

ACTM Regional Algebra II Exam
March 8, 2008

Place the letter of the correct answer in the space provided as well as on the answer sheet. Make sure you attempt the Tie-Breaker questions at the end of the test if you have time.

_____ 1. Perform the indicated operations and simplify: $5x - 2x[7 - (x - 3)]$

A) $30x - 3x^2$

B) $-15x + 2x^2$

C) $12x - 3x^2$

D) $-3x + 2x^2$

_____ 2. Factor completely, relative to the integers: $3x^2 + x - 14$

A) $(3x + 7)(x - 2)$

B) $(3x - 7)(x + 2)$

C) $(3x - 2)(x + 7)$

D) $(3x + 2)(x - 7)$

_____ 3. Perform the indicated operations and simplify: $9 - \frac{x^2}{y^2}$
 $3 + \frac{x}{y}$

A) $\frac{3y - x}{y}$

B) $3 - x$

C) $\frac{9y^2 - x^2}{3y + x}$

D) $\frac{(9y - x)(9y + x)}{y(3y + x)}$

_____ 4. Perform the indicated operations and simplify: $\frac{4x + 8}{9x^3} \div \frac{x^2 - 4}{3x}$

A) $\frac{4(x + 2)^2(x - 2)}{27x^4}$

B) $\frac{4}{3x^2(x - 2)}$

C) $\frac{4x}{x - 2}$

D) $\frac{4}{x^2(x - 2)}$

_____ 5. Simplify and write your answer using positive exponents only: $\left(\frac{4x^{-1}y^8}{12x^2y^6}\right)^{-2}$

A) $\frac{9x^2}{y^4}$

B) $\frac{9x^9}{y^4}$

C) $\frac{y^4}{9x^6}$

D) $\frac{9x^6}{y^4}$

_____ 6. Simplify: $\left(7x^{\frac{1}{2}}\right)\left(5x^{\frac{3}{4}}\right)$

A) $35x^{\frac{3}{8}}$

B) $35x^{\frac{4}{6}}$

C) $35x^{\frac{5}{4}}$

D) $12x^{\frac{2}{3}}$

_____ 7. Solve the equation for x: $\frac{3}{x + 5} + \frac{2}{x - 4} = \frac{13}{(x + 5)(x - 4)}$

A) $x = -5, x = 4$

B) $x = 3$

C) $x = \frac{5}{6}$

D) $x = \frac{12}{5}$

_____ 8. Solve the inequality and write the final answer using interval notation: $|4x + 8| \geq 12$

- A) $(-\infty, -5] \cup [1, \infty)$ B) $(-5, 1)$ C) $(-\infty, -5) \cup (1, \infty)$ D) $[-5, 1]$

_____ 9. Solve the inequality and write the final answer using interval notation: $-7 < -3x + 5 < 11$

- A) $(4, -2)$ B) $(-4, 2)$ C) $(-2, 4)$ D) $\left(\frac{-16}{3}, \frac{2}{3}\right)$

_____ 10. Perform the indicated operations and reduce to lowest terms: $\frac{4+i}{3-2i}$

- A) $\frac{12+11i+2i^2}{9-4i^2}$ B) $\frac{10+11i}{13}$ C) $\frac{14-5i}{5+2i}$ D) 4

_____ 11. Solve the radical equation for x and check your answer: $\sqrt{x+14} = x+2$

- A) $\{-2, 5\}$ B) $\{-5\}$ C) $\{2\}$ D) $\{-5, 2\}$

_____ 12. Solve for x using the quadratic formula: $x^2 + 5x - 2 = 0$

- A) $x = \frac{-5 \pm \sqrt{33}}{2}$ B) $x = \frac{-5 \pm \sqrt{17}}{2}$ C) $x = -5 \pm \frac{\sqrt{33}}{2}$ D) $x = -5 \pm \frac{\sqrt{17}}{2}$

_____ 13. Find the domain of the function and write your answer in interval notation: $f(x) = \sqrt{3x-15}$

- A) $(-\infty, 5]$ B) $[5, \infty)$ C) $(5, \infty)$ D) $(-\infty, 5) \cup (5, \infty)$

_____ 14. If $f(x) = 3x + 4$ and $g(x) = x^2 - 5$, find $(g \circ f)(x)$ and simplify.

- A) $3x^3 + 4x^2 - 15x - 20$ B) $9x^2 + 11$ C) $9x^2 + 24x + 11$ D) $3x^2 - 11$

_____ 15. Determine the vertex of the parabola given by: $f(x) = 3(x+2)^2 - 9$

- A) $(2, 9)$ B) $(-2, -9)$ C) $(2, -9)$ D) $(-2, 9)$

_____ 16. Given that $x = 2$ is a root of $P(x) = x^3 - 3x^2 - 18x + 40$, find the remaining roots.

- A) $x = -5, x = 4$ B) $x = -8, x = 5$ C) $x = -5, x = 8$ D) $x = -4, x = 5$

_____ 17. Write in terms of simplest logarithmic form: $\log_b \left(\frac{x^4 y^2}{z^5} \right)$

A) $\frac{(4 \log_b x)(2 \log_b y)}{(5 \log_b z)}$

B) $4 \log_b x + 2 \log_b y - 5 \log_b z$

C) $\log_b x^4 + \log_b y^2 - \log_b z^5$

D) $\log_b(x^4 + y^2 - z^5)$

_____ 18. Solve for x: $6^{5x+2} = 36^{3x-4}$

A) $x = -1$

B) $x = 10$

C) $x = 6$

D) $x = 2$

_____ 19. Solve for x exactly: $\log_3 x + \log_3(x-8) = 2$

A) $x = -1, x = 9$

B) $x = 9$

C) $x = -1$

D) $x = 1, x = 3$

_____ 20. Solve for x exactly: $\log x - \log 3 = \log 4 - \log(x+4)$

A) $x = -2, x = 6$

B) $x = -6, x = 2$

C) $x = 2$

D) $x = 6$

An urn contains 100 marbles. Fifty are purple, ten are green, fifteen are red, twenty-five are orange. Two marbles are drawn at random one after another without replacement. Calculate the following:

_____ 21. $P(\text{green and green})$

A) $\frac{100}{10000}$

B) $\frac{90}{10000}$

C) $\frac{90}{100}$

D) $\frac{90}{9900}$

_____ 22. Suppose you drive a distance of 147 miles at a speed of 35 miles per hour. How many hours does it take?

A) 3.5 hours

B) 4.2 hours

C) 2.7 hours

D) 0.238 hour

_____ 23. Find the center and radius of the circle given by the equation: $x^2 + y^2 + 2x + 4y = 11$

A) $(-2, -1), r = 4$

B) $(2, 1), r = 4$

C) $(1, 2), r = 4$

D) $(-1, -2), r = 4$

_____ 24. Find the domain for $f(x) = \frac{3}{x-6}$

A) $(-\infty, -6] \cup [-6, \infty)$

B) $(-\infty, 6) \cup (6, \infty)$

C) $(-\infty, -6) \cup (-6, \infty)$

D) $(-\infty, 6] \cup [-6, \infty)$

_____ 25. A box with an open top is to be constructed from a square piece of cardboard, 3 ft wide, by cutting out a square of length x from each of the four corners and bending up the sides. Find the formula for the volume of the box in cubic inches.

A) $V = x^3$

B) $V = 36^3$

C) $V = x(36-2x)^2$

D) $V = x^2(36-2x)$

Tie Breaker #1

Jennifer ran 6 miles in 43.7 minutes. Find her speed in feet per second.

Tie Breaker #2

In how many ways can 11 players be selected to form a football team if each of the eleven players can play any position? You may leave your answer as a combination or permutation, whichever is more appropriate.

Tie Breaker #3

The half-life of cesium-137 is 30 years. Suppose we have a 200 mg sample. (a) Find the mass that remains after t years. (b) How much of the sample remains after 100 years? (c) After how long will only 1 mg remain?

Answers

1. B
2. A
3. A
4. B
5. D
6. C
7. B
8. A
9. C
10. B
11. C
12. A
13. B
14. C
15. B
16. D
17. B
18. B
19. B
20. C
21. D
22. B
23. D
24. B
25. C

Tie Breaker #1

12.08 ft/sec

Tie Breaker #2

${}_{25}C_{11}$ or 4457400

Tie Breaker #3

a) $.5 = e^{30t}$

b) 19.8 mg

c. approximately 229 years ($228 < t < 231$)