Regional Precalculus/Trigonometry Contest 2006

Select the best answer for each of the following questions and mark it on the answer sheet provided. Be sure to read all the answer choices before making your selection. When you are finished with the multiple choice, attempt the tiebreaker questions.

1. Find the real number c that puts the point $(c, 4)$ on the line passing through the points (0,-3) and (5,7).

   a) $c = 2$ b) $c = 2.5$ c) $c = 3$ d) $c = 3.5$ e) $c = 4$.

2. Find the inverse function $f^{-1}(x)$ of $f(x) = \dfrac{3}{x+2}$.

   a) $f^{-1}(x) = \dfrac{x+2}{3}$ b) $f^{-1}(x) = \dfrac{3-2x}{x}$ c) $f^{-1}(x) = -\dfrac{3}{x+2}$ d) $f^{-1}(x) = \dfrac{3}{x} + \dfrac{3}{2}$

   e) $f^{-1}(x) = \dfrac{x}{3} + \dfrac{2}{3}$

3. How long does it take you to double your money if you invest in an account that draws 5% interest compounded continuously.

   a) 6.54 years   b) 12.55 years   c) 13.86 years   d) 14.21 years   e) 15.05 years

4. Which of the following is the vertex of the parabola $y = x^2 + 6x + 4$.

   a) $(-\dfrac{3}{2}, -\dfrac{1}{2})$ b) $(\dfrac{3}{2}, -\dfrac{1}{2})$ c) (0,4)   d) (3,40)   e) (-3,-5)

5. A parabola has vertex (2,0) and passes through the point (3,5). Which of the following points is on the parabola?

   a) (4,20)   b) (0,18)   c) (1,6)   d) (-1,32)   e) (-2,32)

6. The solution set of $|2x + 1| < 11$ is:

   a) $\{x|\ x < 5\}$ b) $\{x|\ x > 5\}$ c) $\{x|\ x > -6\}$ d) $\{x|\ x < -6 \text{ or } x > 5\}$ e) $\{x|\ -6 < x < 5\}$

7. The center of the circle $x^2 + y^2 + 4x - 2y - 11 = 0$ is:

   a) (-2,1)   b) (1,-2)   c) (1,1)   d) (4,-2)   e) (4,1)

8. The slope of the line $4x + 2y = 7$ is:

   a) 4   b) 2   c) 7   d) -2   e) -0.5

9. The complex number $\dfrac{2}{3+i}$ is equal to:

   a) $\dfrac{2}{3} + \dfrac{2}{i}$ b) $3 - i$ c) $\dfrac{3}{5} - \dfrac{1}{5}i$ d) $\dfrac{3}{4} - \dfrac{1}{4}i$ e) $\dfrac{3}{4} + \dfrac{1}{4}i$

10. Given $f(x) = x^2$ and $g(x) = 2x - 3$, what is the value of $f \circ g(4)$?

    a) 16   b) 5   c) 29   d) 25   e) 0

11. If θ is an angle in standard position whose terminal side passes through the point (-3,4), what is the value of $\sin\theta + \tan\theta$?

a) $-\dfrac{8}{15}$　　b) $-\dfrac{29}{15}$　　c) $\dfrac{1}{5}$　　d) 1　　e) 0

12. If α and β are first quadrant angles for which $\sin\alpha = \dfrac{1}{2}$ and $\cos\beta = \dfrac{1}{3}$, what is the value of $\cos(\alpha+\beta)$?

a) $\dfrac{5}{6}$　　b) $\dfrac{3\sqrt{3}+2}{6}$　　c) $\dfrac{\sqrt{3}-2\sqrt{2}}{6}$　　d) $\dfrac{3\sqrt{3}-2}{6}$　　e) $\dfrac{1}{2}$

13. In a triangle with sides of lengths 3 ft, 4 ft, and 6 ft, what is the measure of the angle between the sides of lengths 3 ft and 4 ft?

a) 48.2^o　　b) 60.0^o　　c) 114.2^o　　d) 117.3^o　　e) 122.8^o

14. What is the exact value of $\sec\dfrac{7\pi}{6}$?

a) $\dfrac{2}{\sqrt{3}}$　　b) $\dfrac{3}{\sqrt{2}}$　　c) $-\dfrac{3}{\sqrt{2}}$　　d) -2　　e) $-\dfrac{2}{\sqrt{3}}$

15. The range of $y = 3 + 2\sin(7x+1)$ is

a) $[0, \dfrac{2\pi}{7}]$　　b) $[3, \dfrac{2\pi}{7}]$　　c) $[2,4]$　　d) $(2,4)$　　e) $[1,5]$

16. Which of the following is true about the graphs of $f(x) = \sin x$, $g(x) = \cos(x - \dfrac{\pi}{2})$, and $h(x) = -\sin(x - \pi)$:

a) They all have the same graph.

b) The functions $f(x)$ and $g(x)$ have the same graph, but the graph of $h(x)$ is different.

c) The functions $f(x)$ and $h(x)$ have the same graph, but the graph of $g(x)$ is different.

d) The functions $g(x)$ and $h(x)$ have the same graph, but the graph of $f(x)$ is different.

e) No two of the function's graphs are the same.

17. Which of the following are true for all real numbers x:　(I)　$\sin x = \sin(-x)$,　(II)　$\cos x = \cos(-x)$,　(III)　$\tan x = \tan(x + \pi)$?

a) Only I.　　b) Only II.　　c) Only III.　　d) Only II and III.　　e) Only I and II.

18. If $0 < x < \dfrac{\pi}{2}$, what is the value of $\sin(\tan^{-1}x)$?

a) $\dfrac{x}{\sqrt{x^2+1}}$　　b) $\dfrac{\sqrt{x^2+1}}{x}$　　c) $\dfrac{1}{x}$　　d) $\dfrac{x}{\sqrt{1-x^2}}$　　e) $\dfrac{\sqrt{1-x^2}}{x}$

19. What is the amplitude of $y = 5\sin 2x + 12\cos 2x$?

a) 13　　b) 17　　c) 2　　d) π　　e) 7

20. In a right triangle with a 30° angle how is the hypotenuse related to the shorter leg?

   a) The length of the hypotenuse is $\sqrt{2}$ times the length of the shorter leg.

   b) The length of the hypotenuse is $\sqrt{3}$ times the length of the shorter leg.

   c) The length of the hypotenuse is 2 times the length of the shorter leg.

   d) The length of the hypotenuse is 3 times the length of the shorter leg.

   e) The length of the hypotenuse is 5 times the length of the shorter leg.

21. The expression $(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2})^{20}$ is equal to

   a) $10\sqrt{2} + i10\sqrt{2}$ b) i c) $-i$ d) 1   e) -1

22. If the central angle of a circle of radius 1 intercepts an arc of length 1, what is the measure of the angle to one decimal place?

   a) 180.0^o b) 90.0^o c) 45.0^o d) 57.3^o e) 63.5^o

23. Which of the following is an equation of the line passing through the point (5,1) and perpendicular to the line $y = \frac{1}{2}x - 3$?

   a) $y - 1 = -2(x - 5)$ b) $y - 5 = -2(x - 1)$ c) $y - 1 = 2(x - 5)$ d) $y - 5 = 2(x - 1)$

   e) $y - 1 = -\frac{1}{2}(x - 5)$

24. Which of the following statements is true about the graph of a polynomial function of degree 3.

   a) It has a maximum value and a minimum value.

   b) It is monotonic.

   c) It intersects the x-axis at three distinct points.

   d) It intersects the y-axis at exactly one point.

   e) It has a vertical asymptote.

25. What is the area of a triangle with sides of lengths 6 in, 8 in, and 10 in?

   a) 40 sq in   b) 30 sq in   c) 24 sq in   d) 9 sq in

   e) It cannot be determined from the given information.

Tiebreakers

1. Find the exact value of x that solves the equation $2^x = 3^{x-4}$. Write your answer in terms of natural logarithms.

2. Find a third degree polynomial function $f(x)$ with zeros $1 + i$ and 2 for which $f(0) = -8$. Write your answer in the form $f(x) = ax^3 + bx^2 + cx + d$.

3. Find the exact value of $\sin 7.5^o$.

Key for the Regional Precalculus/Trigonometry Contest 2006

1. D

2. B

3. C

4. E

5. A

6. E

7. A

8. D

9. C

10. D

11. A

12. C

13. D

14. E

15. E

16. A

17. D

18. A

19. A

20. C

21. E

22. D

23. A

24. D

25. C

Tiebreaker 1.

$$x \ln 2 = (x - 4) \ln 3 \ \Rightarrow \ x(\ln 2 - \ln 3) = -4 \ln 3 \ \Rightarrow \ x = \frac{4 \ln 3}{\ln 3 - \ln 2}$$

$$\Rightarrow \ x = \frac{\ln 81}{\ln \frac{3}{2}} \ \text{(or any equivalent expression involving natural logs)}$$

Tiebreaker 2.

$$f(x) = a(x - [1 + i])(x - [1 - i])(x - 2) = a(x^3 - 4x^2 + 6x - 4) \ \Rightarrow \ f(0) = -4a = -8 \ \Rightarrow \ a = 2$$

$$\Rightarrow \ f(x) = 2x^3 - 8x^2 + 12x - 8$$

Tiebreaker 3.

$$\sin 15^o = \sqrt{\frac{1}{2} - \frac{1}{2} \cos 30^o} = \frac{\sqrt{2 - \sqrt{3}}}{2}$$

$$\Rightarrow \ \sin 7.5^o = \sqrt{\frac{1}{2} - \frac{1}{2} \cos 15^o} = \sqrt{\frac{1}{2} - \frac{1}{2} \frac{\sqrt{2 - \sqrt{3}}}{2}} = \frac{\sqrt{2 - \sqrt{2 - \sqrt{3}}}}{2} \ \text{(or any equivalent expresion)}$$