

Regional Precalculus/Trigonometry Contest 2008

Select the best answer for each of the following questions and mark it on the answer sheet provided. Be sure to read all the answer choices before making your selection. When you are finished with the multiple choice, attempt the tiebreaker questions.

1. If a right triangle has a hypotenuse of length 14.563 m and a shorter leg of length 3.680 m, then the length of the longer leg to the nearest hundredth of a meter is

- a) 15.02 m b) 14.64 m c) 14.09 m d) 13.64 m e) 12.82 m

2. If the legs of a right triangle are of lengths 6.000 m and 9.000 m, then the measure of the smaller acute angle of the triangle is closest to

- a) 56.31° b) 42.00° c) 37.65° d) 33.69° e) 28.20°



Figure 1: Triangles for Problems 3-4

3. The length of side a in Figure 1 is closest to

- a) 27.3 m b) 27.5 m c) 33.1 m d) 33.3 m e) 38.0 m

4. The length of side b in Figure 1 is closest to

- a) 3.14 m b) 3.26 m c) 3.68 m d) 3.95 m e) 9.84 m

5. The area of a triangle with sides of lengths 4.00 ft, 7.00 ft, and 10.00 ft, is closest to

- a) 10.93 sq. ft. b) 14.00 sq. ft. c) 35.00 sq. ft. d) 24.50 sq. ft. e) 18.24 sq. ft.

6. If θ is an angle in standard position whose terminal side passes through the point $(-5, 12)$, then the value of $\sin \theta$ is

- a) $\frac{12}{13}$ b) $-\frac{12}{13}$ c) $\frac{5}{13}$ d) $-\frac{5}{14}$ e) $-\frac{5}{12}$

7. The exact value of $\sec \frac{\pi}{6}$ is

- a) $\frac{1}{2}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{2\sqrt{3}}{3}$ d) 2 e) $\sqrt{3}$

8. If θ is a second quadrant angle for which $\sin \theta = \frac{1}{2}$, then the value of $\cos \theta$ is
- a) 0 b) $\frac{\sqrt{3}}{2}$ c) $-\frac{1}{2}$ d) $-\frac{\sqrt{3}}{2}$ e) 1
9. The graph of $y = \cos x$ is nothing more than the graph of $y = \sin x$ shifted
- a) to the right π units b) to the right $\frac{\pi}{2}$ units c) to the right $\frac{\pi}{4}$ units d) to the left $\frac{\pi}{4}$ units e) to the left $\frac{\pi}{2}$ units
10. Two functions that have a range of $(-\infty, \infty)$ are
- a) the sine and cosine b) the tangent and cotangent c) the secant and cosecant
d) the sine and secant e) the cosine and cosecant
11. The function $f(x) = 5 \sin(\pi x - \frac{\pi}{6})$ has
- a) amplitude 5 and period 2 b) amplitude 5 and period 2π c) amplitude 5 and period $\frac{1}{6}$
d) amplitude π and period 5 e) amplitude π and period $\frac{1}{6}$
12. If $\cos x = 0.6$, $\sin y = 0.5$, $\frac{3\pi}{2} \leq x \leq 2\pi$, and $\frac{\pi}{2} \leq y \leq \pi$, then the value of $\sin(x + y)$ is
- a) $\frac{\sqrt{3}}{10}$ b) 0.1 c) 1.1 d) $\frac{3 + 4\sqrt{3}}{10}$ e) $\frac{3 - 4\sqrt{3}}{10}$
13. The graph of the polar coordinates equation $r = \sin 2\theta$ is
- a) a 2 petal rose b) a 4 petal rose c) a cardioid d) a limaçon e) a circle
14. The solution of the equation $\log_2 x = 5$ is
- a) $x = 25$ b) $x = 10$ c) $x = \frac{5}{2}$ d) $x = \frac{2}{5}$ e) $x = 32$
15. The solution of the equation $\log_6 x + \log_6(x + 2) = 0$ is
- a) $1 + \sqrt{2}$ b) $-1 + \sqrt{2}$ c) 4 d) 6 e) 12
16. Suppose R is the Richter number of an earthquake of intensity I and that I_0 is the intensity of Richter's reference quake. Then $R = \log \frac{I}{I_0}$. How many times more intense is a quake of magnitude 6.9 on the Richter scale than a quake of magnitude 6.2 on that scale?
- a) 1.113 b) 0.700 c) 7.000 d) 3.162 e) 5.012
17. The quotient of complex numbers $\frac{3 + 2i}{3 - 4i}$ in $a + bi$ form is
- a) $0.04 + 0.72i$ b) $1.00 - 0.50i$ c) $1.00 + 0.50i$ d) $0.68 - 0.24i$ e) $0.00 + 5.00i$
18. When dividing $x^3 - x^2 + 2x - 2$ by $x - 1$ one obtains
- a) $x - 1 + \frac{3}{x-1}$ b) $x^2 - x + 1 - \frac{3}{x-1}$ c) $x^2 + 2$ d) $x^2 + x + 2$ e) $x^2 + x - 2$

19. The slope of the line with equation $2x + 3y = 6$ is

- a) $\frac{2}{3}$ b) $-\frac{2}{3}$ c) $\frac{3}{2}$ d) $-\frac{3}{2}$ e) $\frac{1}{2}$

20. The vertex of the parabola $y = 2x^2 + 4x + 8$ has y -coordinate

- a) 8 b) 6 c) 4 d) 2 e) 1

21. If the equation $ax^3 + bx^2 + cx + d = 0$, where a, b, c , and d , are real constants has the solution $2 + 3i$, then how many real solutions does the equation have?

- a) none b) exactly one c) exactly two d) exactly three e) more than three

22. The following system of equations has how many solutions

$$\begin{aligned}x + 2y + 3z &= 4 \\2x + 4y + 6z &= 8 \\2x + 2y + 3z &= 5\end{aligned}$$

- a) none b) exactly one c) exactly two d) exactly three e) infinitely many

23. The line that is perpendicular to $4x + 2y = 8$ and passes through the point $(1,3)$ also passes through the point

- a) $(2,1)$ b) $(0,4)$ c) $(6,2)$ d) $(10,6)$ e) $(11,8)$

24. If $f(x) = \ln(3x - 1)$, then the inverse function $f^{-1}(x)$ of $f(x)$ is given by

- a) $f^{-1}(x) = e^{3x+1}$ b) $f^{-1}(x) = e^{\frac{1}{3}x+1}$ c) $f^{-1}(x) = \frac{1}{\ln(3x-1)}$ d) $f^{-1}(x) = \frac{1}{3}(e^x + 1)$

e) $f^{-1}(x) = \frac{1}{3}e^{x+1}$

25. The multiplicative inverse A^{-1} of the matrix $A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ is

- a) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ b) $\begin{pmatrix} 1 & \frac{1}{2} & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ c) $\begin{pmatrix} 1 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ d) $\begin{pmatrix} 1 & -1 & -2 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ e) $\begin{pmatrix} -1 & -2 & -1 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$

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Tiebreakers

1. Find the exact value of $\cos 11.25^\circ$

2. Find the exact value of x that solves the equation $3e^x = 4^x$.

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3. Compute to the nearest hundredth of a degree the smallest angle of a triangle with sides of lengths 9.000 m, 7.000 m, and 4.000 m.

Key for the Regional Precalculus/Trigonometry Contest 2008

1. C
2. D
3. C
4. A
5. A
6. A
7. C
8. D
9. E
10. B
11. A
12. D
13. B
14. E
15. B
16. E
17. A
18. C
19. B
20. B
21. B
22. E
23. E
24. D
25. D

Tiebreaker 1.

$$\begin{aligned}\cos 11.25^\circ &= \cos\left(\frac{22.5^\circ}{2}\right) \\ &= \sqrt{\frac{1}{2} + \frac{1}{2} \cos 22.5^\circ} \\ &= \sqrt{\frac{1}{2} + \frac{1}{2} \sqrt{\frac{1}{2} + \frac{1}{2} \cos 45^\circ}} \\ &= \sqrt{\frac{1}{2} + \frac{1}{2} \sqrt{\frac{1}{2} + \frac{1}{2} \frac{\sqrt{2}}{2}}} \\ &= \frac{\sqrt{2 + \sqrt{2 + \sqrt{2}}}}{2}\end{aligned}$$

Other equivalent answers would be acceptable.

Tiebreaker 2.

$$\begin{aligned}3e^x &= 4^x \\ \ln 3e^x &= \ln 4^x \\ \ln 3 + \ln e^x &= x \ln 4 \\ \ln 3 &= (\ln 4)x - x \\ x &= \frac{\ln 3}{\ln 4 - 1}\end{aligned}$$

Other equivalent answers like $\frac{\log 3}{\log 4 - \log e}$ would be acceptable.

Tiebreaker 3.

$$4^2 = 7^2 + 9^2 - 2(7)(9) \cos \theta$$

so

$$\cos \theta = 0.9047619$$

or $\theta = 25.21^\circ$