ALGEBRA II
2006 ACTM REGIONAL CONTEST

1. If $x \neq 0$, then $x^0 =$

a) 0 b) 1 c) $\frac{1}{x}$ d) undefined

2. What number must be added to $x^2 + 5x$ to complete the square?

a) $\frac{5}{2}$ b) $\frac{5}{4}$ c) $\frac{25}{4}$ d) 25

3. The equation of the line parallel to the line $x = 3y - 2$ and through the point (7, 1) is

a) $y - 1 = \frac{1}{3}(x - 7)$ b) $y - 1 = 2(x - 7)$

c) $y - 1 = -2(x - 7)$ d) $y - 1 = 3(x - 7)$

4. If $i = \sqrt{-1}$ then $i^{15} =$

a) $-i$ b) i c) -1 d) 1

5. The determinant of $\begin{bmatrix} 3 & 4 \\ -2 & 7 \end{bmatrix}$ is

a) 8 b) 12 c) 13 d) 29

6. $2^{\log_2 8} =$

a) 6 b) 8 c) 16 d) 256

7. $\frac{3}{x-2} - \frac{1}{2-x} =$

a) $\frac{2}{0}$ b) $\frac{2}{x^2 - 4}$ c) $\frac{2}{x-4}$ d) $\frac{4}{x-2}$

8. $\dfrac{3}{3-\sqrt{2}} =$

a) $\dfrac{1}{1-\sqrt{2}}$ b) $-(1+\sqrt{2})$ c) $3(3+\sqrt{2})$ d) $\dfrac{3}{7}(3+\sqrt{2})$

9. Let $f(x) = \dfrac{x+2}{x-4}$. The domain of the function f is

a) $(-\infty, -2) \cup (-2, \infty)$ b) $(-\infty, 4) \cup (4, \infty)$

c) $(-\infty, -2) \cup (-2, 4) \cup (4, \infty)$ d) $(-\infty, \infty)$

10. Solve x, $|3x - 2| < 1$

a) $(-\infty, \frac{1}{3}) \cup (1, \infty)$ b) $(\frac{1}{3}, 1)$

c) $(-\infty, \frac{1}{3})$ d) $(1, \infty)$

11. Which of the following describes the "end behavior" of the graph of $y = -3x^{24} + 14x^{11} - 12x^4 + 73$?

a) rises on both the left and right b) falls on both the left and right

c) rises on the left and falls on the right d) falls on the left and rises on the right

12. The function $D(x) = .056057x^2 + 1.06657x$ models the distance, $D(x)$, (in feet) it takes a car traveling x mph to stop. Approximately how fast is a car traveling that takes 300 feet to stop?

a) 64 mph b) 67 mph c) 70 mph d) 75 mph

13. The function f is defined by the table

x	0	1	2	3
$f(x)$	1	3	−1	−4

$f^{-1}(1) =$

a) 0 b) $\dfrac{1}{3}$ c) 1 d) 3

14. Use the following chart to compute $(f \circ g)(3)$.

x	2	3	4
$f(x)$	1	4	6
$g(x)$	3	2	1

a) 1 b) 2 c) 6 d) 8

15. The range of the function $f(x) = 3 - 2^x$ is

a) $(-\infty, 3)$ b) $(-\infty, 3]$ c) $(3, \infty)$ d) $(-\infty, \infty)$

16. If x and y are positive then $\ln \frac{3x^4}{5y^2} =$

a) $\frac{\ln 3 + \ln x^4}{\ln 5 + \ln y^2}$ b) $4 \ln 3x - 2 \ln 5y$

c) $\ln 3 + 4 \ln x - \ln 5 + 2 \ln y$ d) $\ln 3 + 4 \ln x - \ln 5 - 2 \ln y$

17. If $\log_b 2 = .41$ and $\log_b 3 = .64$, then $\log_b 12 =$

a) 0.52 b) 1.46 c) 1.69 d) 2.10

18. If $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x-4}$ then the graph of g is the same as the graph of f shifted 4 units

a) to the right b) to the left c) lower d) raised

19. Suppose the equation $A(t) = 981e^{0.3t}$ models how much money (dollars) students spend for leisure activities during the high school years where $t = 0$ corresponds to the freshman year. Based on this model, how much would you expect a high school junior to spend on leisure activities?

a) \$588.60 b) \$1787.50 c) \$1599.98 d) \$2412.87

20. For a specific area code, how many different phone numbers are possible for a given 3-digit prefix? [Assume that there are no restrictions on the digits following a given prefix.]

a) 3024 b) 5040 c) 6561 d) 10,000

21. If $ax^2 + 3x + 1 = 0$ has exactly 1 distinct solution then a =

a) $\frac{9}{4}$ b) 3 c) 6 d) this equation never has only one distinct solution

22. For what values of x is it true that $\sqrt[3]{x} > x^2$

a) None b) $(-1, 0)$ c) $(0, 1)$ d) $(-1, 1)$

23. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^{-1} =$

a) $\begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{4} \end{bmatrix}$ b) $\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$ c) $\begin{bmatrix} 1 & \frac{3}{2} \\ -2 & -\frac{1}{2} \end{bmatrix}$ d) $\begin{bmatrix} 0 & -1 \\ -3 & -3 \end{bmatrix}$

24. What is the value if \$3000 is invested at 4% compounded continuously for 5 years?

a) \$3199.48 b) \$3600 c) \$22,167.17 d) \$3664.21

25. If $f(x) = \begin{cases} x+3 & \text{if } x < 4 \\ x-5 & \text{if } x \geq 4 \end{cases}$ then for how many pairs of numbers (a, b) is it true that $f(a) = f(b)$?

a) None b) 1 c) 2 d) an infinite number

ALGEBRA II (Tie breakers) **2006 ACTM REGIONAL CONTEST**

Name _____ **School** _____

Instructions: Work the following problems ***in order***. Show all your work.

1. Average daily temperatures (T) in the state of Arkansas range from 26° F to 94°F. Write this fact as a single absolute value inequality.

2. Write a polynomial function of least degree and in the form $y = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$ whose graph has x-intercepts $(-2, 0)$, $(1, 0)$ and $(3, 0)$ and y-intercept $(0, 5)$.

3. A person's body mass index (BMI) is directly proportional to his/her weight in pounds and inversely proportional to the square of his/her height in inches. If a 6-ft tall person weighing 177 pounds has a BMI of 24, find the BMI (to the nearest whole number) of a person 5 ft 4 in tall who weighs 120 pounds.

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ANSWER KEY

1. **b**
2. **c**
3. **a**
4. **a**
5. **d**
6. **b**
7. **d**
8. **d**
9. **b**
10. **b**
11. **b**
12. **a**
13. **a**
14. **a**
15. **a**
16. **d**
17. **b**
18. **a**
19. **b**
20. **d**
21. **a**
22. **c**
23. **b**
24. **d**
25. **d**

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ANSWER KEY

1. $|T - 60| \le 34$

2. $y = a(x+2)(x-1)(x-3)$

$y = a(x^3 - 2x^2 - 5x + 6)$

Let $x = 0$ and $y = 5$ and we get

$5 = a(6)$

$a = \frac{5}{6}$

$y = \frac{5}{6}(x^3 - 2x^2 - 5x + 6)$

$\therefore y = \frac{5}{6}x^3 - \frac{5}{3}x^2 - \frac{25}{6}x + 5$

3. BMI $= \dfrac{kW}{H^2}$

$24 = \dfrac{k(177)}{(72)^2}$

$k = 702.9$

$\therefore$ BMI $= \dfrac{(702.9)(120)}{(64)^2} = 20.6 \sim 21$