2006 ACTM REGIONAL GEOMETRY EXAM

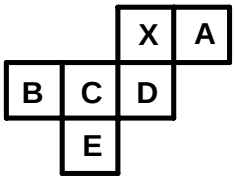
In each of the following choose the best answer and mark the appropriate letter on the answer key. Please note the figures may not be drawn to scale.

1. Jane counted the number of edges in a prism and also in a pyramid and got the same number. Which of the following numbers did she get?

   A. 50 B. 39 C. 42 D. 35

   E. None of these

2. If the net below is folded to form a cube which face will be opposite X?



Problem 2

3. Consider the regular polygons listed below.
   I. Equilateral Triangle II. Square III. Hexagon IV. Octagon

   Which of these regular polygons would tessellate the plane?

   A. I, II, and IV only B. II, III, and IV only C. I, III, and IV only

   D. I, II, and III only E. None of these

4. How many of the Platonic solids have faces that are equilateral triangles?

   A. 4 B. 5 C. 2 D. 6

   E. None of these

5. The contrapositive of the statement “If the quadrilateral ABCD is a parallelogram, then the opposite pairs of sides are congruent” is

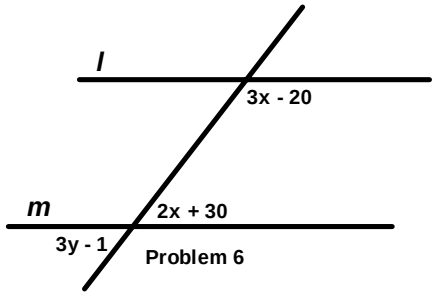
   A. If the quadrilateral ABCD has its opposite pairs of sides congruent then ABCD is a parallelogram.
   B. If the quadrilateral ABCD is not a parallelogram, then the opposite pairs of sides are not congruent.
   C. If the opposite pairs of sides of the quadrilateral ABCD are not congruent, then ABCD is not a parallelogram.
   D. The quadrilateral ABCD is not a parallelogram or the opposite pairs of sides are congruent.
   E. None of these

6. In the figure at the right, lines ***l*** and ***m*** are parallel. What is the value of y?

A. $4\frac{1}{3}$ B. 43

C. 19 D. 33

E. None of these



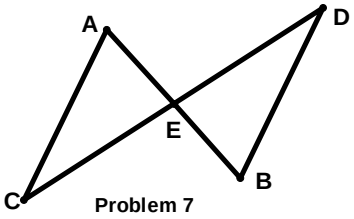
Problem 6

. Suppose in the figure at the right that $\overline{AC} \parallel \overline{BD}$ and that E is the midpoint of $\overline{AB}$. Which of the following theorems would you use to prove $\Delta ACE \cong \Delta BDE$?

A. SSS B. SSA

C. AAS D. SAS

E. None of these



Problem 7

8. Congruent angles are indicated in the accompanying figure. Which of the following can you conclude?

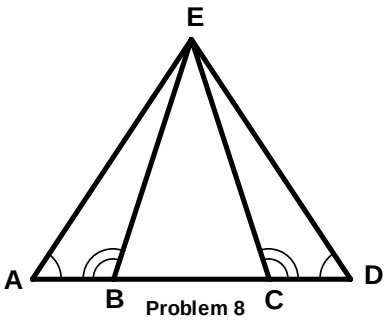
I. ΔADE is isosceles II. $\Delta ABE \cong \Delta DCE$

III. ΔBDE is isosceles IV. $\Delta EBC \cong \Delta ECB$

A. I, II, and IV only B. I, II, and III only

C. II, III, and IV only D. I, III, and IV only

E. None of these



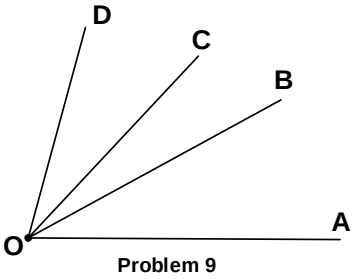
Problem 8

9. From the figure at the right, which of the following pairs of angles are adjacent?

A. $\angle AOC, \angle BOC$ B. $\angle AOC, \angle BOD$

C. $\angle AOB, \angle DOC$ D. $\angle AOC, \angle DOC$

E. None of these



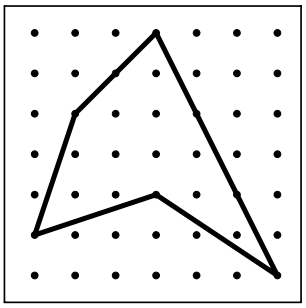
Problem 9

10. The sum of the measures of the interior angles of a polygon is 1260. How many sides does the polygon have?

A. 10 B. 7 C. 11 D. 9

E. None of these

11. Assume the grid on the right represents a 6 x 6 geoboard. What is the area of the polygon represented?

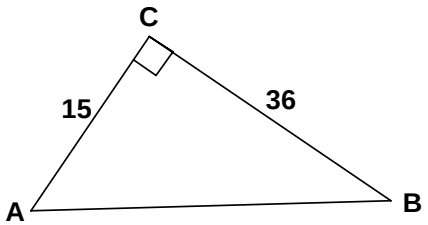


Problem 11

A. 12 ½ B. 14 C. 15

D. 13 ½ E. None of these

12. In the triangle at the right find the value of $\angle A$ to the nearest tenth of a degree.



Problem 12

A. 67.4^o B. 22.6^o

C. 24.6^o D. 65.4^o

E. None of these

13. Javier stands 63 feet from the base of a flag pole that is 25 feet tall. If Javier's eyes are 6 feet above the ground, at what angle must he elevate his line of sight to look directly at a bird on the top of the flag pole?

A. 23.7^o B. 21.6^o C. 73.2^o D. 16.8^o

E. None of these

14. Points P = (–3, 1), Q = (2, 3), and R = (–1, 5) are three coordinates of a parallelogram in the rectangular coordinate plane. Which of the following points could be the fourth vertex of the parallelogram?

A. (0, 0) B. (–6, 3) C. (4, 8) D. (4, 7)

E. None of these

15. Let A = (–5, 12) and B = (7, –4). The perpendicular bisector of the segment $\overline{AB}$ has slope of

A. $-\frac{3}{4}$ B. $\frac{3}{4}$ C. $\frac{4}{3}$ D. $-\frac{4}{3}$

E. None of these

16. The vertices of a triangle are P = (–7, –5), Q = (2, 8) and R = (8, 0). The centroid C of the triangle lies on the median drawn from P. The coordinates of C are

A. (5, 4) B. (–1, –½) C. (1, 1) D. (–3, –2)

E. None of these

17. Which of the following translations has the same effect as the composition of two translations; <–5, –7> followed by <3, 6>, from the origin?

A. <–2, –1> B. <–2, 1> C. <–15, –42>

D. <2, 1> E. None of these

18. The equation of the circle with a diameter whose endpoints are (–5, –1) and (3, 5) is

A. $(x + 1)^2 + (y – 2)^2 = 25$ B. $(x – 1)^2 + (y + 2)^2 = 25$

C. $(x + 1)^2 + (y – 2)^2 = 5$ D. $(x - 1)^2 + (y + 2)^2 = 5$

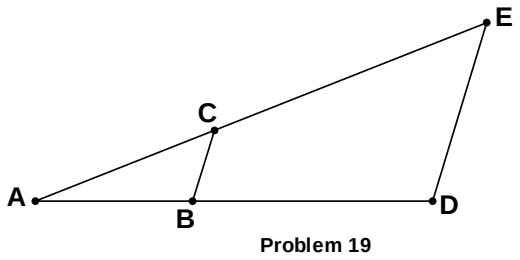
E. None of these

19. In the figure at the right, if $\overline{BC} \parallel \overline{DE}$, AB = 12, BD = 15, and AE = 45 then CE =

A. 20 B. 25

C. 36 D. 9

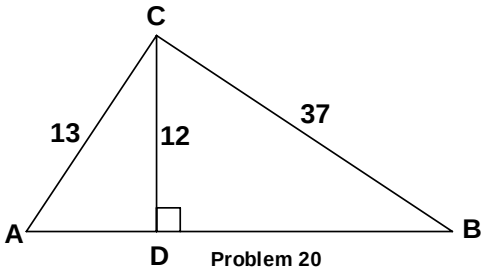
E. None of these



Problem 19

20. In ΔABC, AC = 13, CD = 12 and BC = 37. The area of ΔABC is

A. 215
B. 360
C. 240
D. Not enough information
E. None of these



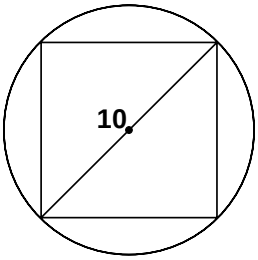
Problem 20

21. The figure at the right is a dart board. If a dart is thrown at and hits the board, what is the probability it lands outside the square and inside the circle?

A. $\frac{\pi - 2}{\pi}$ B. $\frac{1}{\pi}$

C. $\frac{\pi}{2}$ D. $\frac{2}{\pi}$

E. None of these



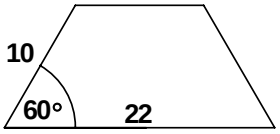
Problem 21

22. The perimeter of the isosceles trapezoid at the right is

A. $44 + 10\sqrt{3}$ B. 54

C. 64 D. 59

E. None of these



Problem 22

23. Which of the following sets of numbers represent the sides of an obtuse triangle?

A. 12, 16, 30 B. 12, 16, 19 C. 12, 16, 20

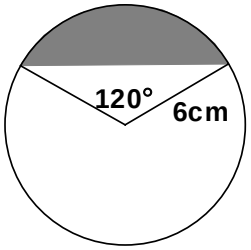
D. 12, 16, 22 E. both A and D

24. Find the area of the shaded part of the circle. Leave your answer in terms of π.

A. $12\pi - 9\sqrt{3}$ B. $12\pi - 18$

C. $18\pi - 9\sqrt{3}$ D. $12\pi - 9$

E. None of these



Problem 24

25. The radius of the base of a right circular cone and a right circular cylinder are the same. The height of the cone is twice the height of the cylinder. The ratio of the volume of the cone to the volume of the cylinder is

A. 2 to 1 B. 1 to 2 C. 2 to 3 D. 3 to 2

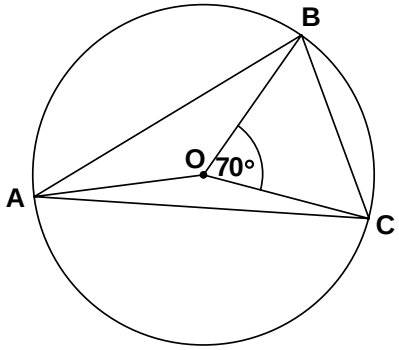
E. None of these

TIE–BREAKER PROBLEMS

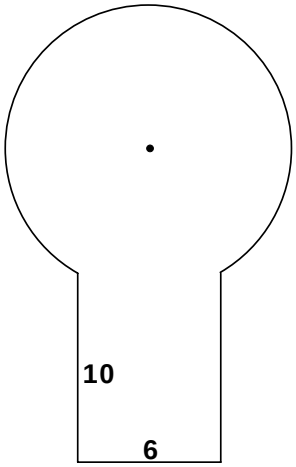
NAME _____ SCHOOL _____
PLEASE PRINT PLEASE PRINT

These problems will be used only in the case of a tie for first, second or third place. They will be used in the order they are given until the tie is broken. A clear explanation of your solution is required.

1. In the circle with center at O, which is longer $\overline{OA}$ or $\overline{BC}$? Give a clear justification of your response.



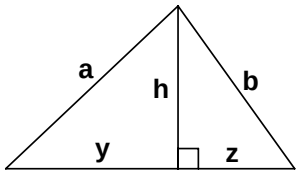
2. The radius of the circle is 6 and the rectangle has an altitude of 10 and a base of 6. What is the perimeter of the figure? Explain.



NAME_____ SCHOOL_____

3. Consider the triangle below with an altitude, h, and sides a and b with a > b. The altitude divides the base into segments y and z.

Prove: $\frac{a-b}{y-z}=\frac{y+z}{a+b}$



KEY – GEOMETRY (REGIONAL)

1. C
2. E
3. D
4. E
5. C
6. D
7. C
8. A
9. D
10. D
11. B
12. A
13. D
14. B
15. B
16. C
17. A
18. A
19. B
20. C
21. A
22. B
23. E
24. A
25. C

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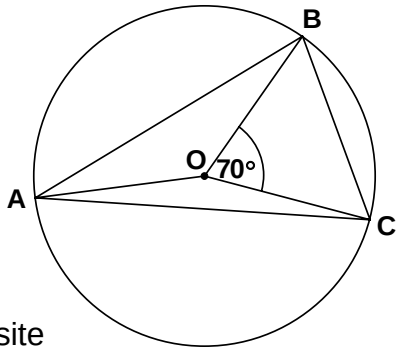
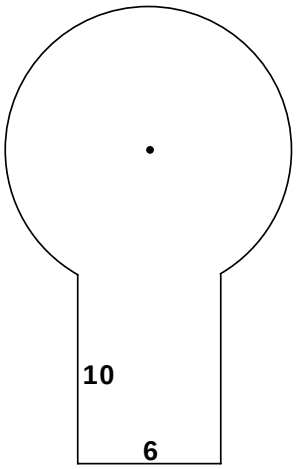


The radius of the circle is the length of the side of a regular hexagon. This radius subtends a central angle of 60^o.
Since $\overline{BC}$ subtends an angle of 70^o, then BC > OA.

Alternate;
ΔBOC is isosceles. The base angles $\angle$B and $\angle$C must equal 55^o. Therefore, BC > OB, since the larger side must lie opposite the larger angle. Thus, BC > OA.

2. The radius of the circle is 6 and the rectangle has an altitude of 10 and a base of 6. What is the perimeter of the figure? Explain.

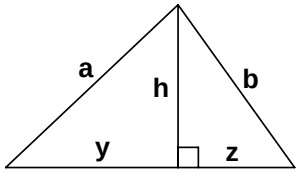


Draw in the two radii. With the missing side of the rectangle an equilateral triangle is formed. The central angle then must be 60^o, so one-sixth of the circle is missing. Since the circumference is 12π, the perimeter of the figure is $10\pi + 10 + 6 + 10 =$ **$10\pi + 26$**.

NAME_____ SCHOOL_____

3. Consider the triangle below with an altitude, h, and sides a and b with a > b. The altitude divides the base into segments y and z.

Prove: $\frac{a-b}{y-z}=\frac{y+z}{a+b}$



$h^2 = a^2 - y^2$

$h^2 = b^2 - z^2$

So,

$a^2 - y^2 = b^2 - z^2$

Therefore, $a^2 - b^2 = y^2 - z^2$.

So, $(a - b)(a + b) = (y - z)(y + z)$.

Thus, $\frac{a-b}{y-z}=\frac{y+z}{a+b}$.