$. A pair of congruent angles is

- A. $\angle 1, \angle 3$
- B. $\angle 2, \angle 3$
- C. $\angle 1, \angle 5$
- D. $\angle 3, \angle 5$
- E. None of these



6. If ABCD is a square and $\triangle ABE$ is an equilateral triangle, then the measure of $\angle BFC$ is

- A. 75° B. 90° C. 105°
 D. 100° E. None of these



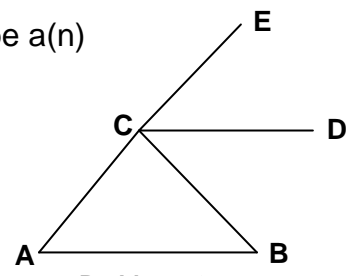
Problem #6

7. The sum of the measures of the interior angles of a polygon is 2160° . The number of diagonals in the polygon is

- A. 14 B. 77 C. 154
 D. 84 E. None of these

8. In the figure, $\overline{CD} \parallel \overline{AB}$ and \overline{CD} bisects $\angle BCE$. $\triangle ABC$ must be a(n)

- A. equilateral triangle B. obtuse triangle
 C. isosceles triangle D. scalene triangle
 E. None of these

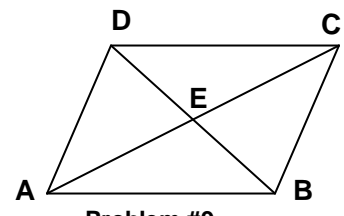


Problem #8

9. ABCD is a parallelogram. If $m \angle BAD = 80^\circ$ which of the following must be true?

- P. $m \angle AED = 90^\circ$
 Q. $\overline{DE} \cong \overline{AE}$
 R. $m \angle ADE = 50^\circ$

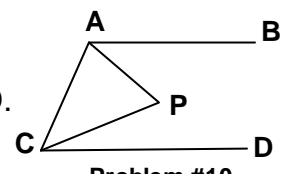
- A. P and Q B. P and R C. Q and R
 D. R E. None of these



Problem #9

10. In the figure $\overline{AB} \parallel \overline{CD}$, \overline{AP} bisects $\angle BAC$ and \overline{CP} bisects $\angle ACD$. Then $\angle APC$ is a(n)

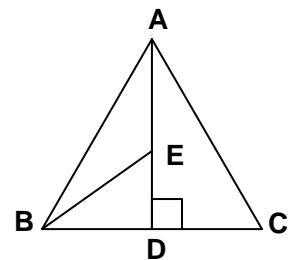
- A. right angle B. acute angle C. obtuse angle
 D. not enough information E. None of these



Problem #10

11. $\triangle ABC$ is an equilateral triangle with $AB = 30$. If \overline{AD} is an altitude and $DE = \frac{1}{3}AD$, then the measure of $\angle ABE$ is

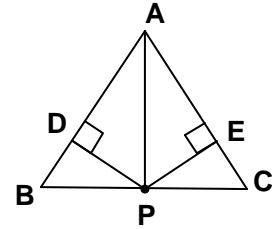
- A. 36° B. 45° C. 120°
 D. 30° E. None of these



Problem #11

12. $\triangle ABC$ is an isosceles triangle with $AB = AC$. P is the midpoint of \overline{BC} , $\overline{PD} \perp \overline{AB}$ and $\overline{PE} \perp \overline{AC}$. Which of the following triangle congruences would be used to prove $\triangle BDP \cong \triangle CEP$?

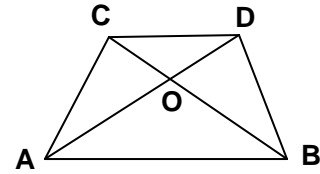
- A. SAS B. SSA C. SSS
D. AAS E. None of these



Problem #12

13. In the figure, $\overline{CD} \parallel \overline{AB}$. To prove $OB(OD) = OA(OC)$ one could prove

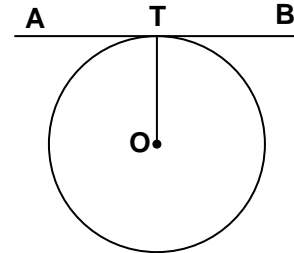
- A. $\triangle AOC \sim \triangle BOD$ B. $\triangle AOB \sim \triangle DOC$
C. $\triangle ACD \sim \triangle DBA$ D. $\triangle COD \sim \triangle BOD$
E. None of these



Problem #13

14. \overline{AB} is tangent to the circle at T . If O is the center of the circle, which of the following is/are true?

- A. \overline{OT} bisects $\angle AOB$
B. \overline{OT} is the perpendicular bisector of \overline{AB}
C. $\overline{OA} \perp \overline{OB}$ D. $\overline{OA} \cong \overline{OB}$
E. None of these



Problem #14

15. A ladder leaning against a wall has a slope of $\frac{3}{4}$. If the top of the ladder is 10 feet above the base of the wall, then how long is the ladder?

- A. 16 feet 8 inches B. 12 feet 4 inches C. 13 feet 4 inches
D. 15 feet 6 inches E. None of these

16. Let $A = (-2, 1)$ and $B = (4, 3)$ be points in the coordinate plane. The equation of the circle that has \overline{AB} for a diameter is

- A. $(x - 1)^2 + (y - 2)^2 = 40$ B. $(x - 1)^2 + (y - 2)^2 = 10$
C. $(x - 1)^2 + (y - 2)^2 = 25$ D. $(x - 1)^2 + (y - 2)^2 = 5$
E. None of these

17. A line that is perpendicular to the line $3x - 2y = 6$ and passes through $(1,0)$ has the equation

A. $y = \frac{2}{3}(x - 1)$

B. $y = -\frac{2}{3}(x - 1)$

C. $y = -\frac{3}{2}(x - 1)$

D. $y = \frac{3}{2}(x - 1)$

E. None of these

18. $\triangle ABC$ is an isosceles triangle with D and E midpoints of \overline{AB} and \overline{AC} , respectively. If $AB = 12$ and the perimeter of trapezoid $BCED$ is 36 then $BC =$

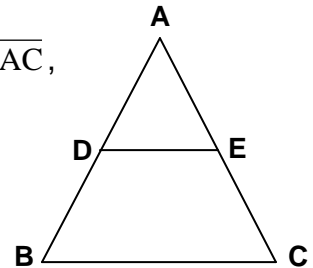
A. 8

B. 16

C. 18

D. 9

E. None of these



Problem #18

19. A regular dodecagon (12 sides) is inscribed in a circle whose diameter is 20 inches. What is the length (to the nearest hundredth of an inch) of a side of the dodecagon?

A. 6.50 inches

B. 13.00 inches

C. 2.59 inches

D. 5.18 inches

E. None of these

20. $ABCDEF$ is a regular hexagon inscribed in a circle with a radius of 6. What is the perimeter of the rectangle $BCEF$?

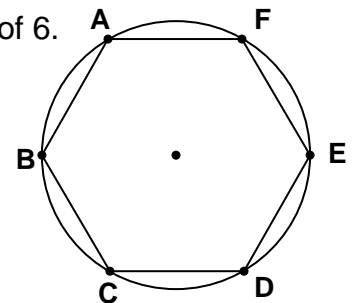
A. $12(1 + \sqrt{3})$

B. $36\sqrt{3}$

C. $12(1 + \sqrt{5})$

D. 24

E. None of these



Problem #20

21. A polygon is circumscribed about a circle. Which of the following is always true?

P The bisectors of the angles of the polygon intersect at the center of the circle.

Q The perpendicular bisectors of the sides intersect at the center of the circle.

R The area of the polygon is one-half the product of the radius of the circle and the perimeter of the polygon.

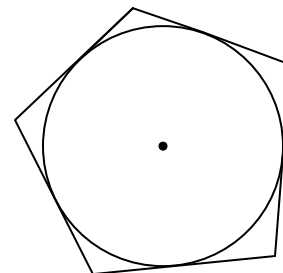
A. P and Q

B. P and R

C. Q and R

D. P, Q, and R

E. None of these



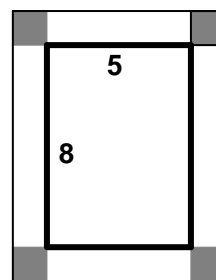
Problem #21

22. Let $A = (5, -2)$ and $B = (8, y)$. If $AB = \sqrt{58}$ and $y \geq 5$, then $y =$

- A. 10 B. 6 C. 5 D. -9
E. None of these

23. From a rectangular sheet of paper, Thomas makes an open rectangular prism by cutting squares from each corner and folding up the sides. If the base of the prism is 5 inches by 8 inches and it has a volume of 80 cubic inches, what is the area of the original sheet of paper?

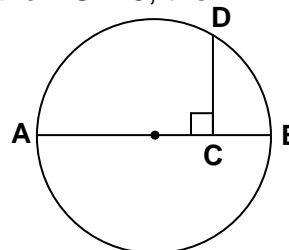
- A. 90 sq in B. 92 sq in
C. 108 sq in D. 56 sq in
E. None of these



Problem #23

24. In the circle \overline{AB} is a diameter and $\overline{CD} \perp \overline{AB}$. If $AB = 13$ and $AC = 9$, then $AD =$

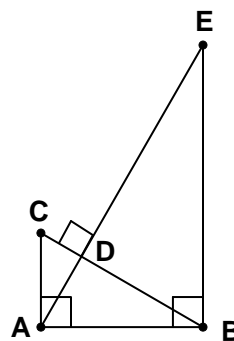
- A. Not enough information B. 10
C. 12 D. $\sqrt{117}$
E. None of these



Problem #24

25. In the figure at the right $m\angle ABD = 30^\circ$, $m\angle BAE = 60^\circ$ and the perpendiculars are as indicated. The ratio of the perimeter of $\triangle ABC$ to the perimeter of $\triangle BDE$ is

- A. 1 : 2 B. $\sqrt{3} : 3$
C. 1 : 3 D. 1 : 6
E. None of these



Problem #25

Be sure you have completed the 25 questions on the exam before you attempt the tie-breaker problems.

Tie Breaker Questions

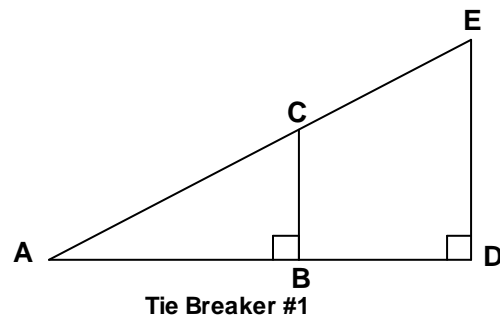
Name _____
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School _____

The tie-breaker questions will be used to break ties between 1st, 2nd, and 3rd should a tie occur. They will be used in the order they are given.

Tie-Breaker #1

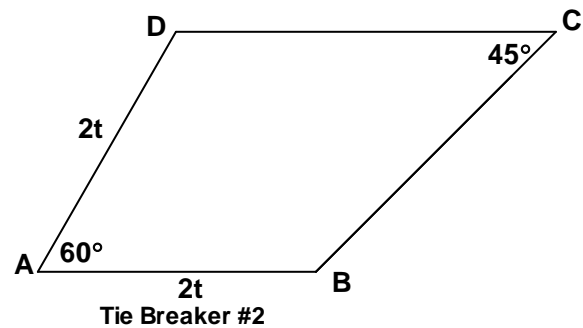
In the figure at the right $AC = 30$, $AD = 42$ and $CB = 18$. Determine the length CE .



Tie Breaker #2

In trapezoid $ABCD$, $\overline{CD} \parallel \overline{AB}$. If $AB = AD = 2t$, express CD in terms of the variable t .

Hint: Draw perpendiculars from C and D to \overline{AB} .

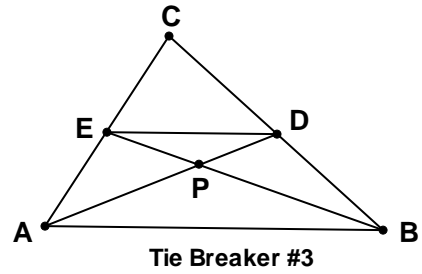


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Tie-Breaker #3

In $\triangle ABC$, D and E are midpoints of \overline{BC} and \overline{AC} , respectively. \overline{AD} and \overline{BE} are medians of the triangle. What is the ratio of $\text{Area}(\triangle ABC)$ to the $\text{Area}(\triangle PED)$? Justify your answer.



Key – 2008 Geometry Exam

- 1. D
- 2. D
- 3. A
- 4. C
- 5. A
- 6. C
- 7. B
- 8. C
- 9. E
- 10. A
- 11. D
- 12. D
- 13. B

- 14. E
- 15. A
- 16. B
- 17. B
- 18. B
- 19. D
- 20. A
- 21. D
- 22. E
- 23. C
- 24. D
- 25. B

Tie Breaker Questions (Key)

Name _____ KEY _____

School _____

Please Print

The tie-breaker questions will be used to break ties between 1st, 2nd, and 3rd should a tie occur. They will be used in the order they are given.

Tie-Breaker #1

In the figure at the right $AC = 30$, $AD = 42$ and $CB = 18$. Determine the length CE .

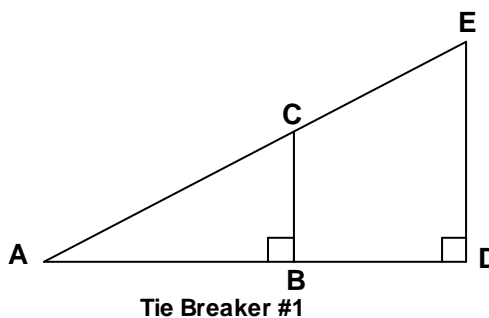
$$AB = \sqrt{30^2 - 18^2} = 24$$

$$BD = 42 - 24 = 18$$

$$\frac{x}{30} = \frac{18}{24}$$

$$24x = 540$$

$$x = 22.5$$



Tie Breaker #2

In trapezoid ABCD, $\overline{CD} \parallel \overline{AB}$. If $AB = AD = 2t$, express CD in terms of the variable t .

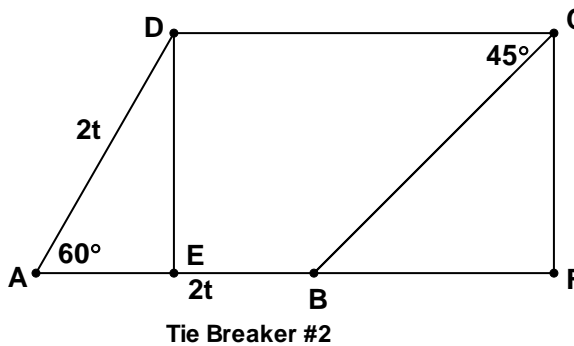
Hint: Draw perpendiculars from C and D to \overline{AB} .

$\triangle ADE$ is a $30^\circ - 60^\circ - 90^\circ$ triangle.

Therefore, $DE = t\sqrt{3}$ and $CE = t\sqrt{3}$

Since $\triangle BCF$ is a $45^\circ - 45^\circ - 90^\circ$ triangle,

$BF = t\sqrt{3}$.



Therefore, $DC = EB + BF = t + t\sqrt{3}$

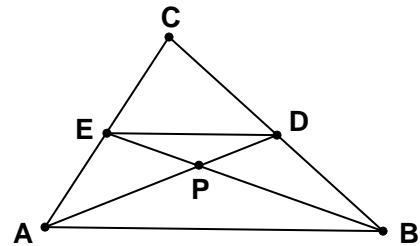
Name KEY
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School _____

Tie-Breaker #3

In $\triangle ABC$, D and E are midpoints of \overline{BC} and \overline{AC} , respectively. \overline{AD} and \overline{BE} are medians of the triangle. What is the ratio of $\text{Area}(\triangle ABC)$ to the $\text{Area}(\triangle PED)$? Justify your answer.

The medians of the triangle intersect in a point that divides the medians into segments that are in a 2:1 ratio.



Tie Breaker #3

Since $\triangle PED \sim \triangle PBA$ then the $4\text{Area}(\triangle PED) = \text{Area}(\triangle PBA)$.

The altitude of $\triangle PBA$ to the side \overline{AB} is one-third the altitude of $\triangle ABC$ to the side \overline{AB} .

Therefore, $3\text{Area}(\triangle PBA) = \text{Area}(\triangle ABC)$.

Thus $12\text{Area}(\triangle PED) = \text{Area}(\triangle ABC)$.

So the ratio, $\text{Area}(\triangle ABC) : \text{Area}(\triangle PED) = 12 : 1$.